HEWLETT hp PACKARD

HP-81
real estate and investment analysis handbook

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HP-81 real estate and investment analysis handbook

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INTRODUCTION

This HP-81 REAL ESTATE AND INVESTMENT ANALYSIS HANDBOOK is a supplement to the HP-81 Operating Guide or Owner's Handbook. It is specifically designed to explain and illustrate how the HP-81 calculator can be used effectively and efficiently to solve a wide variety of recurring problems that confront the real estate practitioner or analyst. It covers as nearly as possible the full spectrum of computational problems related to real estate and investment transactions, derived from actual field experience. Of course these techniques also apply to other areas of financial analysis.

The terminology and symbols used in the examples that illustrate how the HP-81 can help solve real estate problems are those most widely accepted in professional real estate practice. A detailed listing of these symbols and terms is provided in the Appendix. In addition, the formulas used in the keystroke procedures are presented as they are covered throughout the Handbook. However, it is not necessary to memorize these formulas, or even to use them, in order to apply the procedures that are presented. Just follow the steps as shown, and the proper answers will be produced every time.

The HP-81 is a highly flexible calculating instrument. In a number of instances, the operator can enter figures in any order. This means that you can use your own worksheet format instead of being forced into a specific pattern by the machine. Where this is possible with the HP-81, it is noted in the Handbook. In other cases, the steps and keystrokes must be followed exactly. This is also noted when it applies to the specific procedures.

Users of the HP-81 have a printed tape record of every calculation as long as the printer is on. This is a decided advantage in long or involved calculations so that you can check your data inputs as you go along.

Another outstanding distinguishing feature of the HP-81 is that it is preprogrammed to produce many calculations involving several related results automatically. This saves both time and effort (as well as minimizing chances of making a mistake) in producing such important printouts as Depreciation Schedules; Mortgage Amortization Schedules; Mean and Standard Deviation; and Coefficient of Determination, Y-intercept, and Slope in simple linear regression (or trend lines). The several automatic Extended Functions and Operating Modes that produce such tabulations are summarized in the Appendix of this Handbook, as well as being specifically noted throughout the body of the Handbook.

With the HP-81, you no longer need cumbersome printed tables of compound interest factors, or loan reduction schedules, or mortgage payments, or mortgage constants, or depreciation schedules. All of these can be calculated accurately in less time than it normally takes to look them up in a printed book of tables. The same applies to components of mortgage-equity (Ellwood) analysis.

To help you in using this Handbook, and in finding the keystroke procedure applicable to your particular problem, the Table of Contents is organized by general topic areas. Each procedure in that topic area is then listed individually. For example, if you want to find the balance outstanding on a fully-amortized, level-payment mortgage at any time before maturity, look under CHAPTER 4: Simple Mortgages (Fully Amortized) and find the procedure entitled 'Accumulated Interest Paid Between Periods and Remaining Balance'. Or, if you want to construct a declining-balance depreciation schedule, simply look under CHAPTER 11: Depreciation and find the appropriate 'Declining Balance' procedure.

The keystroke procedures presented here have been developed by many individuals in actual practice. They have been field-tested and they work. Many are adapted from HP-80 and HP-70 routines. Our thanks go to all those who contributed suggestions and ideas for these procedures, as well as examples of their use. It is possible that with practice and experience you can add to the practical applications of the HP-81 in solving real estate problems, and perhaps develop short-cuts in some instances. The range of applications is not yet fully explored, although the examples provided here do appear to cover the most important recurring situations that confront professionals in real estate practice.

In compiling and testing the routines and procedures for solving real estate problems that are contained in this manual, the authors received suggestions, assistance and criticisms from a number of sources. We wish to acknowledge publicly our appreciation for their help and advice. In particular, we are especially indebted to our good friend and distinguished colleague, Dr. Stephen D. Messner, who is Director of the Center for Real Estate and Urban Economic Studies as well as Head of the Finance Department at the University of Connecticut. Dr. Messner gave unstintingly of his time and skill in commenting on materials prepared for inclusion in this manual. He is responsible for several of the routines and the ideas that underlie them, most especially in the areas of Rate-of-Return analysis and Investment-Feasibility analysis. The Modified IRR procedures are wholly his invention.

Storrs, Connecticut
March, 1975

William N. Kinnard, Jr.
Byrl N. Boyce

HOW TO READ THIS HANDBOOK

HP-81 Real Estate problem solutions in this book are presented in step-by-step keystroke form. The general procedure is shown first, followed by an example. For ease of understanding, the examples show solution step numbers corresponding to the numbers shown in the general procedure. To further clarify the examples, intermediate results are shown and comments explaining the displayed answer are given where needed. In most cases the resulting tape printout is also shown.

In both the procedure and examples, numbers to be keyed in are shown without boxes while function keys are shown with boxes around them $\boxed{}$. The shift key is shown as $\boxed{}$. Problems and solutions appear as follows:

Example: What is the monthly payment amount for a 30-year, fully amortized, level-monthly payment $40,000 mortgage at 9.25%?

Solution: Enter $\boxed{\text{CLEAR}}$

1. Key (in any order) 40000 $\boxed{\text{PV}}$ **9.25** $\boxed{\text{SAVE}\uparrow}$

12 $\boxed{\div}$ $\boxed{\text{i}}$ **30** $\boxed{\text{SAVE}\uparrow}$ **12** $\boxed{\times}$ $\boxed{\text{n}}$

2. Press $\boxed{\text{PMT}}$

The monthly payment is $329.07.

```
               CLEAR
40000.00      →P
    9.25       ↑
   12.00       ÷
     .77      ◇
              → I
   30.00       ↑
   12.00       ×
  360.00      ◇
              → N
              PMT
  329.07
```

The data entry order indicated in the general procedure must be followed in sequence. Where 'Calculate and Key (in any order)' is indicated, as in Step 1 of the above problem, the data within that step can be entered in any order.

NOTE:

Pressing the green key (shift key) followed by a number from 0-6, will cause the display to be rounded to that number of decimal places. The HP-81 however, retains and uses a full ten digits internally.

Pressing the CLEAR key clears all the registers except storage registers 0-9 and 14-19.

Additional information regarding the operation of your HP-81 calculator is contained in your HP-81 Operating Guide.

REMEMBER--

The following assumptions are made within the preprogrammed financial functions contained in the left column keys.

--Payments occur at the end of the time period.
--The initial amount (present value) occurs at the beginning of the first period.
--The final amount (future value) is the amount at the end of the last period.
--Interest is compounded at the same time periodic payments are due.
--The interest rate per period is entered as a whole percent, not a decimal.

If your problem differs from any of these assumptions, you'll need to vary the keystrokes. For example, if your payments occur at the beginning of the period, the keystroke solution must be adjusted. Different calculating modes can be used, as summarized in the Appendix to this volume, and in the HP-81 Operating Guide. Now you're ready to start saving hours of calculation time!

CHAPTER 1 - PERCENTAGE CALCULATIONS

Percentage calculations play an important role in a great variety of real estate problems. The following procedures and examples illustrate some applications with the HP-81.

ADJUSTMENTS IN DIRECT SALES COMPARISON ANALYSIS

Sales comparisons in appraisal work involve cumulative plus and minus percentage or dollar adjustments to produce an adjusted sales price for each comparable sale. This can be accomplished quickly and easily on the HP-81 to produce the amount of each adjustment, the total net adjustment, the adjusted sales price, and the adjusted sales price per unit of comparison (e.g., sales price per square foot).

Example 1-- A residential property containing 1450 square feet sold for $29,000. To bring it into true comparability with the subject property, it is concluded that the following adjustments are required: Time + 8.5%, Condition - 10%, Age + 5%, Location - 12%, Baths - $750, Garage + $1100. What is the amount of each adjustment in dollars, the total net adjustment in dollars, the adjusted sales price, and the adjusted sales price per square foot?

Enter:

29000 [STO] [0] **.085** [ ] [×] [Σ+]
.10 [CHS] [ ] [×] [Σ+] **.05** [ ] [×] [Σ+]
.12 [CHS] [ ] [×] [Σ+] **750** [CHS] [Σ+]
1100 [Σ+] [◊] [RCL] [0] [+] [◊]
1450 [÷]

The net adjustment is -$2115, the adjusted sales price is $26,885; and the adjusted sales price per square foot is $18.54.

```
                 CLEAR

   29000.00     →0
       .085     N  ×
    2465.00     ◊
    2465.00        Σ+
-       .10     N  ×
-   2900.00     ◊
-   2900.00        Σ+
        .05     N  ×
    1450.00     ◊
    1450.00        Σ+
-       .12     N  ×
-   3480.00     ◊
-   3480.00        Σ+
-    750.00        Σ+
    1100.00        Σ+
-   2115.00     ◊

                   ↑
   29000.00     ←0
                   +
   26885.00     ◊

    1450.00        ÷
      18.54     ◊
```

Example 2 -- A property was purchased for $110,000. It resold 31 months later for $140,000. There were no changes in the property between sales. What was the percentage change in sales price? What time adjustment (annual) does this resale indicate on a straight-line basis? on a compound interest basis?

Enter:

110000 [SAVE ↑] 140000 [ ] [%] [◊]

31 [÷] 12 [×] 10.56

If annual compounding is assumed, Enter

31 [SAVE ↑] 12 [÷] [n] 110000 [PV]

140000 [FV] [i] 9.78

If monthly compounding is assumed, Enter

31 [n] 110000 [PV] 140000 [FV]

[i] 12 [×] 9.37

```
               CLEAR

  110000.00        ↑
  140000.00      Δ %
      27.27      ◊

      31.00        ÷
        .88      ◊
      12.00        ×
      10.56      ◊

      31.00        ↑
      12.00        ÷
       2.58      ◊
                 → N
  110000.00      →P
  140000.00      → F
I N T
       9.78

      31.00      → N
  110000.00      →P
  140000.00      → F
I N T
        .78
      12.00        ×
       9.37      ◊
```

CHAPTER 2- COMPOUND INTEREST AND DISCOUNT FACTORS

Whenever a real estate analyst requires a compound interest or discount factor (any of the six functions of money at interest) for use in real estate problem-solving, it can be produced quickly and accurately on the HP-81. Printed tables of precalculated factors are rendered unnecessary because the HP-81 can produce anything -- and more -- that is available in any printed set of financial tables, no matter how voluminous or detailed they may be. Moreover, the analyst can usually calculate the factor in less time on the HP-81 than it takes to look up the factor in a printed set of tables.

Compounded interest and discount factor calculations on the HP-81 have the following characteristics (and advantages):

1. Factors can be calculated for virtually any number of compounding periods and any rate of interest or discount.
2. Fractional time periods and interest (discount) rates can be employed, with accuracy to 10 decimal places.
3. The printer will provide 0-6 decimal places (see Shift Key Functions in the Appendix). Figures with more than 6 decimal places (7-9) are shown in scientific notation.
4. Regardless of the number of decimal places set and displayed, the HP-81 retains and stores 10 decimal places for further calculations.
5. Unless the problem specifically calls for the factor alone, the HP-81 calculates answers with dollar amounts directly, thereby reducing calculating time as well as opportunities to make an operator error.
6. All compound interest and discount calculations on the HP-81 require using 4 of the 5 function keys in the left-hand column of the keyboard: [n] [i] [PMT] [PV] [FV]
7. With the value of 3 items known, the value of any unknown is calculated. To calculate [PMT], [PV] or [FV], the values of [n] and [i] must be known.
8. The values of [n] (number of periods) or [i] (interest rate per period) can be calculated as unknowns if the other is known.
9. The value of any known item can be varied to calculate the value of the same unknown item.
10. The three known values can be entered in any order.
11. If a different set of known items or a different unknown is used, the [CLEAR] key must be pressed and the entire set of known values re-entered.

12. The entered values of the known items are stored and retained in storage until new values are entered.

    Pressing the CLEAR key does not erase the values stored. The stored values can be recalled for use by pressing the appropriate RCL key combination. (The storage registers in which each value is retained are summarized in the Appendix.)

13. The value keyed into n is the number of compounding periods.

14. The values keyed into i and PMT are the amounts per compounding period.

15. The value keyed into i is always expressed in whole percentage numbers, not a decimal figure; for example, 9.25%, not .0925.

16. Payments and compounding occur at the end of each period.

FUTURE WORTH OF ONE (Compound Amount of One; Accumulation of One)

FV is the unknown. The known values are n , i and PV which may be keyed in any order.

The Formula is: $FV = PV\,(1 + i)^n$

1. Key in (in any order)
   a. Number of compounding periods, press n .
   b. Interest rate per period, press i .
   c. Present value (initial investment), press PV .
2. To calculate future value (future worth), press FV .

Example - An investor purchased a parcel of land 8 years ago for $13,500. Ignoring holding costs, how much must the property resell for in order for the investor to earn 7.65% per year?

Enter:

CLEAR 8 n 7.65 i 13500 PV FV

or

13500 PV 8 n 7.65 i FV $24,347.03

To find the required resale price at 9%

interest, enter 9 i FV $26,899.60

```
            CLEAR

        8.00    → N
        7.65    → I
    13500.00    →P
F   V
    24347.03

            CLEAR

    13500.00    →P
        8.00    → N
        7.65    → I
F   V
    24347.03
        9.00    → I
F   V
    26899.60
```

FUTURE WORTH OF ONE PER PERIOD (Accumulation of One Per Period)

[FV] is the unknown. The known values are [n], [i] and [PMT], which may be keyed in any order.

The Formula is: $FV = PMT\ \dfrac{(1+i)^n - 1}{i}$

1. Key in (in any order)
   a. Number of compounding periods, press [n].
   b. Interest rate per period, press [i].
   c. Payment per period (at end of period), press [PMT].
2. To calculate future value (future worth), press [FV].

Example - An investor holding a parcel of land producing no income paid $385 per year in taxes (at the end of each year). At the end of 8 years, how much must be recovered on resale for the investor to earn 7.65% per year on his payments?

Enter:

[CLEAR] **8** [n] **7.65** [i] **385** [PMT] [FV]

$4043.68

```
            CLEAR

        8.00    → N
        7.65    → I
      385.00    →PT
F   V
     4043.68
```

To combine the two previous examples and derive the resale price required to earn 7.65% on the original investment of $13,500 and the 8 annual tax payments of $385:

Enter:

[CLEAR] **8** [n] **7.65** [i] **13500** [PV] [FV]

[STO] [0]

[CLEAR]

[RCL] [n] [n] [RCL] [i] [i] **385** [PMT]

[FV] [RCL] [0] [+] [◊]

Total required resale price is $28,390.71.

```
            CLEAR

        8.00    → N
        7.65    → I
    13500.00    →P
F   V
    24347.03
                →0
            CLEAR

        8.00    ← N
                → N
        7.65    ← I
                → I
      385.00    →PT
F   V
     4043.68
                 ↑
    24347.03    ←0
                 +
    28390.71    ◊
```

SINKING FUND FACTOR (Payment Amount For a Sinking Fund)

PMT is the unknown. The known values are n, i and FV, which may be keyed in any order.

The Formula is: $$\text{PMT} = \text{FV} \frac{i}{(1 + i)^n - 1}$$

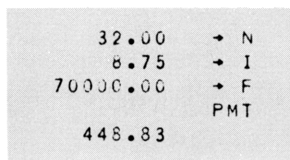
1. Key in (in any order)
   a. Number of compounding periods, press n.
   b. Interest rate per period, press i.
   c. Future value (future worth), press FV.
2. To calculate sinking fund payment, press PMT.

Example - An investor paid $70,000 for a building with an estimated remaining economic life of 32 years. What amount must be set aside annually at the end of each year to recover the full investment in the building over the remaining economic life, if the annual payments can accumulate at 8.75%, compounded annually at the end of each year?

Enter:

32 n 8.75 i 700000 FV PMT

The annual sinking fund payment is $448.83.



PRESENT WORTH OF ONE (Reversion Factor)

PV is the unknown. The known values are n, i and FV, which may be keyed in any order.

The Formula is: $$\text{PV} = \text{FV} \frac{1}{(1 + i)^n}$$

1. Key in (in any order)
   a. Number of compounding periods, press n.
   b. Interest rate per period, press i.
   c. Future value (future worth), press FV.
2. To calculate present value (present worth), press PV.

Example 1 - An income property is forecast to be worth $250,000 ten years hence. If 14% is regarded as an appropriate annual rate of return to compensate an investor for waiting and risk-taking, what should an investor pay for it today?

Enter:

10 $\boxed{\text{n}}$ **14** $\boxed{\text{i}}$ **250000** $\boxed{\text{FV}}$ $\boxed{\text{PV}}$

```
         CLEAR

    10.00     → N
    14.00     → I
250000.00     → F
P V
 67435.95
```

The investor should pay $67,435.95.

Example 2 - A parcel of land recently sold for $8500. Market evidence indicates that competitive land values have been increasing at 1.25% per month. What was it worth 2 years and 5 months ago, when the then-owner died?

Enter:

2 $\boxed{\text{SAVE}\uparrow}$ **12** $\boxed{\times}$ **5** $\boxed{+}$ $\boxed{\text{n}}$ **1.25** $\boxed{\text{i}}$

8500 $\boxed{\text{FV}}$ $\boxed{\text{PV}}$

```
     2.00     ↑
    12.00     ×
    24.00  ◇
     5.00     +
              → N
     1.25     → I
  8500.00     → F
P V
  5928.75
```

The land was worth $5928.75.

PRESENT WORTH OF ONE PER PERIOD (Level Annuity; Inwood Factor)

PV is the unknown. The known values are n , i and PMT , which may be keyed in any order.

The Formula is: $PV = PMT\dfrac{(1+i)^n - 1}{i\,(1+i)^n}$

1. Key in (in any order)
   a. Number of compounding periods, press $\boxed{\text{n}}$.
   b. Interest rate per period, press $\boxed{\text{i}}$.
   c. Payment per period, press $\boxed{\text{PMT}}$.
2. To calculate present value (present worth), press $\boxed{\text{PV}}$.

Example - A 15-year lease calls for monthly rental payments of $525, payable at the end of each month. What is the present worth of the rental stream, discounted at 11.45%?

Enter:

15 $\boxed{\text{SAVE}\uparrow}$ **12** $\boxed{\times}$ $\boxed{\text{n}}$ **11.45** $\boxed{\text{SAVE}\uparrow}$

12 $\boxed{\div}$ $\boxed{\text{i}}$ **525** $\boxed{\text{PMT}}$ $\boxed{\text{PV}}$

```
         CLEAR
    15.00     ↑
    12.00     ×
   180.00  ◇
              → N
    11.45     ↑
    12.00     ÷
      .95  ◇
              → I
   525.00     →PT
P V
 45063.87
```

The present worth of the rental stream is $45,063.87.

INSTALLMENT TO AMORTIZE ONE (Amortization Payment)

[PMT] is the unknown. The known values are [n], [i] and [PV], which may be keyed in any order.

The Formula is: $$PMT = PV\frac{i(1+i)^n}{(1+i)^n - 1}$$

1. Key in (in any order)
   a. Number of compounding periods, press [n].
   b. Interest rate per period, press [i].
   c. Present value (present worth), press [PV].
2. To calculate payment per period, press [PMT].

Example 1 - What monthly payment (principal plus interest) will fully amortize a mortgage of $45,500 in 22 years and 8 months, at 8.5% interest?

Enter:

22 [SAVE↑] 12 [×] 8 [+] [n] 8.5 [SAVE↑] 12 [÷] [i] 45500 [PV] [PMT]

The monthly payment is $377.67.

```
        CLEAR
   22.00      ↑
   12.00      ×
  264.00   ◇
    8.00      +
           → N
    8.50      ↑
   12.00      ÷
     .71   ◇
           → I
45500.00   →P
           PMT
  377.67
```

Example 2 - What is the mortgage constant (F) on a 20-year, level-monthly payment, fully amortized mortgage with a 9.25% interest rate?

Enter: [ ] [6]

20 [SAVE↑] 12 [×] [n] 9.25 [SAVE↑] 12 [÷] [i] 1 [PV] [PMT] 12 [×]

The annual constant is .109904 or 10.9904%.

```
           CLEAR
 20.000000       ↑
 12.000000       ×
240.000000   ◇
             → N
  9.250000       ↑
 12.000000       ÷
   .770833   ◇
             → I
  1.000000   →P
             PMT
   .009159
 12.000000       ×
   .109904   ◇
```

CONVERSION TO PAYMENTS IN ADVANCE (Payments in Beginning of Period)

1. Multiplication by Base Factor: FV and PV

The 'Base Factor' is 'One' plus the interest rate per period, expressed as a decimal. For example, if the interest rate is 7.65% and the compounding period is one year, the Base Factor is 1.0765. If the compounding period is one month, the Base Factor is 1 + (.0765 [÷] 12) or 1.006375.

6 1 [SAVE↑] 7.65 [SAVE↑] 12
[÷] [%] [+] [◊]

```
        CLEAR

  1.000000        ↑
  7.650000        ↑
 12.000000        ÷
   .637500    ◊
                  %
   .006375        +
  1.006375    ◊
```

The easiest and quickest way to convert a [PV] or [FV] calculation based on End-of-Period payments and compounding, to payments in advance, is to multiply it by the Base Factor. This is most readily accomplished by the following keystrokes after [PV] or [FV] has been calculated with the foregoing procedures:

[RCL] [i] [%] [+] [◊]

Thus, in the examples provided in earlier sections with End-of-Period payments, the results for Beginning-of-Period payments would be:

Future Worth of One Per Period -

Enter:

8 [n] 7.65 [i] 385 [PMT] [FV] [RCL]
[i] [%] [+] [◊]

$4353.02

```
            CLEAR

          8.00    → N
          7.65    → I
        385.00    →PT
 F   V
       4043.68
          7.65    → I
                    %
        309.34      +
       4353.02    ◊
```

Present Worth of One Per Period -

Enter:

15 SAVE↑ **12** × n **11.45** SAVE↑
12 ÷ i **525** PMT PV RCL i
% + ◊

$45,493.86

```
        CLEAR
   15.00      ↑
   12.00      ×
  180.00    ◊
            → N
   11.45      ↑
   12.00      ÷
     .95    ◊
            → I
  525.00    →PT
P   V
45063.87
     .95    ← I
              %
  429.98      +
45493.86    ◊
```

NOTE:

This procedure has the advantage of printing out the FV *or* PV *value based on End-of-Period payments, so that the impact of payments in advance can be readily calculated. This advantage also applies to the Beginning-of-Period calculations for* PMT *illustrated below.*

NOTE:

These procedures do not apply to Future Worth of One or Present Worth of One calculations. Since there is no periodic payment, there can be no beginning-of-period payment.

2. Division by Base Factor: PMT

With payments in advance (Beginning-of-Period) for PMT calculations, the PMT calculation for End-of-Period payments and compounding is divided by the Base Factor. For the examples used in preceding sections, the keystrokes would be as follows:

Sinking Fund Payment -

Enter:

CLEAR **32** n **8.75** i **70000** FV PMT
1 SAVE↑ RCL i % + ÷

$412.72

```
        CLEAR
   32.00    → N
    8.75    → I
70000.00    → F
            PMT
  448.83
    1.00      ↑
    8.75    ← I
              %
     .09      +
              ÷
  412.72    ◊
```

Installment to Amortize One -

Enter:

CLEAR 22 SAVE↑ 12 × 8 + n 8.5

SAVE↑ 12 ÷ i 45500 PV PMT

1 SAVE↑ RCL i % + ÷

$375.01

```
          CLEAR

   22.00       ↑
   12.00       ×
  264.00       ◊
    8.00       +
              → N
    6.50       ↑
   12.00       ÷
     .71       ◊
              → I
45500.00      →P
              PMT
  377.67
    1.00       ↑
     .71      ← I
               %
     .01       +
               ÷
  375.01       ◊
```

SOLVING FOR NUMBER OF PERIODS (n)

The number of payments or the number of compounding periods can be calculated quickly and easily on the HP-81. $\boxed{n}$ is the unknown. $\boxed{i}$ must be one of the known values. Two from among $\boxed{PV}$, $\boxed{FV}$ and $\boxed{PMT}$ are the other known values.

Example 1 - An investor purchased a parcel of land for $43,875. He forecasts that land values will increase at 8% per year. How long must he hold the land in order to double his investment (ignoring holding costs and disposition expenses)?

Enter:

43875 PV 2 × FV 8

i n 4 ◊

or

43875 STO 0 PV 2 × FV

8 i n 4 ◊

He must hold it 9 years and 1 month.

```
           CLEAR

  43875.00      →P
      2.00        ×
  87750.00      ◊
                → F
      8.00      → I
N
      9.01
      9.0065    ◊
  43875.00      →0
                →P
      2.00      N ×
  87750.00      ◊
                → F
      8.00      → I
N
      9.01
      9.0065    ◊
```

SOLVING FOR INTEREST RATE, DISCOUNT RATE, RATE OF RETURN (i)

The HP-81 automatically calculates the interest rate (or rate of discount, or rate of return) per period, as an Internal Rate of Return.

The unknown is [i]. One of the known values must be [n]. Two from [PV] [FV], and [PMT] are the other two known values.

Example 1 - Future Worth of One, Present Worth of One -
A house was purchased for $41,990 four years ago. It just resold for $53,500. What annual rate of interest did the owner earn (ignoring holding and disposition costs)?

Enter:

41990 [PV] 53500 [FV] 4 [n] [i]

6.24% per year

```
              CLEAR
   41990.00    →P
   53500.00    → F
       4.00    → N
INT
       6.24
```

Example 2 - Future Worth of One Per Period, Sinking Fund Payment -
An investment in a building was $170,000. The investor has been setting aside $1600 per year to provide for full recovery of the $170,000 in 25 years. What is the indicated level-annuity Capital Recovery Rate (or implicit reinvestment rate)?

Enter:

170000 [FV] 1600 [PMT] 25 [n] [i]

10.51% per year

```
              CLEAR
  170000.00    → F
    1600.00    →PT
      25.00    → N
INT
      10.51
```

Example 3 - Present Worth of One Per Period; Installment To Amortize One
A $75,000 fully amortized loan has level monthly payments of $637.13. The maturity is 22 years 5 months; What is the annual interest rate?

Enter:

75000 [PV] 637.13 [PMT] 22 [SAVE ↑]

12 [×] 5 [+] [n] [i] 12 [×]

8.75% annual interest rate

```
              CLEAR
   75000.00    →P
     637.13    →PT
      22.00     ↑
      12.00     ×
     264.00    ◇
       5.00     +
               → N
INT
        .73
      12.00     ×
       8.75    ◇
```

LOGARITHMS

A logarithm is the exponent of a base number represented by a natural number; it is the number of times a base number is multiplied by itself to produce a given number. It is included here because a logarithm is 'n' in the expression $x = (1+i)^n$, where 'x' is the number whose logarithm is sought, and '1+i' is the base number.

1. Logarithms to the Base 10

Most logarithms are expressed as an exponent of the base number 10. On the HP-81, base 10 logarithms are calculated automatically with the following keystrokes:

Key in number whose logarithm is sought, press $\boxed{}$ $\underset{\text{log}}{\boxed{-}}$.

Example - What is $\log_{10} 40$?

Enter:

40 $\boxed{}$ $\underset{\text{log}}{\boxed{-}}$ $\boxed{}$ **6** $\boxed{\Diamond}$

```
          CLEAR

    40.00    L
     1.60    ◇
 1.602060    ◇
```

2. Natural Logarithms (base e)

Some mathematical procedures require logarithms to the base 'e', which is approximately 2.71828. On the HP-81, $\log_e X$ is solved by finding 'n' where '1+i' = 2.71828. Since 'i' is a percentage entered as a whole percentage number, 'i' is entered as 171.828. The keystrokes are:

Enter:

1. **Number whose natural logarithm is sought, press $\boxed{FV}$.**
2. **Enter 1, press $\boxed{PV}$.**
3. **Enter 171.828, press $\boxed{i}$.**
4. **To calculate natural logarithm, press $\boxed{n}$.**

Example - What is $\log_e 40$?

Enter:

40 $\boxed{FV}$ **1** $\boxed{PV}$ **171.828** $\boxed{i}$ $\boxed{n}$ $\boxed{}$ **6** $\boxed{\Diamond}$

```
          CLEAR

     40.00    → F
      1.00    →P
   171.828    → I
N
      3.69
  3.688882    ◇
```

CHAPTER 3 - STATISTICS

Many statistical measures frequently used in solving real estate problems can be calculated automatically on the HP-81. In addition, those requiring additional computations can be handled easily, often using the automatic calculation functions as time-saving components.

MEAN AND STANDARD DEVIATION (Ungrouped Data)

The formula for Arithmetic Mean is: $\bar{x} = \frac{\Sigma x}{n}$

The formula for Standard Deviation is:

$$s = \sqrt{\frac{\Sigma x^2 - \frac{(\Sigma x)^2}{n}}{n - 1}}$$

To calculate the arithmetic mean and the standard deviation from a sample of numerical observations:

1. Press CLEAR .
2. Key in first number (x_1), press $\Sigma+$.
3. Key in second number (x_2), press $\Sigma+$.
4. Press $\bar{x}$.

The HP-81 prints out, in order, Standard Deviation (shown as σ), Number of Entries (shown as N), and Arithmetic Mean shown as $\bar{x}$).

The HP-81 also retains useful calculated values in storage as follows:

Storage Register 10: Σx^2 (Sum of the squared values of x)
Storage Register 11: n (Number of observations)
Storage Register 12: Σx (Sum of the values of x)

If these stored values are to be retained for further use, they should be transferred to other storage registers. Otherwise, they will be erased when the CLEAR key is pressed.

NOTE:

To erase a wrong entry, key in the number again and press $\Sigma+$ ($\Sigma-$)

Example - A survey of 10 apartment property sales reveals the following set of Gross Rent Multipliers: 5.7, 5.9, 6.3, 6.2, 6.0, 5.9, 6.2, 6.1, 6.3, 6.1. What is the mean GRM, and what is its standard deviation?

Enter:

[CLEAR] 5.7 [Σ+] 5.9 [Σ+] 6.3 [Σ+]
6.2 [Σ+] 6.0 [Σ+] 5.9 [Σ+] 6.2 [Σ+]
6.1 [Σ+] 6.3 [Σ+] 6.1 [Σ+] [$\bar{x}$]

The Mean is 6.07, and the Standard Deviation is 0.19.

(To find Σx^2, press [RCL] [•] [0])

(To find Σx, press [RCL] [•] [2])

```
            CLEAR
      5.70      Σ+
      5.90      Σ+
      6.30      Σ+
      6.20      Σ+
      6.00      Σ+
      5.90      Σ+
      6.20      Σ+
      6.10      Σ+
      6.30      Σ+
      6.10      Σ+
σ
       .19
N
     10.00
x̄
      6.07
                 ↑
    368.79     ←0*
                 ↑
     60.70     ←2*
```

MEAN AND STANDARD DEVIATION (Grouped Data)

Grouped data are presented in Frequency distributions, to save time and effort in writing down (or entering) each observation individually.

The formula for arithmetic mean is: $\bar{x} = \dfrac{\Sigma fx}{n}$, where 'f' is the frequency or number of times each value of x is included in the sample.

The formula for Standard Deviation is: $s = \sqrt{\dfrac{\Sigma fx^2 - \dfrac{(\Sigma fx)^2}{n}}{n - 1}}$

To calculate the Arithmetic Mean

1. Key in x_1, press [SAVE ↑], key in f_1, press [×]
2. Key in x_2, press [SAVE ↑], key in f_2, press [×] [+]
3. Continue until all paired values of x and f have been entered.
4. Press [◊]. This prints the value of Σfx.

   Press [STO] [1]. This stores Σfx for future use.
6. Key in the number of observations*, press [STO] [2] [÷].

**NOTE:*

The value of n also equals Σf.

To calculate the Standard Deviation

1. Press [CLEAR] .
2. Key in x_1, press [SAVE ↑] [×] , key in f_1, press [×].
3. Key in x_2, press [SAVE ↑] [×] , key in f_2, press [×] [+] .
   Continue until all values of x and f have been entered.
4. Press [◊] . This prints the value of Σx^2.
5. Press [RCL] [1] . (This recalls Σfx from the Mean calculation.)
6. Press [SAVE ↑] [×] . This produces ($\Sigma fx)^2$.
7. Press [RCL] [2] [÷] [−] .
8. Press [RCL] [2] [1] [−] [÷] .
9. Press [▒] (or enter .5, press [y^x]).

Example - A survey of 266 one-bedroom apartment rentals reveals that 54 rent for \$170 per month unfurnished, 32 rent for \$175 per month, 88 rent for \$180 per month, and 92 rent for \$186 per month. What are the Mean monthly rental and the Standard Deviation? (n or Σ f = 266)

Enter:

170 [SAVE ↑] 54 [×] 175 [SAVE ↑] 32 [×]
[+] 180 [SAVE ↑] 88 [×] [+] 186 [SAVE ↑]
92 [×] [+] [◊] [STO] [1] (This is Σ fx.)
266 [STO] [2] [÷] (This is $\bar{x}$.)
[CLEAR]
170 [SAVE ↑] [×] 54 [×] 175 [SAVE ↑] [×]
32 [×] [+] 180 [SAVE ↑] [×] 88 [×] [+] 186
[SAVE ↑] [×] 92 [×] [+]
[◊] (This is Σfx^2.)
[RCL] [1] [SAVE ↑] [×] [RCL] [2] [÷] [−]
[RCL] [2] 1 [−] [÷] [▒]

$\bar{x}$ = \$179.44
s = \$5.97

```
          CLEAR

  170.00       ↑
   54.00       ×
 9180.00    ◊
  175.00       ↑
   32.00       ×
 5600.00    ◊
               +
  180.00       ↑
   88.00       ×
15840.00    ◊
               +
  186.00       ↑
   92.00       ×
17112.00    ◊
               +
47732.00    ◊

              →1
  266.00      →2
               +
  179.44    ◊


          CLEAR

  170.00       ↑
               ×
28900.00    ◊
```

54.00	×
1560600.00	◇
175.00	↑
	×
30625.00	◇
32.00	×
980000.00	◇
	+
180.00	↑
	×
32400.00	◇
88.00	×
2851200.00	◇
	+
186.00	↑
	×
34596.00	◇
92.00	×
3182832.00	◇
	+
8574632.00	◇
	↑
47732.00	←1
	↑
	×
2.278343824+09	◇
	↑
266.00	←2
	÷
8565202.35	◇
	−
	↑
266.00	←2
1.00	−
	÷
35.58	◇
	√
5.97	◇

AVERAGE DEVIATION

The formula for Average Deviation is: $\text{A.D.} = \dfrac{\Sigma\,|x - \bar{x}|}{n}$

$|x - \bar{x}|$ means the absolute difference between each observed value of x and the Mean, without regard to sign.

To calculate Average Deviation,

1. Key in the value of $\bar{x}$, press STO 0
2. Key in the value of x_1, press RCL 0 −.

   If the result is negative, press CHS.

3. Press Σ+. Repeat for all observations.

4. Press $\boxed{\bar{x}}$.

Example - A sample of 10 apartment house sales revealed Gross Rent Multipliers of 5.7, 5.9, 6.3, 6.2, 6.0, 5.9, 6.2, 6.1, 6.3 and 6.1. The Arithmetic Mean is 6.07. What is the Average Deviation from that mean?

Enter:

6.07 [STO] [0] **5.7** [RCL] [0] [−] [CHS] [Σ+] **5.9** [RCL] [0] [−] [CHS] [Σ+] **6.3** [RCL] [0] [−] [Σ+] **6.2** [RCL] [0] [−] [Σ+] **6.0** [RCL] [0] [−] [CHS] [Σ+] **5.9** [RCL] [0] [−] [CHS] [Σ+] **6.2** [RCL] [0] [−] [Σ+] **6.1** [RCL] [0] [−] [Σ+] **6.3** [RCL] [0] [−] [Σ+] **6.1** [RCL] [0] [−] [Σ+] [$\bar{x}$]

The Average Deviation is 0.16.

```
          CLEAR

    6.07     →0
    5.70      ↑
    6.07     ←0
              -
            S ⇄
     .37     Σ+
    5.90      ↑
    6.07     ←0
              -
            S ⇄
     .17     Σ+
    6.30      ↑
    6.07     ←0
              -
     .23     Σ+
    6.20      ↑
    6.07     ←0
              -
     .13     Σ+
    6.00      ↑
    6.07     ←0
              -
            S ⇄
     .07     Σ+
    5.90      ↑
    6.07     ←0
              -
            S ⇄
     .17     Σ+
    6.20      ↑
    6.07     ←0
              -
     .13     Σ+
    6.10      ↑
    6.07     ←0
              -
     .03     Σ+
    6.30      ↑
    6.07     ←0
              -
     .23     Σ+
    6.10      ↑
    6.07     ←0
              -
     .03     Σ+
σ
     .10
N
   10.00
x̄
     .16
```

STANDARD ERROR OF THE MEAN

The Standard Error of the Mean is a measure of how reliable the Mean of a Sample ($\bar{x}$) is as an estimator of the Mean of the Population (y) from which the sample was drawn.

The formula is: $s_{\bar{x}} = \dfrac{s}{\sqrt{n}}$

To calculate the Standard Error of the Mean

1. Key in* the value of s , press [SAVE ↑] .
2. Key in* the value of n , Press [ ] [y^x] .
3. Press [÷] .

** NOTE:*

The values of s and n may be recalled from the apropriate storage register if they are in active storage.

Example - A sample of 266 one-bedroom apartments shows a Mean monthly rental of $179.44 and a Standard Deviation from that mean of $5.97. What is the indicated Standard Error of the Mean?

Enter:

5.97 [SAVE ↑] 266 [ ] [y^x] [÷]

$s_{\bar{x}}$ = $0.37

```
        CLEAR

   5.97       ↑
 266.00       √
  16.31       ◇
              ÷
    .37       ◇
```

NOTE:

The appropriate z or t value can be applied to $\bar{x}$ and $s_{\bar{x}}$ to indicate how reliable an estimator $\bar{x}$ is of the Population Mean. In this case, the 99% Confidence Interval is $\bar{x} \pm 2.575\ s_{\bar{x}}$. This means that there is a 99% probability that the true average monthly rental for one-bedroom apartments in this market is within the range of $179.44 $\pm$ (2.575 x $0.37). The calculation for the range is:

.37 [SAVE ↑] 2.575 [×] [STO] [0]

179.44 [RCL] [0] [−] [◊]

179.44 [RCL] [0] [+] [◊]

The 99% Confidence Interval for the Population Mean is from $178.49 to $180.39.

```
        CLEAR

    .37         ↑
  2.575         ×
    .95     ◊
               →0
 179.44         ↑
    .95        ←0
                -
 178.49     ◊

 179.44         ↑
    .95        ←0
                +
 180.39     ◊
```

LINEAR TREND LINE

This automatic procedure calculates the linear regression (by least-squares) of a series of values with equally spaced data points. This means that the values of the independent variable (x) must be equidistant from one another.

The formula for the regression line is: $Y_c = a + bx$.

Y_c is the calculated value of the dependent variable (y).

a is the "constant", the calculated value of y when x = 0.

b is the "regression coefficient", the slope of the line. This is the calculated number of units change in the calculated value of y for a one-unit change in the value of x. The sign (+ or -) of b indicates the direction of the change in the calculated value of y.

r^2 is the "coefficient of determination", the percentage of variance in y associated with (or explained by) changes in the value of x.

Time series linear trends are most frequently calculated with this procedure, where x is the number of the time period in which a corresponding value of y is recorded or observed. x-values must be evenly spaced and continuous. The data must be entered in chronological sequence. The value of x may not be 0.

To calculate r^2, b and a:

1. Press [CLEAR].
2. Key in the first value of y (y_1), press [TL].
3. Key in y_2, press [TL].

Continue for all values of y, in sequence.

4. Press [ ] (TL). See printout or r^2 (shown as FACTR) b (shown as s), and a (shown as y).

To find the calculated value of y for any x,

5. Key in appropriate time-period number (x), press (n).

6. Press (TL). This prints Y_c.

Example - The following population figures have been obtained for Nobletown:

Year	Population
1960 (Census)	14,623
1961	15,500
1962	17,100
1963	18,200
1964	19,450
1965	21,600
1966	23,100
1967	24,400
1968	25,500
1969	26,800
1970 (Census)	27,846
1971	28,450
1972 (Estimate)	30,000

a. What is the trend line equation for Nobletown's population?
b. What is the population estimate for 1980?
c. How good an estimate is the 1972 figure of 30,000?
d. What is the calculated population for 1965?

NOTE:

Population is y, Date is x. 1960 = x_1, 1965 = x_6, 1972 = x_{13}, 1980 = x_{21}.

Enter:

a. [CLEAR] **14623** [TL] **15500** [TL] **17100** [TL] **18200** [TL] **19450** [TL] **21600** [TL] **23100** [TL] **24400** [TL] **25500** [TL] **26800** [TL] **27846** [TL] **28450** [TL] **30000** [TL] [ ] [TL] CAL

Y_c **= 13247.77 + 1322.51 x**

b. **21** [n] [TL]

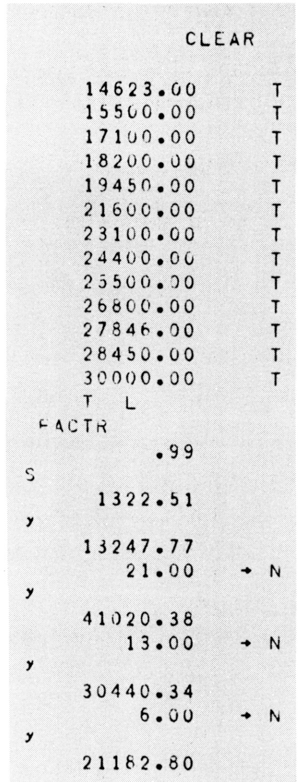
1980: 41,020

c. **13** [n] [TL]

Y_c **1972: 30,440 (Difference is +440 or 1.47%.)**

d. **6** [n] [TL]

1965: 21,183 (vs. 21,600)



EXPANDED TREND LINE (simple Linear Regression)

This procedure allows for complete variability in the values of x as well as y. The data must be entered in pairs: each x-value with its associated y-value. The first value of x entered may not be 0.

To calculate a, b and r^2 in a simple linear regression,

1. Press [CLEAR].
2. Key in the value of x_1, press [STO] [TL].

   Key in the value of y_1, press [TL].
3. Repeat Step 2 for each pair of values of x and y.
4. Press [ ] [TL] to print r^2, b and a.

To find any value of Y_c, given the value of x,

1. Key in the value of x, press [n].

2. Press (TL) to print Y_c.

Example - The sales price (y) and square foot area (x) of 8 residential properties were as follows:

Sale No.	Area (sq. ft.)	Sales Price
1	1000	$15,500
2	1200	$17,500
3	1500	$22,000
4	1300	$19,000
5	1400	$21,500
6	1150	$16,000
7	1350	$20,000
8	1200	$18,000

a. What is the linear regression equation with Area as the independent variable and Sales Price as the dependent variable? What is the coefficient of determination (r^2)?

b. What is the forecast (calculated) sales price Y_c when area is 1150 square feet? 1250 square feet? 1525 square feet?

Enter:

a. (CLEAR) **1000** (STO) (TL) **15500** (TL) **1200** (STO) (TL) **17500** (TL) **1500** (STO) (TL) **22000** (TL) **1300** (STO) (TL) **19000** (TL) **1400** (STO) (TL) **21500** (TL) **1150** (STO) (TL) **16000** (TL) **1350** (STO) (TL) **20000** (TL) **1200** (STO) (TL) **18000** (TL) ( ) (TL)

b. **1150** (n) (TL) **1250** (n) (TL) **1525** (n) (TL)

```
                CLEAR
  1000.00        +3*
 15500.00          T
  1200.00        +3*
 17500.00          T
  1500.00        +3*
 22000.00          T
  1300.00        +3*
 19000.00          T
  1400.00        +3*
 21500.00          T
  1150.00        +3*
 16000.00          T
  1350.00        +3*
 20000.00          T
  1200.00        +3*
 18000.00          T
T L
```

a. The linear regression equation is:

$Y_c = 113.31 + 14.71\ x$

Predicted Sales Price = $113.31 + $14.71 times Area in Square feet

$r^2 = .94$

b. When x = 1150 sq. ft., Y_c = \$17,032.
When x = 1250 sq. ft., Y_c = \$18,504.
When x = 1525 sq. ft., Y_c = \$22,549.

```
FACTR
            .94
S
          14.71
y
         113.31
        1150.00      → N
y
       17032.37
        1250.00      → N
y
       18503.60
        1525.00      → N
y
       22549.46
```

NOTE:

For future reference and analysis, as explained below, transfer a from storage register 13 to storage register 6; transfer b from storage register 10 to storage register 7.

[RCL] [•] [3] [STO] [6] [RCL] [•] [0] [STO] [7]

The number of observations (n) is in storage register 14.

LINEAR REGRESSION: ANALYTICAL FORMAT

The linear regression line, a point estimate of Y_c and the regression coefficient (b) can all be tested for their statistical significance and statistical reliability by calculating associated Standard Errors. The two most useful and commonly employed measures are the Standard Error of the Estimate (s_{yx}) and the Standard Error of the Regression Coefficient (s_b).

s_{yx} measures the variability (dispersion) around the Regression Line and points on that line.

The formula is: $$s_{yx} = \sqrt{\frac{\Sigma y^2 - a\Sigma y - b\,\Sigma xy}{n-2}}$$

s_b measures the dispersion around the Regression Coefficient (b).

The formula is: $$s_b = \frac{s_{yx}}{\sqrt{\Sigma x^2 - \dfrac{(\Sigma x)^2}{n}}}$$

s_b is also used to test the statistical significance of b, through the 't-test', where $t = \frac{b}{s_b}$.

To calculate these measures, the analyst needs the values of Σx, Σx^2, Σy, Σy^2, Σxy and n. As it happens, all these values are in 'temporary' storage in registers 10-13 and in the stack in the HP-81 before the operator presses [ ] [TL] in the Expanded Trend Line procedure illustrated in the immediately preceding section.

If the analyst desires to calculate s_{yx}, s_b, t or any other measure requiring these values, it is necessary to [CLEAR] after calculating a, b and r , and re-enter the entire data set as before.

1. Press [CLEAR].
2. Key in x_1, press [STO] [TL].
   Key in y_1, press [TL].
3. Repeat Step 2 for all paired values of x and y.

(This is the same as before. Now stop here for a moment.)

There are values stored and available as follows:

Stack y	Σy^2
Stack z	Σy
Storage Register 10	Σx
Storage Register 11	Σx^2
Storage Register 12	Σxy
Storage Register 14	n

Since all the foregoing but storage register 14 are erased when [CLEAR] is pressed, it is necessary to transfer these values to other storage registers. a is already in register 6; b is in register 7.

1. For Σy^2, press [R↓] , press [STO] [1] .
2. For Σy, press [R↓] , press [STO] [2] .
3. For Σx^2, press [RCL] [•] [1] , press [STO] [3] .
4. For Σx, press [RCL] [•] [0] , press [STO] [4] .
5. For Σxy, press [RCL] [•] [2] , press [STO] [5] .

Now all the necessary ingredients to calculate s_{yx}, s_b and t are stored, as follows:

Measure	Storage Register
Σy^2	1
Σy	2
Σx^2	3
Σx	4
Σxy	5
a	6
b	7
n	14

To calculate s_{yx}:

1. Press [CLEAR].

2. Press [RCL] [1] [SAVE ↑] [RCL] [6] [RCL] [2] [×] [−] [RCL] [7] [RCL] [5] [×] [−] [RCL] [•] [4] [2] [−] [÷] [ ] [y^x]

   627.24

To calculate s_b:

Go right on from S_{yx}.

1. Press [RCL] [3] [SAVE ↑] [RCL] [4] [SAVE ↑] [×] [RCL] [•] [4] [÷] [−] [ ] [y^x] [÷]

   1.50

To calculate t:

Go right on from s_b.

1. Press [RCL] [7] [$x \rightleftarrows y$] [÷]

   9.78

```
                CLEAR

         1000.00     →3*
        15500.00       T
         1200.00     →3*
        17500.00       T
         1500.00     →3*
        22000.00       T
         1300.00     →3*
        19000.00       T
         1400.00     →3*
        21500.00       T
         1150.00     →3*
        16000.00       T
         1350.00     →3*
        20000.00       T
         1200.00     →3*
        18000.00       T
  2.833750000+09       ↓
                     →1
       149500.00       ↓
                     →2
                       ↑
     12925000.00     ←1*
                     →3
                       ↑
        10100.00     ←0*
                     →4
                       ↑
  1.913000000+08     ←2*
                     →5

                CLEAR

  2.833750000+09     ←1
                       ↑
          113.31     ←6
                       ↑
       149500.00     ←2
                       ×
     16939747.83      ◇
                       −
                       ↑
           14.71     ←7
                       ↑
  1.913000000+08     ←5
                       ×
  2.814449641+09      ◇
                       −
                       ↑
            8.00     ←4*
            2.00       −
                       +
       393435.33      ◇
                      √
          627.24      ◇
                       ↑
     12925000.00     ←3
                       ↑
        10100.00     ←4
```

```
                 ↑
                 ×
1.020100000+08◇
                 ↑
      8.00    ←4*
                 +
12751250.00   ◇
                 -
              √
     416.83   ◇
                 ÷
       1.50   ◇
                 ↑
      14.71   ←7
       1.50   ⇄
                 +
       9.78   ◇
```

LOGARITHMIC TREND (Curvilinear Regression)

The Trend Line and Linear Regression procedures illustrated above can also be employed with the logarithms of the values of y. The y-values are entered as: y_1 [−], y_2 [−] and so on. All log y values must be calculated first before the Trend Line or Linear Regression routine is started.

A straight line with log-values of y_c is one with a constant percentage change in y for a unit change in x.

To convert a log y_c value to a natural number (antilog),

1. Key in 10, press [SAVE ↑].

2. Key in log value of y_c, press [y^x] .

CHAPTER 4: SIMPLE MORTGAGES (FULLY AMORTIZED)

PERIODIC PAYMENT AMOUNT, ANNUAL DEBT SERVICE

This calculation uses the Installment to Amortize One Format presented in Chapter 2. It produces the periodic payment in dollars that will fully amortize the principal amount of the mortgage at the indicated contract interest rate. Annual debt service is obtained by multiplying by the number of payments per year. (See NOTE below.)

Example 1 - What is the monthly payment on a fully amortized mortgage loan of $45,500 with a maturity of 20 years and interest at 9.25%?

Enter:

[CLEAR] 12 [STO] [0] 20 [×] [n] 9.25

[ ] [÷] [i] 45500 [PV] [PMT]

$416.72

```
                CLEAR

        12.00     →0
        20.00      ×
       240.00     ◇
                  → N
         9.25     N +
          .77     ◇
                  → I
     45500.00     →P
                  PMT
       416.72
```

Example 2 - What is Annual Debt Service on the foregoing mortgage loan?

Enter:

(Immediately after pressing [PMT]

, DO NOT [CLEAR] .)

[ ] [×]

$5000.63

```
                  N ×
      5000.63     ◇
```

NOTE:

The calculated PMT is shown rounded to two decimal places (nearest cent), but retained in the calculator to 10 decimals places. For realistic dollars and cents results, ADS is calculated as:

Enter:

Reenter PMT amount as calculated [ ] [×]

416.72 [ ] [×]

$5000.64

```
       416.72     N ×
      5000.64     ◇
```

Example 3 - Payments in Advance

Divide PMT by the Base Factor, after the PMT calculation is completed. For a 20-year 9.25% monthly payment loan of $45,500, the payment in advance is:

Enter:

[CLEAR] **12** [STO] [0] **20** [×] [n] **9.25** [ ]

[÷] [i] **45500** [PV] [PMT] **1** [SAVE ↑]

[RCL] [i] [%] [+] [÷]

413.53

	CLEAR
12.00	→0
20.00	×
240.00	◇
	→ N
9.25	N ÷
.77	◇
	→ I
45500.00	→P
	PMT
416.72	
1.00	↑
.77	← I
	%
.01	+
	÷
413.53	◇

MORTGAGE CONSTANT

This is Annual Debt Service expressed as a percentage of Mortgage Principal; it is often expressed in decimal form.

The formula is: $f = \dfrac{ADS}{P}$

The Mortgage Constant is calculated with the Installment to Amortize One Format using either dollar or decimal figures.

Example - What is the Mortgage Constant for a $63,000 mortgage loan at 8.75% interest, with level monthly payments over 22 years 5 months?

For Dollar Amounts, Enter:

[CLEAR] 12 [STO] [0] 22 [×] 5 [+] [n]

8.75 [░] [÷] [i] 63000 [PV] [PMT]

[░] [×] [RCL] [PV] [÷] [░] [6] [◊]

f = .101941

```
            CLEAR
   12.00     →0
   22.00       ×
  264.00     ◊
    5.00       +
             → N
    8.75     N ÷
     .73     ◊
             → I
63000.00     →P
             PMT
  535.19
             N ×
 6422.26     ◊
63000.00     →P
               ÷
     .10     ◊
 .101941     ◊
```

For Decimal Figures, Enter:

[CLEAR] [░] [6]

12 [STO] [0] 22 [×] 5 [+] [n]

8.75 [░] [÷] [i] 1 [PV] [PMT]

[░] [×] [RCL] [PV] [÷]

f = .101941

```
              CLEAR
  12.000000    →0
  22.000000      ×
 264.000000    ◊
   5.000000      +
               → N
   8.750000    N ÷
    .729167    ◊
               → I
   1.000000    →P
               PMT
    .008495
               N ×
    .101941    ◊
   1.000000    →P
                 ÷
    .101941    ◊
```

NUMBER OF PAYMENTS TO FULL AMORTIZATION

Since periodic payments are rounded to the nearest cent (and often rounded more), These payments will not always pay off the full amount of the mortgage over the nominal maturity. This procedure (solving for [n]) identifies the number of payments until full amortization of loan principal.

Example - A $63,750 mortgage loan has a nominal maturity of 18 years 6 months. The interest rate is 8.5%. Monthly payments are $570.60. Exactly how many months are required for full amortization?

Enter:

CLEAR

63750 PV **570.60** PMT **8.5** SAVE↑

12 ÷ i n

222.04 months

```
          CLEAR

 63750.00    →P
   570.60    →PT
     8.50    ↑
    12.00    ÷
      .71    ◇
             → I
N
   222.04
```

If monthly payments were changed to $570.75, how many payments would be required for full amortization?

Enter:

Do not Clear after foregoing calculation.

570.75 PMT n

221.90 months

```
   570.75    →PT
N
   221.90
```

NUMBER OF PAYMENTS TO A SPECIFIED BALANCE

While a mortgage may be scheduled to be fully amortized, the borrower often intends to pay off or refinance the loan prior to maturity, at some specified remaining balance or equity position. Given the interest rate per period, payment per period, total periods to full amortization and specified remaining balance, the number of periods can be found by:

1. Calculating n to amortize the specified balance, and
2. Subtracting the calculated value in Step 1 from total periods to full amortization.

Example - An \$80,000 mortgage loan has monthly payments of \$671.36 at 9% interest. How long will it be before the mortgage balance is \$50,000?

Enter:

[CLEAR]

50000 [PV] 671.36 [PMT] 9 [SAVE ↑] 12 [÷] [i] [n] [STO] [0] (109.44 payments to amortize \$50,000), 80000 [PV] [n] (300 payments to amortize \$80,000), [RCL] [0] [−] [◊]

190.56 months (15 years, 11 months)

```
          CLEAR

   50000.00    →P
     671.36    →PT
       9.00     ↑
      12.00     ÷
        .75    ◇
               → I
N
     109.44
               →0
   80000.00    →P
N
     300.00
                ↑
     109.44    ←0
                -
     190.56    ◇
```

PRESENT WORTH OF A MORTGAGE

To find the present worth or principal amount of a mortgage at any rate of discount or interest, use the [PV] procedure illustrated in Chapter 2.

Example - A prospective home buyer can afford \$250 per month in debt service. He can obtain a 30-year monthly payment mortgage loan at 8.75% interest. Can he afford a \$41,500 house if the maximum down payment he can make is 20%?

Enter:

[CLEAR] 12 [STO] [0] 30 [×] [n] 8.75 [ ] [÷] [i] 250 [PMT] [PV] Maximum mortgage amount is \$31,778.30.

41500 [SAVE ↑] 20 [%] [−] [◊]

Mortgage required is \$33,200.

[−] [◊] No, he is \$1,421.70 short.

```
          CLEAR

      12.00    →0
      30.00     ×
     360.00    ◇
               → N
       8.75    N ÷
        .73    ◇
               → I
     250.00    →PT
P    V
   31778.30
   41500.00     ↑
      20.00     %
    8300.00     -
   33200.00    ◇

                -
-   1421.70    ◇
```

EFFECTIVE INTEREST RATE (Effective Yield) - NO FEES

Effective annual interest rate or yield is important to the borrower so he can make meaningful comparisons among alternative loan opportunities, as well as know what the true interest cost is as an annual rate. It is important for the lender to know because he is required to report the effective Annual Percentage Rate under the terms of Regulation Z, Truth in Lending.

When no fees are charged the borrower for making the loan (e.g., discount points), the effective interest rate per period is found by solving for i using the procedure outlined in Chapter 2. The annual rate is found by multiplying the effective interest rate per period by the number of periods per year.

Example - A 25-year mortgage with a principal of $52,500 has monthly payments of $422.75. What is the annual rate of interest?

Enter:

CLEAR 12 STO 0 25 × n

422.75 PMT

52500 PV

i ×

8.50%

```
                CLEAR

          12.00      →0
          25.00       ×
         300.00      ◇
                     → N
         422.75      →PT
       52500.00      →P
I  N  T
            .71
                     N ×
           8.50      ◇
```

EFFECTIVE INTEREST RATE (Effective Yield) - FEES CHARGED

Borrowers are sometimes charged fees in connection with the issuance of mortgage loans (discount "points", for example), which raises the effective interest rate or Annual Percentage Rate. The actual amount received by the borrower (PV) is reduced, while periodic payments remain the same. Using the compound interest procedures outlined in Chapter 2, the steps involved in calculating effective interest rate are:

1. Calculate the payment on the nominal contract terms. [STO] [PMT]
2. Calculate the principal amount less fees, press [PV] .
3. Press [CLEAR], then press [RCL] [n] [n] enter PMT [PMT] [RCL] [PV] [PV] [i] . This gives interest rate per period.
4. Multiply by number of payments per period.

NOTE:

Step 3. [RCL] [PMT] [PMT] *can be used if rounded payment is not used.*

Example - Fees as a Percentage of Principal
A $52,500 mortgage at 8.5% interest is fully amortized in monthly payments over 25 years. The lender charges 4.5 points to the borrower. What is the effective annual interest rate if the loan is held to maturity?

Enter:

[CLEAR]

12 [STO] [0] **25** [×] [n] **8.5** [░] [÷] [i]

52500 [PV] [PMT]

[STO] [PMT]

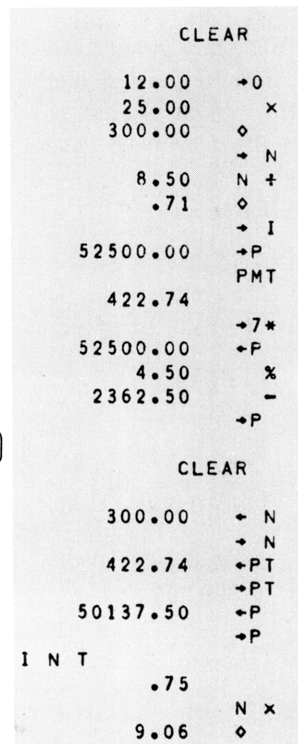
[RCL] [PV] **4.5** [%] [−] [PV]

[CLEAR]

[RCL] [n] [n] [RCL] [PMT] [PMT] [RCL] [PV]

[PV] [i] [░] [×]

9.06%


```
          CLEAR
    12.00     →0
    25.00      ×
   300.00     ◇
             → N
     8.50    N ÷
      .71     ◇
             → I
 52500.00    →P
             PMT
   422.74
             →7*
 52500.00    ←P
     4.50      %
  2362.50      −
             →P
          CLEAR
   300.00    ← N
             → N
   422.74    ←PT
             →PT
 50137.50    ←P
             →P
I N T
      .75
             N ×
     9.06     ◇
```

ACCUMULATED INTEREST PAID BETWEEN PERIODS AND REMAINING BALANCE

This routine calculates the total amount of interest which is paid between two given payments, and the balance of the mortgage loan remaining after the latter period.

Rounding to the nearest cent affects the results of the remaining balance calculation.

NOTE:

This can be a lengthy calculation. It should not be used merely to calculate remaining mortgage balance.

The keystrokes are:

1. Key in the beginning payment number, press [STO] [1].
2. Key in last payment number, press [STO] [2].
3. Key in interest rate per period, press [i].
4. Key in amount of payment per period, press [PMT].
5. Key in principal amount, press [PV].
6. Press [Σ+].

Example - A mortgage loan of $126,500 at 7.5% interest was originated 10 years 5 months ago. It has monthly payments of $934.80. How much interest has been paid since it was originated, and what is the remaining balance today?

Enter:

[CLEAR]

1 [STO] [1] **10** [SAVE ↑]

12 [STO] [0] [×] **5** [+]

[STO] [2] **7.5** [ ] [÷] [i] **934.80** [PMT]

126500 [PV] [Σ+]

Total interest paid is $89,655.14. Balance is $99,305.14.

```
          CLEAR
     1.00      →1
    10.00       ↑
    12.00      →0
                ×
   120.00      ◇
     5.00       +
               →2
     7.50      N ÷
      .63      ◇
               → I
   934.80      →PT
126500.00      →P
                Σ+
P  V
  99305.14
I N T
  89655.14
```

AMORTIZED LOAN SCHEDULE

This routine calculates the amount of interest paid and the remaining loan balance for each payment period over the span of payments specified. It also shows the total amount of interest paid over the span of payments covered. All the calculated amounts are printed out in a schedule for each period.

Rounding the payment figure to the nearest cent affects the calculated results of this routine.

NOTE:
This can be a lengthy calculation.

The keystrokes are:

1. Press [CLEAR].
2. Key in the number of the first payment, press [STO] [1].
3. Key in the number of the last payment, press [STO] [2].
4. Key in the principal amount, press [PV].
5. Key in the interest per payment period, press [i].
6. Key in the payment amount per period, press [PMT].
7. Press [EXT( )] [6].

Example - With the mortgage loan terms from the foregoing example under Accumulated Interest Paid and Remaining Balance (Principal - $126,500; Monthly Payment - $934.80; Interest Rate - 7.5%), calculate the loan amortization schedule for the first year.

Enter:

[CLEAR]

1 [STO] [1] 12 [STO] [2]

126500 [PV] 7.5 [SAVE↑]

12 [÷] [i] 934.80 [PMT] [EXT( )] [6]

```
                   CLEAR

          1.00      →1
         12.00      →2
     126500.00      →P
          7.50       ↑
         12.00       ÷
           .63      ◇
                    → I
        934.80      →PT
1
        790.63
        144.17
     126355.83
2
        789.72
        145.08
     126210.75
3
        788.82
        145.98
     126064.77
4
        787.90
        146.90
     125917.87
5
        786.99
        147.81
     125770.06
6
        786.06
        148.74
     125621.32
```

```
7
        785.13
        149.67
     125471.65
8
        784.20
        150.60
     125321.05
9
        783.26
        151.54
     125169.51
10
        782.31
        152.49
     125017.02
11
        781.36
        153.44
     124863.58
12
        780.40
        154.40
     124709.18
I N T
       9426.78
```

MORTGAGE BALANCE OUTSTANDING AT END OF SPECIFIED PERIOD

It is often necessary in real estate investment and appraisal analysis to calculate the remaining balance outstanding on a mortgage loan after a specified period less than the full maturity or amortization term. This is especially the case in using Mortgage-Equity or Ellwood analysis. (See Chapter 10.)

To obtain full accuracy it is necessary first to calculate the exact number of periods to amortize the mortgage loan fully. If this is not done, and the nominal maturity is used, the calculated balance outstanding will be slightly off, depending on the extent to which the payment per period is a rounded figure.

The keystrokes required are the same as those shown for Example 3 under Effective Interest Rate above.

1. Press [CLEAR].
2. Calculate interest rate per period, press [i].
3. Key in payment per period, press [PMT].
4. Key in mortgage principal, press [PV].
5. Press [n]. (This gives exact number of periods.)

6. Calculate number of payments for specified time period, press [–]. (This gives exact number of periods remaining to full maturity.)
7. Press [n]. (This enters the figure calculated in Step 6 into storage, Register 19.)
8. Press [CLEAR].
9. Press [RCL] [n] [n].
10. Press [RCL] [i] [i].
11. Press [RCL] [PMT] [PMT] (Steps 9-11 recall and enter the values retained in storage for [n], [i] and [PMT] for further calculation.)
12. Press [PV]. (This is the remaining balance.)

Example - A $126,500 mortgage loan has monthly payments of $934.80. The interest rate in 7.5% and the contract maturity is 25 years. What is the balance outstanding after 10 years?

Enter:

[CLEAR]

7.5 [SAVE↑] **12** [STO] [0] [÷] [i] **934.80**

[PMT]

126500 [PV] [n]

300.02 monthly payments

10 [ ] [×] [–] [n]

180.02 payments remaining

[CLEAR]

[RCL] [n] [n] [RCL] [PMT] [PMT] [RCL]

[i] [i] [PV]

Balance $100,846.89

```
                CLEAR
          7.50        ↑
         12.00       →0
                      ÷
           .63       ◇
                     → I
        934.80       →PT
     126500.00       →P
 N
        300.02
         10.00       N ×
        120.00       ◇
                      –
                     → N
        180.02       ◇

                CLEAR
        180.02       ← N
                     → N
        934.80       ←PT
                     →PT
           .63       ← I
                     → I
 P   V
     100846.89
```

NOTE:

Other methods may be used to calculate remaining balance but may be more time-consuming than the above routine. Slightly different answers may be obtained due to the rounding of calculated values.

a. The Accumulated Interest Paid and Remaining Balance routine illustrated earlier could be applied in this case as follows:

Enter:

[CLEAR]

1* [STO] [1] **12** [STO] [0] **10** [×] [STO] [2]

7.5 [ ] [÷] [i] **934.80** [PMT] **126500**

[PV] [Σ+]

The remaining balance is $100,846.83.

***Can be any period number up to the total stored in** [STO] [2] **.**

```
              CLEAR

         1.00   →1
        12.00   →0
        10.00    ×
       120.00   ◇
                →2
         7.50   N ÷
          .63   ◇
                → I
       934.80   →PT
    126500.00   →P
                Σ+
  P  V
    100846.83
 I N T
     86522.83
```

b. The Amortized Loan Schedule routine illustrated could be applied in this case, entering the number of payments in the specified time span in both storage registers 1 and 2, as follows:

Enter:

[CLEAR]

12 [STO] [0] **10** [×] [STO] [1] **10** [ ]

[×] [STO] [2] **7.5** [ ] [÷] [i] **934.80**

[PMT] **126500** [PV] [EXT( )] [6]

The remaining balance is $100,846.83.

```
              CLEAR

        12.00   →0
        10.00    ×
       120.00   ◇
                →1
        10.00   N ×
       120.00   ◇
                →2
         7.50   N ÷
          .63   ◇
                → I
       934.80   →PT
    126500.00   →P
 120
       632.18
       302.62
    100846.83
 I N T
       632.18
```

NOTE:

The effects of using rounded payment amounts without calculating and using the exact number of payment periods are illustrated by the following calculation:

Enter:

[CLEAR]

12 [STO] [0] 25 [×] 10 [ ] [×] [−]

[n] 7.5 [ ] [÷] [i] 934.80 [PMT] [PV]

```
                CLEAR
        12.00    →0
        25.00     ×
       300.00    ◇
        10.00    N ×
       120.00    ◇
                  -
                 → N
         7.50    N ÷
          .63    ◇
                 → I
       934.80    →PT
P   V
    100840.08
```

The indicated remaining balance is \$100,840.08, as opposed to \$100,846.89 using the exact number of payment periods, as opposed to \$100,846.83 using the Amortized Loan Schedule & Remaining balance Routines.

SCHEDULE OF ANNUAL INTEREST AND PRINCIPAL PAYMENTS FROM DEBT SERVICE

It is often necessary to develop a schedule of annual payments of mortgage interest and principal, when mortgage payment periods are less than one year (monthly, quarterly, semi-annual). This is especially useful in calculating taxable income for deriving After-Tax Cash Flow.

The routine is a modification of the Accumulated Interest Paid and Remaining Balance procedure. The amount of interest paid in each successive year is calculated with this routine for payment periods 1-12, 13-24, 25-36 and so forth over the number of years in the time span included in the analysis.

To obtain annual principal payments, the year's annual interest payments are subtracted from annual debt service. In the Accumulated Interest Paid and Remaining Balance Routine, interest paid is stored in Register 13. The keystrokes are:

1. Key in periodic payment, press [SAVE ↑]; key in the number of payments per year, press [×] [STO] [0]. (This stores Annual Debt Service.)
2. Key in 1, press [STO] [1].
3. Key in 12, press [STO] [2].
4. Key in interest rate per period, press [i].
5. Key in payment per period, press [PMT].
6. Key in principal amount, press [PV].
7. Press [Σ+]. (This calculates and prints Interest Paid and Remaining Balance for Year 1.)

8. Press [RCL] [0] [RCL] [•] [3] [−] [◊]

   (This calculates and prints out Principal Amortization for Year 1.)

9. Key in 13, press [STO] [1] .
10. Key in 24, press [STO] [2] .
11. Press [Σ+] . (This calculates and prints Interest Paid and Remaining Balance for Year 2.)
12. Press [RCL] [0] [RCL] [•] [3] [−] [◊]

   (This calculates and prints out Principal Amortization for Year 2.)

Repeat Steps 9-12 for each successive year in the time span covered by the analysis. Use 25 and 36, 37 and 48, and so forth as successive values entered in Steps 9 and 10.

Example - A $126,500 mortgage loan has monthly payments of $934.80, with interest at 7.5%. Construct the schedule of annual interest and principal payments over the first four years of the loan term.

Enter:

[CLEAR]

934.80 [SAVE ↑] 12 [×] [STO] [0]

(Annual debt service is $11,217.60.)

```
               CLEAR

      934.80       ↑
       12.00       ×
    11217.60       ◊
                  →0
```

Year 1:

1 [STO] [1] 12 [STO] [2] 7.5 [SAVE ↑]

12 [÷] [i] 934.80 [PMT] 126500 [PV]

[Σ+] [RCL] [0] [RCL] [•] [3] [−] [◊]

```
        1.00      →1
       12.00      →2
        7.50       ↑
       12.00       ÷
         .63       ◊
                 → I
      934.80     →PT
   126500.00      →P
                  Σ+
P  V
   124709.18
I  N  T
     9426.78
                   ↑
    11217.60      ←0
                   ↑
     9426.78     ←3*
                   −
     1790.82       ◊
```

Year 2:

13 [STO] [1] **24** [STO] [2] [Σ+] [RCL] [0]

[RCL] [·] [3] [−] [◊]

```
       13.00    →1
       24.00    →2
                 Σ+
P   V
   122779.31
I N T
     9287.73
                  ↑
    11217.60    ←0
                  ↑
     9287.73    ←3*
                  -
     1929.87     ◇
```

Year 3:

25 [STO] [1] **36** [STO] [2] [Σ+] [RCL] [0]

[RCL] [·] [3] [−] [◊]

```
       25.00    →1
       36.00    →2
                 Σ+
P   V
   120699.62
I N T
     9137.91
                  ↑
    11217.60    ←0
                  ↑
     9137.91    ←3*
                  -
     2079.69     ◇
```

Year 4:

37 [STO] [1] **48** [STO] [2] [Σ+] [RCL] [0]

[RCL] [·] [3] [−] [◊]

```
       37.00    →1
       48.00    →2
                 Σ+
P   V
   118458.48
I N T
     8976.46
                  ↑
    11217.60    ←0
                  ↑
     8976.46    ←3*
                  -
     2241.14     ◇
```

Since 10 years is a frequently employed income projection period, especially in Mortgage-Equity or Ellwood analysis (See Chapter 10), for illustrative purposes the 10-year calculated schedule of annual interest and principal payments for the mortgage loan in this example is as follows:

Year	Debt Service	Interest	Principal	End-Of-Year Balance
1	$11,217.60	$9,426.78	$1,790.82	$124,709.18
2	$11,217.60	$9,287.73	$1,929.87	$122,779.31
3	$11,217.60	$9,137.91	$2,079.69	$120,699.62
4	$11,217.60	$8,976.46	$2,241.14	$118,458.48
5	$11,217.60	$8,802.50	$2,415.10	$116,043.38
6	$11,217.60	$8,614.99	$2,602.61	$113,440.77
7	$11,217.60	$8,412.93	$2,804.67	$110,636.10
8	$11,217.60	$8.195.21	$3,022.39	$107,613.71
9	$11,217.60	$7,960.59	$3,257.01	$104,356.70
10	$11,217.60	$7,707.73	$3,509.87	$100,846.83

Note that the remaining balance at the end of Year 10 is the same as that calculated in the earlier remaining balance.

AMOUNT OF LAST PAYMENT

When the periodic payment amount is rounded, the full amount of the mortgage principal may not be fully amortized over the contract maturity term - or it may be more than amortized. As a result, the last payment will be larger or smaller than the other periodic payments. To calculate the amount of the last payment, it is necessary first to calculate the mortgage balance at the end of the contract maturity term and then add the amount of the periodic payment.

The basic routine is that used and illustrated in the foregoing section on Mortgage Balance Outstanding at End of Specified Period. In this case, the specified period is the contract maturity.

Example 1 - What is the last monthly payment on a $76,750 mortgage at 8.75% interest, with monthly payments of $678.00 and a contract maturity of 20 years?

Enter:

[CLEAR]

8.75 [SAVE↑] **12** [STO] [0] [÷] [i]

678. [PMT] **76750** [PV] [n]

240.24 payments

20 [ ] [×] [−] [n]

[CLEAR]

[RCL] [n] [n] [RCL] [i] [i] [RCL]

[PMT] [PMT] [PV]

160.47 balance

[RCL] [PMT] [+] [◊]

838.47 final payment

```
              CLEAR
          8.75        ↑
         12.00      →0
                      ÷
           .73      ◊
                    → I
        678.00      →PT
      76750.00      →P
  N
        240.24
         20.00      N ×
        240.00      ◊
                      -
                    → N
              CLEAR
           .24      ← N
                    → N
           .73      ← I
                    → I
        678.00      ←PT
                    →PT
P   V
        160.47
        678.00      ←PT
                      +
        838.47      ◊
```

These calculations can be worked with the routines for Accumulated Interest Paid and remaining Balance and Amortized Loan Schedule, but they can be lengthy.

Example 1 above worked with the Accumulated Interest Paid and Remaining Balance routine is as follows:

Enter:

CLEAR

12 STO 0 20 × STO 1 240 STO

2 8.75 ÷ i 678. PMT

76750 PV Σ+ RCL PMT 160.60

+ ◊

$838.60

```
              CLEAR

        12.00     →0
        20.00      ×
       240.00     ◊
                  →1
       240.00     →2
         8.75     N ÷
          .73     ◊
                  → I
       678.00     →PT
     76750.00     →P
                  Σ+
P  V
       160.60
I N T
         6.07
       678.00     ←PT
       160.60      +
       838.60     ◊
```

NOTE:

Slightly different answers may be obtained due to the rounding of calculated values.

CHAPTER 5 - BALLOON-PAYMENT MORTGAGES (PARTIALLY AMORTIZED)

Mortgage loans which are not fully amortized over the contract maturity term require a large lump-sum repayment of principal at maturity. This lump-sum payment is called a "Balloon" payment.

In addition, as indicated in several examples in Chapter 4, a simple or fully amortized mortgage loan may be prepaid by the borrower - with or without prepayment penalty. When this occurs, the outstanding balance of the mortgage loan is due and payable as a Balloon.

Real Estate problems involving partially amortized mortgage loans require somewhat more complicated calculations and procedures than do those with simple mortgages. This is primarily because the income stream to the lender consists of two parts: the periodic payment and the lump sum payment at the end of the income projection period (or contract maturity). These complications can be readily handled on the HP-81 with the following routines.

PERIODIC PAYMENT AMOUNT; ANNUAL DEBT SERVICE

This routine calculates the level payment of principal and interest per period for a partially amortized (balloon payment or prepaid) mortgage loan. It also produces Annual Debt Service by multiplying the periodic payment by the number of payments per year.

The important variation in this routine, as opposed to that for fully amortized mortgages, is that the amount to be amortized over the contract maturity is the mortgage principal less the present worth of the balloon payment. Thus it is necessary to calculate the present worth of the balloon payment first.

The keystrokes are:

1. Press [CLEAR] .
2. Key in number of payments per year, press [STO] [0] ; key in number of years to contract maturity, press [×] [n] .
3. Key in annual interest rate, press [ ] [÷] [i] .
4. Key in amount of balloon payment, press [FV] .
5. Press [PV] . (This is the present worth of the balloon payment.)
6. Key in principal amount, press [x⇄y] [−] [PV] .
7. Press [CLEAR] .

8. Press [RCL] [n] [n] [RCL] [i] [i] [RCL] [PV] [PV] [PMT] .
   (This gives the periodic payment amount.)

9. Press [▒] [×] . (This gives Annual Debt Service.)

Example - A mortgage loan of $60,000 has a maturity of 20 years. The interest rate is 8.5%. There is to be a balloon payment of $23,507.58 at maturity. What are the monthly payment and Annual Debt Service?

Enter: [CLEAR]

12 [STO] [0] **20** [×] [n] **8.5** [▒] [÷] [i]

23507.58 [FV] [PV] **60000** [$x \rightleftarrows y$] [−] [PV]

[CLEAR]

[RCL] [n] [n] [RCL] [i] [i] [RCL] [PV]

[PV] [PMT] **$483.20 per month**

[▒] [×] **$5798.42 Annual Debt Service**

```
              CLEAR
       12.00     →0
       20.00        ×
      240.00     ◇
                 →  N
        8.50     N  ÷
         .71     ◇
                 →  I
    23507.58     →  F
P   V
     4320.26
    60000.00
     4320.26        ⇄
                    −
                 →P
              CLEAR
      240.00     ←  N
                 →  N
         .71     ←  I
                 →  I
    55679.74     ←P
                 →P
                 PMT
      483.20
                 N  ×
     5798.42     ◇
```

MORTGAGE CONSTANT

The mortgage constant (F) is calculated with the same routine as above, except that Annual Debt Service is expressed as a percentage of the principal amount of the mortgage:

$$F = \frac{ADS}{P}$$

The keystrokes to obtain the mortgage constant are:

1. Calculate Annual Debt Service using Steps 1-9 in the Periodic Payment Amount; Annual Debt Service routine given above.

2. Key in principal amount, press [÷] [▒] [6] [◊].

3. Press 100 [×] .

Example - Using the mortgage loan illustrated in the preceding section, what is the mortgage constant?

Enter:

60000 [÷] [ ] [6] [◊] **F = .096640**

100 [×] **F = 9.664035%**

```
   5798.42      ◊

  60000.00      ÷
       .10      ◊
   .096640      ◊

100.000000      ×
  9.664035      ◊
```

NOTE:

ADS of $5798.42 shows in the printout.

NUMBER OF PAYMENTS TO A SPECIFIED BALANCE

This routine is exactly the same as that for simple mortgage. The fact of a balloon payment has no influence on the procedure or keystrokes, except that the specified balance must be no lower than the amount of the balloon payment.

The keystrokes are:

1. Key in amount of specified balance, press [PV] .
2. Key in interest rate per period, press [i] .
3. Key in payment per period, press [PMT] .
4. Press [n] [STO] [0] . (This gives the number of payments required to amortize the specified balance; it is the number of payments "left".)
5. Key in the principal of the mortgage, press [PV] .
6. Press [n] . (This gives the total number of payments required to amortize the principal amount of the mortgage.)
7. Press [RCL] [0] [−] [◊] . (This gives the number of payments required to reach the specified balance.)

Example 1 - The $60,000 mortgage with interest at 8.5% has monthly payments of $483.20. How long will it be until the mortgage balance is down to $23,507.58?

Enter:

CLEAR 23507.58 PV 8.5 SAVE↑ 12 ÷

i 483.20 PMT n STO 0 60000 PV

n RCL 0 − ◇ 240 months

12 ÷ 20 years (This confirms the earlier calculations.)

```
              CLEAR
  23507.58      →P
      8.50       ↑
     12.00       ÷
       .71       ◇
                → I
    483.20      →PT
N
     59.86
                →0
  60000.00      →P
N
    299.86
                 ↑
     59.86      ←0
                 −
    240.00       ◇

     12.00       ÷
     20.00       ◇
```

Example 2 - The investor in the property with the mortgage in Example 1 has given an option to sell a half interest in the property for $40,000. How long will it be before the proceeds of that sale will cover the outstanding balance of the mortgage, I.E. how many periods before the outstanding balance is $40,000?

Enter:

40000 PV 8.5 SAVE↑ 12 ÷ i 483.20

PMT n STO 0 60000 PV n RCL

0 − ◇ 174.79 months

12 ÷ 14.57 years (14 years 7 months)

```
              CLEAR
  40000.00      →P
      8.50       ↑
     12.00       ÷
       .71       ◇
                → I
    483.20      →PT
N
    125.07
                →0
  60000.00      →P
N
    299.86
                 ↑
    125.07      ←0
                 −
    174.79       ◇

     12.00       ÷
     14.57       ◇
```

PRESENT WORTH OF A MORTGAGE

This routine requires two procedures: calculating the present worth of the periodic payments over the contract maturity period, and calculating the present worth of the balloon payment. Both are discounted at the appropriate interest rate, and added together to derive the present worth or market value of the mortgage loan.

NOTE:

The routine illustrated in the following section assumes that the balloon payment is due and payable in the same period as, and in addition to, the final periodic payment.

This routine is useful in situations when the contract or nominal rate of interest is different (higher or lower) from the prevailing market rate. The calculation provides the market value of the mortgage loan at the market rate.

The keystrokes are:

1. Key in the number of payments to contract maturity, press $\boxed{\text{n}}$.
2. Key in the market rate of interest per period, press $\boxed{\text{i}}$.
3. Key in the periodic payment amount, press $\boxed{\text{PMT}}$.
4. Press $\boxed{\text{PV}}$ $\boxed{\text{STO}}$ $\boxed{\text{0}}$. (This calculates and prints the present worth of the balloon payment.)
5. Press $\boxed{\text{CLEAR}}$
6. Press $\boxed{\text{RCL}}$ $\boxed{\text{n}}$ $\boxed{\text{n}}$ $\boxed{\text{RCL}}$ $\boxed{\text{i}}$ $\boxed{\text{i}}$.
7. Key in amount of balloon payment, press $\boxed{\text{FV}}$.
8. Press $\boxed{\text{PV}}$. (This calculates and prints the present worth of the balloon payment.)
9. Press $\boxed{\text{RCL}}$ $\boxed{\text{0}}$ $\boxed{+}$ $\boxed{\Diamond}$. (This calculates and prints the present worth of the mortgage loan.

Example 1 - A $60,000 mortgage loan with a balloon payment of $23,507,58 has monthly payments of $483.20. The contract interest rate is 8.5%. It was originated 3 years ago. The market interest rate is now 9.5 %. What are the present worth (market value) of this mortgage, its book value, and the discount or loss to the lender if the loan is sold on the open market?

Enter: $\boxed{\text{CLEAR}}$

a. Calculate number of periods remaining to contract maturity.

23507.58 $\boxed{\text{PV}}$ **8.5** $\boxed{\text{SAVE}\uparrow}$ **12** $\boxed{\div}$

```
            CLEAR

23507.58     →P
    8.50      ↑
   12.00      ÷
     .71     ◊
             → I
  483.20     →PT
```

483.20 PMT n STO 0 **60000** PV
n RCL 0 − **3** SAVE↑ **12** ×
− n **204 months**

```
N
        59.86
                 →0
     60000.00    →P
N
       299.86
                  ↑
        59.86    ←0
                  −
         3.00     ↑
        12.00     ×
        36.00    ◊
                  −
                 → N
       204.00    ◊
```

b. Calculate book value at 8.5%.

CLEAR RCL n n RCL i i
RCL PMT PMT PV STO 0 CLEAR
RCL n n RCL i i **23507.58** FV
PV RCL 0 + ◊ STO 1

Book Value - $57622.96

```
              CLEAR
       204.00    ← N
                 → N
          .71    ← I
                 → I
       483.20    ←PT
                 →PT
P    V
     52052.97
                 →0
              CLEAR
       204.00    ← N
                 → N
          .71    ← I
                 → I
     23507.58    → F
P    V
      5569.99
                  ↑
     52052.97    ←0
                  +
     57622.96    ◊
                 →1
```

c. **Calculate present worth at 9.5%.**

CLEAR RCL n n **9.5** SAVE↑ **12** ÷
i RCL PMT PMT PV STO 0
CLEAR RCL n n RCL i i
RCL FV FV PV RCL 0 + ◊

Market Value - $53,524.37

```
              CLEAR
       204.00    ← N
                 → N
         9.50     ↑
        12.00     ÷
          .79    ◊
                 → I
       483.20    ←PT
                 →PT
```

```
P   V
    48819.23
                   →0

              CLEAR

         204.00    ← N
                   → N
            .79    ← I
                   → I
    23507.58       ← F
                   → F
P   V
     4705.14
                     ↑
    48819.23       ←0
                     +
    53524.37       ◇
```

d. Calculate discount or book loss.

[RCL] [1] [−] [◊] **Dollar loss - $4,098.59**

[RCL] [1] [÷] [ ] [6] [◊] **Percentage loss - 7.11%**

```
                     ↑
    57622.96       ←1
                     −
-    4098.59       ◇
                     ↑
    57622.96       ←1
                     ÷
-        .07       ◇
-    .071128       ◇
```

PRESENT WORTH OF MORTGAGE - BALLOON PAYMENT ONE PERIOD AFTER LAST PAYMENT

Sometimes the balloon payment is due and payable one period after the last periodic payment in the contract maturity. If this is encountered, the present worth of the mortgage loan can be calculated simply by calculating the present worth of the balloon payment for n + 1 periods, and adding it to the present worth of the periodic payments for n periods. With this minor adjustment, the same routine as that illustrated in the preceding section is used.

Example - Suppose the mortgage loan in the preceding section called for balloon payment to be made one month after the last monthly payment to be made one month after the last monthly payment of $483.20. What is the present worth or market value of that mortgage loan at an 9.50% yield?

Enter:

CLEAR 12 STO 0 20 × n 9.5 [shift]

÷ i 483.20 PMT PV STO 0

CLEAR RCL n 1 + n RCL i i

23507.58 FV PV RCL 0 + ◊

```
                    CLEAR

          12.00       →0
          20.00        ×
         240.00        ◊
                      → N
           9.50       N ÷
            .79        ◊
                      → I
         483.20       →PT
  P   V
      51838.20
                      →0
                    CLEAR

         240.00       ← N
           1.00        +
                      → N
            .79       ← I
                      → I
       23507.58       → F
  P   V
        3514.58
                       ↑
       51838.20       ←0
                       +
       55352.77        ◊
```

The market value of the mortgage is $55,352.77, as compared with $53,524.37 if the balloon payment is payable with the last periodic payment.

EFFECTIVE INTEREST RATE - NO FEES

This routine calculates the effective interest rate or yield to the mortgage lender if a balloon payment mortgage loan is held to maturity, or paid off at par in advance of maturity. It requires setting the Bond Mode and Annual Coupon Mode (see Appendix and HP-81 Operating Guide for details) and then following the required steps in specified sequence.

The balloon payment is entered as an amount separate and distinct from the last periodic payment.

The routine is a Bond Yield-to-Call routine which produces effective interest rate or yield to maturity per payment period. An annual effective rate is produced by multiplying the interest rate per period by the number of payments per year.

The keystrokes are:

1. Set Bond Mode by pressing [shift] EXT( ) 8 .
2. Set Annual Coupon Mode by pressing [shift] EXT( ) 2

3. Key in amount of balloon payment, press [FV] .

4. Key in number of payments, press [SAVE↑] 365 [×] [STO] [DAY]

5. Key in monthly payment amount, press [PMT] .

6. Key in amount of mortgage principal, press [PV] .

7. Press [░] [i] . (This calculates and prints interest rate per period.)

NOTE:
This is a time-consuming calculation. Do not press any keys until it is finished.

8. Key in the number of payments per year, press [×] . (This calculates and prints the effective interest rate on an annual basis.)

Example - A $75,500 mortgage with interest at 7.5%has 12 years 7 months remaining on its contract maturity, at which time a balloon payment of $47,003.37 is due. Monthly payments are $557.94.

An investor has just been offered the opportunity to purchase this mortgage loan for $60,000. Should the investor accept, assuming the minimum acceptable yield to maturity is 10%?

Enter:

[CLEAR] [░] [EXT( )] [8] [░]

[EXT( )] [2] **47003.37** [FV] **12** [STO] [0] **12**

[×] **7** [+] [SAVE↑] **365** [×] [STO] [DAY]

557.94 [PMT] **60000** [PV] [░] [i] [░] [×]

```
               CLEAR
BON                8
                 AN2
  47003.37      → F
     12.00      →0
     12.00        ×
    144.00      ◇
      7.00        +
                  ↑
    365.00        ×
  55115.00      ◇
                →4*
    557.94      →PT
  60000.00      →P
YIELD
TO           CALL
       .86
                N ×
     10.31      ◇
```

Yes, the indicated yield to maturity or effective interest rate is 10.31%, which more than meets the minimum standard of acceptability.

EFFECTIVE INTEREST RATE - FEES OR DISCOUNT CHARGED

In this situation, the only difference is that the amount entered as [PV] is the principal of the mortgage less the amount of fees or discount "points" charged the borrower. The borrower receives less than the face amount of the mortgage, but repays that amount. The earlier the mortgage is paid off, the higher the effective interest rate or yield to the lender.

The routine is the same as that illustrated in the preceding section with the one exception of the value entered as [PV] .

Example - A $60,000 mortgage loan has a contract maturity of 20 years. Monthly payments are $483.20. The balloon payment is $23,507.58. The borrower is charged 5 points on the loan. What is the effective yield to the lender if the loan is held to maturity?

Enter:

[CLEAR] [ ] [EXT( )] [8] [ ] [EXT( )]

[2] **23507.58** [FV] **12** [STO] [0] **20** [×]

[SAVE↑] **365** [×] [STO] [DAY] **483.20** [PMT]

60000 [SAVE↑] **5** [%] [−] [PV] [ ] [i]

[ ] [×]

```
                    CLEAR
BON                     8
                      AN2
        23507.58      → F
           12.00      →0
           20.00        ×
          240.00        ◇
                        ↑
          365.00        ×
        87600.00        ◇
                      →4*
          483.20      →PT
        60000.00        ↑
            5.00        %
         3000.00        -
                       →P
YIELD
TO             CALL
             .76
                      N ×
            9.13        ◇
```

9.13% (This compares with the 8.50% nominal or contract rate illustrated previously for these mortgage loan terms without the discount.)

ACCUMULATED INTEREST PAID AND REMAINING BALANCE

The routine for a balloon payment mortgage to calculate total interest paid during a specified number of time periods and the balance remaining at the end of the last period is precisely the same as that for simple mortgages. The fact of the balloon payment makes no difference. However, since the balloon payment amount exactly pays off the loan entirely, it is not necessary first to calculate the exact number of payment periods.

Example - A $60,000 mortgage loan has a contract maturity of 20 years. Its monthly payments are $483.20 at 8.5% interest. The balloon payment in 20 years is $23,507.58. What is the accumulated interest paid in 10 years, and what is the remaining balance at the end of 10 years?

Enter: [CLEAR]

1 [STO] [1] **120** [STO] [2] **8.50** [SAVE ↑]

12 [÷] [i] **483.20** [PMT] **60000** [PV] [Σ+]

Remaining balance - $49,050.32
Accumulated interest paid - $47,034.32

```
             CLEAR

       1.00     →1
     120.00     →2
       8.50      ↑
      12.00      ÷
        .71    ◇
               → I
     483.20    →PT
   60000.00    →P
                Σ+
P    V
   49050.32
I N T
   47034.32
```

AMOUNT OF BALLOON PAYMENT

To calculate the amount of the balloon payment, the same routine is used as that for remaining balance at a specified period for simple mortgages. This routine also indicates what the full maturity term of the mortgage would be in the absence of partial amortization and a balloon payment. Since balloon payment loans are often expressed in terms of their full as well as contract maturity (for example, a 20-year loan with a 15-year maturity), this is a useful calculation.

Example 1 - A $60,000 mortgage loan has a contract maturity of 20 years. Monthly payments are $483.20 at 8.5% interest. What is the amount of the balloon payment? What would be the full amortization term?

Enter: [CLEAR]

8.5 [SAVE ↑] **12** [÷] [i] **483.20** [PMT]

60000 [PV] [n]

The full amortization term is 299.86 months.

20 [SAVE ↑] **12** [×] [−] [n] [CLEAR] [RCL]

[n] [n] [RCL] [i] [i] [RCL] [PMT] [PMT]

[PV]

The balloon payment is $23,508.66.

```
             CLEAR

       8.50      ↑
      12.00      ÷
        .71    ◇
               → I
     483.20    →PT
   60000.00    →P
N
     299.86
      20.00      ↑
      12.00      ×
     240.00    ◇
                 −
               → N
```

```
        CLEAR

   59.86    ← N
            → N
     .71    ← I
            → I
  483.20    ←PT
            →PT
P   V
23508.66
```

This routine can also be used to calculate the amount and percentage of mortgage paid off (p in Mortgage-Equity or Ellwood analysis) as well as the remaining balance.

Example 2 - Using the mortgage loan terms from Example 1, what are the remaining mortgage balance, the amount of mortgage paid off, and the percentage of mortgage paid off at the end of 10 years?

Enter: [CLEAR]

8.50 [SAVE ↑] **12** [÷] [i] **483.20** [PMT]

60000 [PV] [n] **120** [−] [n] [CLEAR]

[RCL] [n] [n] [RCL] [i] [i] [RCL]

[PMT] [PMT] [PV]

Remaining balance - $49,050.34

[RCL] [PV] [$x \rightleftarrows y$] [−] [◊]

$10,949.66 paid off

[RCL] [PV] [÷]

18% paid off (p = .18)

```
        CLEAR

    8.50      ↑
   12.00      ÷
     .71      ◊
            → I
  483.20    →PT
60000.00    →P
N
  299.86
  120.00      −
            → N

        CLEAR

  179.86    ← N
            → N
     .71    ← I
            → I
  483.20    ←PT
            →PT
P   V
49050.34
60000.00    ←P
49050.34      ⇄
              −
10949.66      ◊

60000.00    ←P
              ÷
     .18      ◊
```

The percentage of mortgage balance remaining and the percentage of mortgage paid off can also be calculated without reference to dollar amounts.

Example 3 - A mortgage loan at 8.5% interest will be fully amortized in monthly payments over 25 years. What is the percentage of the loan remaining as a balance (b) in 10 years, and what is the percentage of the loan paid off (p)?

Enter: [CLEAR]

[ ] [6] **12** [STO] [0] **25** [×] [n] **8.5**

[ ] [÷] [i] **1** [PV] [PMT] [STO] [PMT]

Monthly payment factor - .008052

[CLEAR] [RCL] [n] **10** [ ] [×] [−] [n]

[RCL] [i] [i] [RCL] [PMT] [PMT] [PV]

b = .817706

1 [$x \rightleftarrows y$] [−] [◊]

p = .182294

```
            CLEAR

12.000000    →0
25.000000     ×
300.000000   ◊
            → N
8.500000    N ÷
.708333      ◊
            → I
1.000000    →P
            PMT
.008052
            →7*
            CLEAR

300.000000  ← N
10.000000   N ×
120.000000   ◊
              −
            → N
.708333     ← I
            → I
.008052     ←PT
            →PT
P  V
.817706
1.000000
.817706       ⇄
              −
.182294      ◊
```

NOTE:

The routines for balloon payment loans to calculate the Mortgage Amortization Schedule and Schedule of Annual Interest and Principal Payments are exactly the same as those for simple mortgages.

In addition, there is no point to illustrating a routine for Last Payment Amount for balloon mortgages. If the balloon payment is payable in the last payment period, the Last Payment Amount is the sum of the monthly payment plus the balloon payment. If the balloon payment is payable one period after the last payment period, then the balloon payment is the Last Payment Amount.

In addition, there is no point to illustrating a routine for Last Payment Amount for balloon mortgages. If the balloon payment is payable in the last payment period, the Last Payment Amount is the sum of the monthly payment plus the balloon payment. If the balloon payment is payable one period after the last payment period, then the balloon payment is the Last Payment Amount.

CHAPTER 6: MORTGAGES WITH EQUAL PRINCIPAL PAYMENTS

Sometimes mortgage loans call for equal amounts of payment of principal (a constant amount paid toward principal in each payment principal), plus interest each period on the outstanding balance. Each periodic payment declines by the amount of interest reduced from the previous period's repayment of principal.

This type of loan is typically used for financing raw acreage purchases or land for development. They are usually relatively short-term loans, with annual or semiannual payments.

PAYMENT SCHEDULE

This routine produces a schedule of periodic total payments, and also permits the analyst to obtain the total payment amount for any specified payment number. Required information includes: principal amount, maturity in years, payments per year, and annual interest rate.

The keystrokes are:

1. Press [CLEAR].
2. Key in principal amount, press [SAVE↑]; key in number of years to maturity, press [SAVE↑]; key in number of payments per year, press [×] [÷] [STO] [0] [SAVE↑]
3. Key in principal amount, press [+]
4. Key in annual interest rate, press [SAVE↑]; key in number of payments per year, press [÷] [STO] [1] [%] .
5. Press [RCL] [0] [+] [STO] [·] [3] .
6. Press [RCL] [0] [RCL] [1] [%] [CHS] [STO] [·] [0] [SAVE↑] [SAVE↑].
7. Key in 1, press [n].
8. Press [TL] to obtain total payment for Period 1. Continue pressing [TL] for each successive total payment.

Example - An investor has just purchased 10 acres of land for development for a price of $100,000. He has obtained a $60,000 land loan with a maturity of 6 years. Principal repayment is in equal semi-annual installments with interest at 10% on the remaining balance. What is the schedule of total payments (principal and interest)?

Enter: [CLEAR]

60000 [SAVE ↑] **6** [SAVE ↑] **2** [×] [÷] [STO]

[0] [SAVE ↑] **60000** [+] **10** [SAVE ↑] [2] [÷]

[STO] [1] [%] [RCL] [0] [+] [STO] [·] [3]

[RCL] [0] [RCL] [1] [%] [CHS] [STO] [·]

[0] **Interest Decline** [SAVE ↑] [SAVE ↑] **1** [n]

[TL]	**$8000**	**Payment 1**
[TL]	**$7750**	**Payment 2**
[TL]	**$7500**	**Payment 3**
[TL]	**$7250**	**Payment 4**
[TL]	**$7000**	**Payment 5**
[TL]	**$6750**	**Payment 6**
[TL]	**$6500**	**Payment 7**
[TL]	**$6250**	**Payment 8**
[TL]	**$6000**	**Payment 9**
[TL]	**$5750**	**Payment 10**
[TL]	**$5500**	**Payment 11**
[TL]	**$5250**	**Payment 12**

```
                   CLEAR
     60000.00        ↑
         6.00        ↑
         2.00        ×
        12.00     ◇
                     ÷
      5000.00     ◇
                  →0
                     ↑
     60000.00        +
        10.00        ↑
         2.00        ÷
         5.00     ◇
                  →1
                     %
      3250.00        ↑
      5000.00     ←0
                     +
                  →3*
                     ↑
      5000.00     ←0
                     ↑
         5.00     ←1
                     %
-      250.00     →0*
                     ↑
                     ↑
         1.00     →  N
y
      8000.00
y
      7750.00
y
      7500.00
y
      7250.00
y
      7000.00
y
      6750.00
y
      6500.00
y
      6250.00
y
      6000.00
y
      5750.00
y
      5500.00
y
      5250.00
```

The payment for any period can be obtained by keying in the payment number, then pressing [n] [TL].

Example - To find the amount of payment number 12, in the preceding example,

Enter:

12 [n] [TL]

```
        12.00   → N
y
      5250.00
```

LOAN REDUCTION SCHEDULE

This routine calculates and prints the total payment, principal reduction payment, amount of interest and remaining balance for each successive mortgage payment period.

The keystrokes are:

1. Press [CLEAR].
2. Key in principal amount, press [SAVE↑]; key in loan maturity in years, press [SAVE↑]; key in number of payments per year, press [×] [÷] [STO] [0]. (This calculates, prints and stores the periodic payment of principal.)
3. Key in annual interest rate, press [SAVE↑]; key in number of payments per year, press [÷] [SAVE↑] [SAVE↑], key in principal amount.
4. Press [$x \rightleftarrows y$] [%] [◊].
5. Press [RCL] [0] [+] [◊]. (This calculates and prints the total payment for the period.
6. Press [R↓] [RCL] [0] [−] [◊]. (This calculates and prints the principal balance remaining at the end of the period.
7. Repeat Steps 4-6 for each successive period.

Example - A $60,000 land loan at 10% interest calls for equal principal payments semi-annually over a 6-year maturity. What is the loan reduction schedule for the first two years?

Enter:

[CLEAR] 60000 [SAVE↑] 6 [SAVE↑] 2 [×]

[÷] [STO] [0] 10 [SAVE↑] 2 [÷] [SAVE↑]

[SAVE↑] 60000 [$x \rightleftarrows y$]

[%] **Interest Payment 1**

[RCL] [0] **Principal Payment 1**

[+] [◊] **Total Payment 1**

[R↓] [RCL] [0] [−] [◊] **Balance 1st 6 Months**

[$x \rightleftarrows y$] [%] **Interest Payment 2**

[RCL] [0] **Principal Payment 2**

[+] [◊] **Total Payment 2**

[R↓] [RCL] [0] [−] [◊] **Balance 1st Year**

[$x \rightleftarrows y$] [%] **Interest Payment 3**

[RCL] [0] **Principal Payment 3**

[+] [◊] **Total Payment 3**

[R↓] [RCL] [0] [−] [◊] **Balance 1 Year 6 Months**

[$x \rightleftarrows y$] [%] **Interest Payment 4**

[RCL] [0] **Principal Payment 4**

[+] [◊] **Total Payment 4**

[R↓] [RCL] [0] [−] [◊] **Balance 2nd Year**

```
              CLEAR
   60000.00       ↑
       6.00       ↑
       2.00       ×
      12.00     ◇
                  ÷
    5000.00     ◇
                →0
      10.00       ↑
       2.00       ÷
       5.00     ◇
                  ↑
                  ↑
   60000.00
       5.00       ⇄
                  %
    3000.00       ↑
    5000.00     ←0
                  +
    8000.00     ◇
   60000.00       ↓
                  ↑
    5000.00     ←0
                  −
   55000.00     ◇
       5.00       ⇄
                  %
    2750.00       ↑
    5000.00     ←0
                  +
    7750.00     ◇
   55000.00       ↓
                  ↑
    5000.00     ←0
                  −
   50000.00     ◇
       5.00       ⇄
                  %
    2500.00       ↑
    5000.00     ←0
                  +
    7500.00     ◇
   50000.00       ↓
                  ↑
    5000.00     ←0
                  −
   45000.00     ◇
       5.00       ⇄
                  %
    2250.00       ↑
    5000.00     ←0
```

```
                 +
   7250.00    ◇

  45000.00       ↓
                 ↑
   5000.00    ←0
                 -
  40000.00    ◇
```

CHAPTER 7: CANADIAN MORTGAGES

In Canada, interest is compounded semi-annually with payments made monthly. This results in a different monthly mortgage factor than is used in the United States, and is programmed into the HP-81. This difference can be handled readily on the HP-81 by converting to the Canadian mortgage factor. Then for any calculation requiring an input for [i], the Canadian mortgage factor is calculated first. This value is entered for [i].

The keystrokes to calculate the Canadian mortgage factor are:

1. Press [CLEAR].
2. Key in 6, press [n]; key in 1, press [PV].
3. Key in the annual interest rate, press [SAVE↑].
4. Key in 200, press [÷]; key in 1, press [+] [FV] [i].
5. Press [STO] [i].

The Canadian mortgage factor is now stored in [i] (Register 16) for future use.

With the adjustment completed and the Canadian mortgage factor stored, it is possible to apply the routines illustrated in Chapters 4 and 5. The following are examples of these applications.

Example 1 - Periodic Payment Amount
What is the monthly payment required to amortize fully a 25-year mortgage loan of $75,500 at 8.75% interest.

Enter: [CLEAR]

6 [n] 1 [PV] 8.75 [SAVE↑] 200 [÷]

1 [+] [FV] [i] (The Canadian mortgage

factor is .72) [STO] [i] [CLEAR] 25 [SAVE↑]

12 [×] [n] [RCL] [i] [i] 75500 [PV] [PMT]

Monthly Payment - $612.77

```
              CLEAR

        6.00     → N
        1.00     →P
        8.75       ↑
      200.00       ÷
         .04     ◇
        1.00       +
                 → F
I N T
         .72
                 →6*
              CLEAR

       25.00       ↑
       12.00       ×
      300.00     ◇
                 → N
         .72     ← I
```

```
                    → I
      75500.00      →P
                    PMT
        612.77
```

Example 2 - Balance Remaining at End of Specified Period

A Canadian mortgage has monthly payments of $612.77 at 8.75% interest. The principal amount is $75,500. What will be the outstanding balance remaining at the end of 10 years?

Enter: CLEAR

6 n **1** PV **8.75** SAVE↑ **200** ÷

1 + FV i STO i

(The Canadian mortgage factor is .72.)

CLEAR

RCL i i **75500** PV **612.77** PMT n

(The exact number of payments to full amortization is 300.)

10 SAVE↑ **12** × − STO n

CLEAR

RCL n n RCL i i RCL

PMT PMT PV

The remaining balance outstanding at the end of 10 years is $61,877.19

```
                 CLEAR

          6.00      → N
          1.00      →P
          8.75      ↑
        200.00      ÷
           .04      ◇
          1.00      +
                    → F
 I N T
           .72
                    →6*
                 CLEAR

           .72      ← I
                    → I
      75500.00      →P
        612.77      →PT
   N
        300.00
         10.00      ↑
         12.00      ×
        120.00      ◇
                    −
                    →9*
                 CLEAR

        180.00      ← N
                    → N
           .72      ← I
                    → I
        612.77      ←PT
                    →PT
  P   V
      61877.19
```

Example 3 - Effective Interest Rate (Yield)

A Canadian mortgage has monthly payments of $612.77 with a maturity of 25 years. The principal amount is $75,500. What is the annual interest rate?

Enter: CLEAR

12 SAVE↑ **25** × n **612.77** PMT

75500 PV i

(The Canadian mortgage factor is .72.)

STO i CLEAR

RCL i i **6** n **1** PV FV

1 − **200** ×

The annual interest rate is 8.75%

```
                 CLEAR
           12.00       ↑
           25.00       ×
          300.00     ◇
                     → N
          612.77     →PT
        75500.00     →P
I N T
             .72
                     →6*
                 CLEAR
             .72     ← I
                     → I
            6.00     → N
            1.00     →P
  F V
            1.04
            1.00       −
          200.00       ×
            8.75     ◇
```

CHAPTER 8:
PRESENT WORTH (PRESENT VALUE) ESTIMATES

Many real estate problems require an estimate of Present Worth or Value of a particular investment position in the real estate. Present Worth estimates for mortgages (both Simple and Balloon Payment loans) are covered in Chapters 4 and 5. The routines presented here focus on calculating or estimating Present Worth of the total property and/or the equity investment position in the property. That Present Worth figure represents Market Value if market data are used and the other conditions of Market Value estimation are met. Otherwise, it represents Investment Value.

The procedures and routines for estimating Present Worth presented here are all applications of Income Capitalization Analysis, the so-called "Income Approach" to value estimation. Most of the routines have already been explained in Chapters 2, 4 and 5, so only their applications to Present Worth estimation for total property investment or equity investment position are covered here. Only the Net Present Value and Variable Annuity (Discounted Cash Flow) routines are explained in detail.

Particular emphasis is placed on Present Worth estimates for income streams at the beginning of the period (payments in advance) because, unlike mortgage payments covered in Chapters 4 - 7, rental or lease payments are frequently receivable at the beginning of the period.

PRESENT WORTH OF LEVEL ANNUITY (END-OF-PERIOD PAYMENTS)

This is the basic Present Worth of One Per Period or Level Annuity described in Chapter 2, and illustrated in Chapters 4 and 5. It calculates the Present Worth of a stream of equal payments, equally spaced in time, receivable at the end of each period for a specified number of periods, discounted at a specified rate per period.

Example - An investor has the opportunity to purchase an investment property which is forecast to produce Net Operating Income of $21,750 annually for 10 years. Annual Debt Service is $14,653. What is the Present Worth of the Cash Throw-Off to Equity if the investor is seeking a 14% Equity Yield Rate (rate of return on the equity investment)?

Enter: [CLEAR]

10 [n] **14** [i] **21750** [SAVE↑] **14653** [−] [◊]

[PMT] **(This prints annual CTO.)** [PV]

Present Worth - $37,018.77

```
           CLEAR
      10.00   → N
      14.00   → I
   21750.00   ↑
   14653.00   -
    7097.00   ◊
            →PT
P   V
   37018.77
```

PRESENT WORTH OF LEVEL ANNUITY PLUS REVERSION (END-OF-PERIOD PAYMENTS)

This procedure combines the Present Worth of One per Period and Present Worth of One routines from Chapter 2. The Present Worth of the income stream of equal end-of-period payments is added to the Present Worth of the forecast reversion at the end of the Income Projection Period (n).

Example - A property is forecast to produce NOI of $21,750 annually for 10 years, with Annual Debt Service of $14,653. The property is expected to resell for $230,000 in 10 years. Sales commission and other disposition expenses paid by the seller in 10 years are estimated to be 7% of sales price. The mortgage balance in 10 years will be $122,175. If a purchaser can assume the existing mortgage, how much should he pay in cash for the equity investment position so as to reach 14% on that investment?

Enter: [CLEAR]

10 [n] 14 [i] 21750 [SAVE ↑] 14653 [−]

[PMT] [PV] [STO] [1]

(This stores the $37,018.77 Present Worth of the 10 year stream of CTO'

[CLEAR] [RCL] [n] [n] [RCL] [i] [i]

230000 [SAVE ↑] 7 [%] [−] [◊]

(This prints out Proceeds of Resale.)

122175 [−] [◊] (This prints out Net Cash Proceeds of Resale.) [FV] [PV] (PW of NCPR is $24,742.25.) [RCL] [1] [+] [◊]

```
        CLEAR
     10.00   → N
     14.00   → I
  21750.00   ↑
  14653.00   −
            →PT
P V
  37018.77
             →1
        CLEAR
     10.00   ← N
             → N
     14.00   ← I
             → I
 230000.00   ↑
      7.00   %
  16100.00   −
 213900.00   ◇
 122175.00   −
  91725.00   ◇
             → F
P V
  24742.25
             ↑
  37018.77   ←1
             +
  61761.02   ◇
```

Present Worth of the equity investment position is $61,761.02. He should pay no more than this.

Alternatively, the Net Present Value Routine discussed in subsequent sections of Chapter 8 can be used, inserting 0 for PV' The steps and keystrokes are:

1. Press [CLEAR].

2. Key in amount of last years income payment, press [SAVE ↑]; key in amount of reversion, press [+] [STO] [9] .

This calculates and stores the amount of the last year's receipts: income plus payment.

3. Press [CLEAR].

4. Key in rate of discount per period, press [i] .

5. Key in 0, press [PV] .

6. Key in annual cash flow, press [STO] [1] [Σ+] .

7. For each successive year except the last, press [RCL] [1] [Σ+] Continue until the last period (year).

8. For the last year, press [RCL] [9] [Σ+] .

NOTE:

To find how many cash flow entries have been made at any stage during the calculation, press [R↓] [R↓]. *Then press* [R↓] [R↓] *before continuing the calculation further.*

Example - Using the figures from the preceding example, what is the present worth of the equity investment position?

Enter: [CLEAR]

21750 [SAVE ↑] **14653** [−] [◊] [STO] [1]

CTO = $7,097

230000 [SAVE ↑] **7** [%] [−] **122175** [−] [◊]

NCPR = $91,725

[RCL] [1] [+] [◊] [STO] [9]

Year 10 Cash Flow = $98,822

```
            CLEAR
      21750.00     ↑
      14653.00     −
       7097.00     ◊
                  →1
     230000.00     ↑
          7.00     %
      16100.00     −
     122175.00     −
      91725.00     ◊
                   ↑
       7097.00    →1
                   +
      98822.00     ◊
                  →9
```

CLEAR

14 i **0** PV RCL 1

Σ+ RCL 1 Σ+ RCL

1 Σ+ RCL 1 Σ+

RCL 1 Σ+ RCL 1

Σ+ RCL 1 Σ+ RCL

1 Σ+ RCL 1 Σ+

RCL 9 Σ+

PW = $61,761.02

This is the same figure as that derived with the first method.

R↓ R↓ **= 10 entries**

```
         CLEAR

   14.00     → I
     .00     →P
               ↑
 7097.00     ←1
 7097.00     Σ+
 6225.44     ◇
               ↑
 7097.00     ←1
 7097.00     Σ+
11686.35     ◇
               ↑
 7097.00     ←1
 7097.00     Σ+
16476.62     ◇
               ↑
 7097.00     ←1
 7097.00     Σ+
20678.62     ◇
               ↑
 7097.00     ←1
 7097.00     Σ+
24364.58     ◇
               ↑
 7097.00     ←1
 7097.00     Σ+
27597.87     ◇
               ↑
 7097.00     ←1
 7097.00     Σ+
30434.10     ◇
               ↑
 7097.00     ←1
 7097.00     Σ+
32922.02     ◇
               ↑
 7097.00     ←1
 7097.00     Σ+
35104.40     ◇
               ↑
98822.00     ←9
98822.00     Σ+
61761.02     ◇
   14.00       ↓
   10.00       ↓
```

PRESENT WORTH OF VARIABLE ANNUITY (END-OF-PERIOD PAYMENTS)

Since not all income streams have the characteristics of a level annuity, the general method to calculate the Present Worth of any series of payments is to treat each payment as a separate lump-sum reversion, discount it via the appropriate Present Worth of One Factor, and sum the resulting series of individual Present Worth estimates.

The procedure to accomplish this is the Discounted Cash Flow or Net Present Value routine in the HP-81 Operating Guide, modified so that 0 is entered in [PV] as the Capital Outlay. (Interested readers may wish to check this routine by applying the figures used in the Present Worth of a Level Annuity example provided above.)

NOTE:

If there is a lump-sum reversion receivable at the end of the final payment period in addition to the final payment of the variable-annuity income stream, the amounts of the reversion and the final payment must first be summed and keyed in as the final payment amount in this routine.

The keystrokes are:

1. Press [CLEAR].

2. Key in rate of discount per period, press [i].

3. Key in 0, press [PV].

4. Key in amount of first payment, press [Σ+]. Repeat Step 4 for each successive payment.

The figure printed after pressing [Σ+] following the entry of the last payment amount is the Present Worth of the variable annuity (or cash flow.)

Example 1 - An investment property is forecast to produce the following CTO stream, plus reversion:

Year	End-of-Year CTO
1	$10,000
2	$9,500
3	$9,000
4	$8,500
5	$8,000
6	$7,500
7	$7,000
8	$6,500
9	$6,000
10	$5,000
Reversion (Year 10)	$125,000

The total payment at the end of Year 10 is thus $130,000. What is the present worth of this equity investment at a 15% yield?

Enter: CLEAR

125000 SAVE ↑ **5000** + STO 1

(This calculates and stores the full amount of the final payment.)

CLEAR **15** i **0** PV **10000** Σ+ **9500** Σ+ **9000** Σ+ **8500** Σ+ **8000** Σ+ **7500** Σ+ **7000** Σ+ **6500** Σ+ **6000** Σ+ RCL 1 Σ+

The Present Worth is $72,472.44.

```
                CLEAR
125000.00           ↑
  5000.00           +
                   →1
                CLEAR
    15.00         → I
      .00          →P
 10000.00          Σ+
  8695.65          ◇
  9500.00          Σ+
 15879.02          ◇
  9000.00          Σ+
 21796.66          ◇
  8500.00          Σ+
 26656.57          ◇
  8000.00          Σ+
 30633.98          ◇
  7500.00          Σ+
 33876.44          ◇
  7000.00          Σ+
 36508.00          ◇
  6500.00          Σ+
 38632.86          ◇
  6000.00          Σ+
 40338.43          ◇
                    ↑
130000.00          →1
130000.00          Σ+
 72472.44          ◇
```

ALTERNATIVE METHOD - REVERSE ENTRY

An alternative method to calculate the Present Worth of a variable annuity is to enter the payments in reverse order, divide by the Future Worth of One (One plus the interest rate per period), add the next preceding payment, divide by the Future Worth of One, and continue for the entire number of payments. This method was developed for use on simple calculators by Mr. L. W. Ellwood. It has no advantage over the foregoing HP-81 routine, except that many practitioners are familiar with its use. It produces the same results as those provided above.

The keystrokes are:

1. Press CLEAR.
2. Key in 1, press SAVE ↑ ; key in interest rate per period, press % + STO 0 (This calculates and stores 1 + i.)
3. Key in last payment amount, press ÷ .

4. Key in next preceding payment amount, press [+] [ ] [÷].

   Repeat Step 4 for each successive preceding payment amount.

   Final printed figure is the Present Worth of the variable annuity.

Example 2 - Using the same income and interest rate figures presented in Example 1 above, what is the Present Worth of the equity investment position?

Enter: [CLEAR]

1 [SAVE ↑] **15** [%] [+] [STO] [0]

125000 [SAVE ↑]

5000 [+] [ ] [÷]

6000 [+] [ ] [÷]

6500 [+] [ ] [÷]

7000 [+] [ ] [÷]

7500 [+] [ ] [÷]

8000 [+] [ ] [÷]

8500 [+] [ ] [÷]

9000 [+] [ ] [÷]

9500 [+] [ ] [÷]

10000 [+] [ ] [÷]

CLEAR	
1.00	↑
15.00	%
.15	+
	→0
125000.00	↑
5000.00	+
	N ÷
113043.48	◇
6000.00	+
	N ÷
103516.07	◇
6500.00	+
	N ÷
95666.15	◇
7000.00	+
	N ÷
89274.91	◇
7500.00	+
	N ÷
84152.10	◇
8000.00	+
	N ÷
80132.26	◇
8500.00	+
	N ÷
77071.53	◇
9000.00	+
	N ÷
74844.81	◇
9500.00	+
	N ÷
73343.31	◇
10000.00	+
	N ÷
72472.44	◇

The Present Worth is $72,472.44. This is precisely the same as the figure derived with the NPV routine.

NEGATIVE PAYMENT AMOUNTS

It frequently happens that in a forecast, future income stream or cash flow there are negative amounts (net cash out flows) in one or more periods after a positive payment(s) is received. The two foregoing routines can accommodate this phenomenon easily, simply by keying in the amount and pressing [CHS].

Example - A property is forecast to produce the following stream of NOI payments at the end of each year:

Year	NOI
1	$10,000
2	$ 5,000
3	- $ 2,500
4	$ 1,500
5	- $ 3,000
6	$60,000

What is its Present Worth if the Discount Rate (r) is 12.75%?

a. NPV Method

[CLEAR] **12.75** [i] **0** [PV] **10000** [Σ+]

5000 [Σ+] **2500** [CHS] [Σ+] **1500** [Σ+]

3000 [CHS] [Σ+] **60000** [Σ+]

Present Worth - $39,544.52

```
              CLEAR

          12.75    → I
            .00    →P
       10000.00     Σ+
        8869.16    ◇
        5000.00     Σ+
       12802.30    ◇
-       2500.00     Σ+
       11058.12    ◇
        1500.00     Σ+
       11986.29    ◇
-       3000.00     Σ+
       10339.87    ◇
       60000.00     Σ+
       39544.52    ◇
```

b. Alternative Method

[CLEAR] **1** [SAVE ↑] **12.75** [%] [+]

[STO] [0] **60000** [ ] [÷] **3000** [−]

[ ] [÷] **1500** [+] [ ] [÷] **2500**

[−] [ ] [÷] **5000** [+] [ ] [÷]

```
              CLEAR

           1.00        ↑
          12.75        %
            .13        +
                     →0
       60000.00    N ÷
       53215.08    ◇
        3000.00        -
                   N ÷
       44536.65    ◇
        1500.00        +
                   N ÷
```

10000 [+] [▒] [÷]

Present Worth - $39,544.52

```
40830.74    ◊
 2500.00    -
          N ÷
33996.22    ◊
 5000.00    +
          N ÷
34586.45    ◊
10000.00    +
          N ÷
39544.52    ◊
```

PRESENT WORTH OF LEVEL ANNUITY (BEGINNING OF PERIOD)

An annuity with payments in advance (beginning of period) is also called an Annuity Due. As explained in Chapter 2, the Present Worth of a level annuity with payments at the beginning of each period is obtained by multiplying either the Present Worth for end-of-period payments or the end-of-period payment by the Base Factor. The Base Factor is One plus the interest rate per period (1 + i).

Example - A 6-year lease calls for rental payments of $1475 per month in advance. The Market Rental is $2000 per month. What is the Present Worth of the leasehold if the annual rate of discount is 20%? (The monthly rental advantage to the lessee is $ 525.)

a. Base Factor times PW for end-of-month payments.

Enter: [CLEAR]

12 [STO] [0] 6 [×] [n] 20 [▒] [÷] [i]

525 [PMT] [PV] [RCL] [i] [%] [+] [◊]

Present Worth - $22,283.38

```
        CLEAR

   12.00    →0
    6.00     ×
   72.00    ◊
           → N
   20.00   N ÷
    1.67    ◊
           → I
  525.00   →PT
P V
21918.08
    1.67   ← I
             %
  365.30     +
22283.38    ◊
```

b. Base Factor times end-of-month payment amount.

Enter: [CLEAR]

12 [STO] [0] 6 [×] [n] 20 [▒] [÷] [i]

525 [RCL] [i] [%] [+] [◊] [PMT] [PV]

```
        CLEAR

   12.00    →0
    6.00     ×
   72.00    ◊
           → N
   20.00   N ÷
    1.67    ◊
           → I
```

Present Worth - $22,283.38

```
          1.67    ← I
                    x
          8.75      +
        533.75    ◇

                  →PT
P    V
    22283.38
```

PRESENT WORTH OF LEVEL ANNUITY WITH BEGINNING-OF-PERIOD PAYMENTS PLUS REVERSION

A reversion is still receivable at the end of the income projection period, so the procedure here is to calculate the Present Worth of the level annuity with beginning-of-period payments, and add it to the Present Worth of the reversion.

Example - What is the Present Worth of the leased fee (lessor's interest) in a rental property with an absolutely net lease calling for rental payments of $1,475 per month for 6 years, if monthly payments are payable in advance, the property reversion is forecast to be $120,000, and the annual rate of discount for the leased fee interest is 11.75%?

Enter: [CLEAR]

12 [STO] [0] **6** [×] [n] **11.75** [▒] [÷]

[i] **1475** [PMT] [PV] [RCL] [i] [%] [+]

[◇] [STO] [1]

(This calculates, prints and stores the PW of the rental payments.)

[CLEAR]

[RCL] [n] [n] [RCL] [i] [i] **120000**

[FV] [PV] **(This calculates and stores the PW of the reversion.)**

[RCL] [1] [+] [◇]

PW of the leased fee is $136,191.34.

```
                  CLEAR
         12.00    →0
          6.00      ×
         72.00    ◇
                  → N
         11.75    N ÷
           .98    ◇
                  → I
       1475.00    →PT
P    V
     75950.94
           .98    ← I
                    %
        743.69      +
      76694.63    ◇
                  →1
                  CLEAR
         72.00    ← N
                  → N
           .98    ← I
                  → I
     120000.00    → F
P    V
     59496.71
                    ↑
      76694.63    ←1
                    +
     136191.34    ◇
```

PRESENT WORTH OF A VARIABLE ANNUITY WITH BEGINNING-OF-PERIOD PAYMENTS

To convert a variable annuity (annuity due) to beginning-of-payments, and then to find its Present Worth, three possible methods are available to the analyst:

a. Using the Net Present Value (Discounted Cash Flow) routine, key Payment Number One into PV and then continue the routine as before;

b. Using the NPV routine, key in 0 to PV, calculate present worth for end-of-period payments, and then multiply by the Base Factor;

c. Using the Alternative Method (reverse entry of cash flows), do not press ÷ after Payment Number One has been added. Instead, press ◊ to print out Present Worth.

Example - What is the Present Worth of the following cash flow stream (variable annuity) discounted at 15%, if payments are receivable at the beginning of each period?

Year	BOP Payment
1	$8,652
2	$7,677
3	$6,593
4	$5,321
5	$4,108
6	$5,623
7	$6,172
Reversion (End of Year 7)	$35,650

a. Enter: CLEAR

15 i 8652 PV 7677 Σ+ 6593 Σ+

5321 Σ+ 4108 Σ+ 5623 Σ+ 6172 Σ+

35650 Σ+

Present Worth - $45,026.42.

```
           CLEAR
   15.00        → I
 8652.00        →P
 7677.00        Σ+
15327.65        ◊
 6593.00        Σ+
20312.91        ◊
 5321.00        Σ+
23811.55        ◊
 4108.00        Σ+
26160.31        ◊
 5623.00        Σ+
28955.94        ◊
 6172.00        Σ+
31624.26        ◊
35650.00        Σ+
45026.42        ◊
```

b. Enter: [CLEAR]

15 [i] **0** [PV] **8652** [Σ+] **7677** [Σ+]

6593 [Σ+] **5321** [Σ+] **4108** [Σ+] **5623**

[Σ+] **6172** [Σ+] **35650** [Σ+] [RCL] [i]

[%] [+] [◊]

Present Worth - $45,026.42

```
              CLEAR
      15.00    → I
        .00    →P
    8652.00     Σ+
    7523.46    ◊
    7677.00     Σ+
   13328.39    ◊
    6593.00     Σ+
   17663.40    ◊
    5321.00     Σ+
   20705.70    ◊
    4108.00     Σ+
   22748.10    ◊
    5623.00     Σ+
   25179.08    ◊
    6172.00     Σ+
   27499.36    ◊
   35650.00     Σ+
   39153.41    ◊
      15.00    ← I
                 %
    5873.01      +
   45026.42    ◊
```

c. Enter: [CLEAR]

1 [SAVE↑] **15** [%] [+] [STO] [0]

35650 [ ] [÷] **6172** [+] [ ] [÷]

5623 [+] [ ] [÷] **4108** [+] [ ] [÷]

5321 [+] [ ] [÷] **6593** [+] [ ] [÷]

7677 [+] [ ] [÷] **8652** [+] [◊]

Present Worth - $45,026.42

```
              CLEAR
       1.00      ↑
      15.00      %
        .15      +
               →0
   35650.00    N ÷
   31000.00    ◊
    6172.00      +
               N ÷
   32323.48    ◊
    5623.00      +
               N ÷
   32996.94    ◊
    4108.00      +
               N ÷
   32265.16    ◊
    5321.00      +
               N ÷
   32683.62    ◊
    6593.00      +
               N ÷
   34153.58    ◊
    7677.00      +
               N ÷
   36374.42    ◊
    8652.00      +
   45026.42    ◊
```

DEFERRED ANNUITIES

Leases often call for periodic contractual adjustments of rental payments. For example, a 10-year lease may call for rentals of $200 per month for 3 years, $250 per month for the next 3 years, and $300 per month for the last 4 years. This situation illustrates what is called a "step-up" lease. A "step-down" lease is similar, except that rental payments are decreased periodically according to the lease contract.

In the example cited, the rental payment streams for years 4-6 and years 7-10 are "deferred annuities". They start at some time in the future. To find their Present Worth, the Present Worth at the start of the payment period is first calculated, and then discounted to the present (time period 0) by multiplying by the appropriate Present Worth of One factor.

The keystrokes are:

1. Press [CLEAR].
2. Calculate Present Worth of payments for time span 1 (n1), press [STO] [1].
3. Calculate Present Worth of payments for time span 2 (n2), press [STO] [2].
4. Calculate Present Worth of payments for time span 3 (n3), press [STO] [3]. Continue for each successive time span.
5. Key in n1, press [n]; press [RCL] [i] [i]; press [RCL] [2] [FV] [PV]

   [STO] [2] (This calculates and stores the present worth of the deferred payments in time span 2.) Continue for each successive time span (deferred annuity).
6. Press [RCL] [1] [+] [RCL] [2] [+] and continue for each successive deferred annuity preceding the last one calculated in Step 5.
7. Press [◊] (This prints out the Present Worth of the entire set of income payments.)

Example - A lease calls for monthly payments (end-of-month) of $500 per month for the first 5 years, $600 per month for the next 4 years, and $750 per month for the next 3 years. What is the present worth of this rental income stream at 13.5%?

Enter: [CLEAR]

12 [STO] [0] **5** [×] [n] **13.5** [ ] [÷]

[i] **500** [PMT] [PV] [STO] [1]

(PW of rental for years 1-5 is $21,729.83.)

4 [ ] [×] [n] **600** [PMT] [PV] [STO] [2]

(PW of rental for years 6-9 is $22,159.58.)

3 [ ] [×] [n] **750** [PMT] [PV] [STO] **3**

(PW of rental for years 10-12 is $22,100.89.)

[CLEAR]

5 [ ] [×] [n] [RCL] [i] [i] [RCL]

[2] [FV] [PV] [STO] [2] **(PW of $600 monthly deferred 5 years is $11,325.29.)**

4 [SAVE ↑] [ ] [×] [RCL] [n] [+] [n]

[RCL] [3] [FV] [PV] **(PW of $750 monthly deferred 9 years is $6602.19.)**

[RCL] [2] [+] [RCL] [1] [+] [◊] **Present Worth of the lease payments is $39,657.31.**

```
            CLEAR
   12.00    →0
    5.00     ×
   60.00    ◊
            → N
   13.50    N ÷
    1.13    ◊
            → I
  500.00    →PT
P  V
21729.83
            →1
    4.00    N ×
   48.00    ◊
            → N
  600.00    →PT
P  V
22159.58
            →2
    3.00    N ×
   36.00    ◊
            → N
  750.00    →PT
P  V
22100.89
            →3
            CLEAR
    5.00    N ×
   60.00    ◊
            → N
    1.13    ← I
            → I
              ↑
22159.58    ←2
            → F
P  V
11325.29
            →2
    4.00      ↑
            N ×
   48.00    ◊
   60.00    ← N
              +
            → N
              ↑
22100.89    ←3
            → F
P  V
 6602.19
              ↑
11325.29    ←2
              +
              ↑
21729.83    ←1
              +
39657.31    ◊
```

DEFERRED FACTOR

Sometimes it is necessary to calculate the "deferred factor" applicable to a deferred annuity. This "deferred factor" is the product of the Present Worth of One per Period factor for the number of periods in the deferred annuity times the Present Worth of One factor for the number of periods to the beginning of the deferred annuity (the number of periods the annuity is deffered).

Example - A step-up lease has monthly payments of $250 for the first 4 years, $350 for the next 3 years, and $500 for the next 5 years. What are the deferred factors for the 3 years of $350 payments, and the 5 years of $500 payments? The rate of discount is 12.75%.

Enter: [CLEAR]

[6] **12** [STO] [0] [3] [×] [n]

12.75 [÷] [i] [1] [PMT] [PV]

[STO] [FV]

```
            CLEAR
12.000000     →0
 3.000000      ×
36.000000      ◇
              → N
12.750000     N ÷
 1.062500      ◇
              → I
 1.000000     →PT
P  V
29.785252
              →5*
```

[CLEAR]

4 [×] [n] [RCL] [i] [i]

[RCL] [FV] [FV] [PV]

The 3-year factor deferred 4 years is 17.934096.

```
            CLEAR
 4.000000     N ×
48.000000      ◇
              → N
 1.062500     ← I
              → I
29.785252     ← F
              → F
P  V
17.934096
```

[CLEAR]

5 [×] [n] [RCL] [i]

[i] [1] [PMT] [PV] [STO] [FV]

```
            CLEAR
 5.000000     N ×
60.000000      ◇
              → N
 1.062500     ← I
              → I
 1.000000     →PT
P  V
44.198308
              →5*
```

CLEAR

7 × n RCL i

i RCL FV FV PV

The 5-year factor deferred 7 years is 18.190411.

```
                CLEAR

     7.000000    N ×
    84.000000      ◊
                 → N
     1.062500    ← I
                 → I
    44.198308    ← F
                 → F
P  V
    18.190411
```

NET PRESENT VALUE

Net Present Value is the difference between the Present Worth of an income stream (stream of cash flows) at a specified rate of discount and the Capital Outlay (initial investment in time period 0) required to acquire the property that will produce that forecast income stream.

The formula is: NPV = PW - CO

This measure shows the "profit" or "loss" associated with a given investment amount (CO) and the forecast future income stream discounted at the required or desired rate of return. If NPV is positive, there is a "profit" forecast and the investment should be made; if NPV is negative, this indicates that the investment is not worth the Capital Outlay required to obtain it, at the rate of discount specified. When NPV = 0, the investment is barely "feasible", since PW exactly equals CO.

a. Net Present Value for Level Annuities

If the forecast income stream is a level annuity, the procedure to calculate NPV is simply to derive PW with the Present Worth of One Per Period routine, and then subtract CO.

Example - An investor has an opportunity to purchase a leasehold interest for $180,000. The lease has 8 years to run. The favorable rental differential is $3,500 per month, with payments at the beginning of the month. The investor is looking for a 20% return on his investment. Should he buy the leasehold?

Enter: CLEAR

12 STO 0 8 × n 20

÷ i 3500 RCL i % + PMT PV

180000 − ◊

```
                CLEAR

     12.00       →0
      8.00         ×
     96.00         ◊
                 → N
     20.00       N ÷
      1.67         ◊
                 → I
```

No, NPV is: – $10,177.10.

```
       1.67   → I
              %
      58.33   +
             →PT
P  V
  169822.90
  180000.00   –
–  10177.10   ◊
```

b. Net Present Value for Level Annuity plus Reversion

In this case, the Present Worth is calculated by summing the results of the Present Worth of One per Period and the Present Worth of One routines, then Capital Outlay is subtracted to produce Net Present Value.

Example - An income property is forecast to produce NOI of $17,895 per year (end-of-year) for 17 years. The proceeds of resale in 17 years are forecast at $105,000. The indicated Discount Rate (r) is 11.85%. The property is for sale today for $140,000. Is it an attractive investment?

Enter: CLEAR

17 n **11.85** i **17895** PMT PV

STO 1

CLEAR

RCL n n RCL i i **105000**

FV PV RCL 1 + ◊

PW is $144,156.74.

140000 – ◊

Yes, NPV is +$4,156.74.

```
             CLEAR
      17.00   → N
      11.85   → I
   17895.00  →PT
P  V
  128511.67
             →1
             CLEAR
      17.00   ← N
              → N
      11.85   ← I
              → I
  105000.00   → F
P  V
   15645.07
              ↑
  128511.67  ←1
              +
  144156.74   ◊
  140000.00   –
    4156.74   ◊
```

c. Net Present Value for Variable Annuity

This case employs the Net Present Value (Discounted Cash Flow) routine in the HP-81 Operating Guide. Capital Outlay is entered as a negative number in PV.

The keystrokes are:

1. Press [CLEAR].

2. Key in rate of discount per period, press [i].

3. Key in Capital Outlay, press [CHS] [PV].

4. Key in amount of first payment, press [Σ+].

   Repeat Step 4 for each successive payment.

The figure printed is NPV

Example - An investor has an opportunity to purchase a property for $51,500. The After Tax Cash Flows (including reversion) are forecast as follows:

Year	ATCF
1	$11,846
2	$9,673
3	$8,217
4	$6,743
5	$5,018
6	$3,716
7	$2,284
8	$ 867
Reversion (EOY 8)	$51,883

If the going rate of return on this type of investment property is 13.75%, should the investor purchase the property?

Enter: [CLEAR]

51883 [SAVE ↑] 867 [+] [STO] [1] (This calculates and stores total receipts for Year 8.)

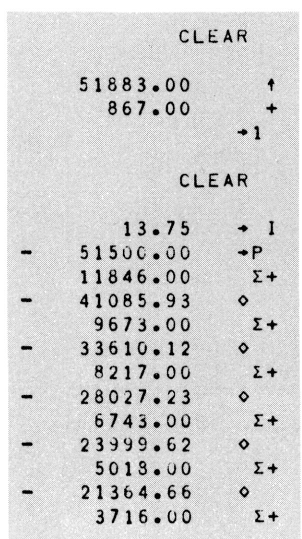
[CLEAR]

13.75 [i] 51500 [CHS] [PV] 11846 [Σ+]

9673 [Σ+] 8217 [Σ+] 6743 [Σ+] 5018 [Σ+]

3716 [Σ+] 2284 [Σ+] [RCL] [1] [Σ+]

Yes, NPV is +$97.29



```
            CLEAR

   51883.00       ↑
     867.00       +
                 →1
            CLEAR
      13.75      → I
-  51500.00      →P
   11846.00      Σ+
-  41085.93     ◇
    9673.00      Σ+
-  33610.12     ◇
    8217.00      Σ+
-  28027.23     ◇
    6743.00      Σ+
-  23999.62     ◇
    5018.00      Σ+
-  21364.66     ◇
    3716.00      Σ+
```

-	19649.26	◇
	2284.00	Σ+
-	18722.35	◇
		↑
	52750.00	←1
	52750.00	Σ+
	97.29	◇

NOTE:

To find Payment Amount to Full Amortization and Number of Periods to Full Amortization for level annuities, use the routines illustrated in Chapter 4. To find Payment Amount to a Specified Balance and Number of Periods to a Specified Balance for level annuities plus reversion, use the routines illustrated in Chapter 5.

CHAPTER 9 - RATES OF RETURN AND RATES OF CAPITALIZATION

There are three types of rates of capitalization used in real estate appraisal and investment analysis.

The first group includes Rates of Return on the investment. These are the Discount Rate or Basic Rate (r) on the total property investment; the Equity Yield Rate (y) on the equity investment; and the Mortgage Interest Rate (i) on the mortgage investment or principal. These are all calculated as Internal Rates of Return over a specified time span, using either yield-to-maturity or yield-to-call routines.

The second category includes Capital Recovery Rates, providing for recovery of forecast capital loss over a specified time span. All are calculated as a Sinking Fund Factor at a specified rate of discount (IRR), which may be the Rate of Return, a lower rate, or zero.

The third category are Capitalization Rates, which are the sum of a given Rate of Return and its associated Capital Recovery Rate. They are rates used to capitalize level-annuity income streams to Present Worth or Value figures. They include the Capitalization Rate, applied to NOI for property value estimates; the Equity Dividend Rate (e), applied to CTO for equity investment valuation; and the Mortgage Constant (f), applied to Annual Debt Service for mortgage value estimation. All are calculated using the Installment to Amortize One routine.

All rates of return and rates of capitalization for real estate problems are calculated and expressed as annual rates.

Because the derivation of rates for mortgages (i, SFF and f) have already been explained and illustrated in Chapters 4 - 7, no mortgage examples are repeated here.

DISCOUNT RATE - BAND OF INVESTMENT

When non-amortized mortgages are involved, the Discount Rate or Basic Rate (r) is found by the following formula:

$$r = m\,i + (1 - m)\,y$$

In this formula, m is the loan-to-value ratio of the mortgage,
i is the annual mortgage interest rate
(1-m) is the equity investment (down payment) ratio,
y is the equity yield rate

Example - An investor can obtain a 65% mortgage on a property he is planning to purchase, with interest at 9.5%. The investor is looking for at least a 14.25% return on his equity. What Discount or Basic Rate is required to meet these standards?

NOTE: Enter all figures as decimals.

Enter: CLEAR

6 **.65** SAVE↑ **.095** × **1**

SAVE↑ **.65** − **.1425** × + ◊

r = .111625 or 11.1625%

```
                CLEAR

       .650000      ↑
       .095000      ×
       .061750   ◊
      1.000000      ↑
       .650000      -
       .142500      ×
       .049875   ◊
                    +
       .111625   ◊
```

CAPITAL RECOVERY RATE

Capital Recovery Rates apply particularly to investments in improvements (buildings) in total property valuation and analysis. They may be calculated on a straight-line basis (which is the same as the Sinking Fund Factor at a zero rate), a "sinking fund'' basis (which is the Sinking Fund Factor at the "safe" rate), and a level-annuity basis (which is the Sinking Fund Factor at the Discount or Base Rate.)

The procedures all use the Sinking Fund Factor routine illustrated in Chapter 2.

Example - An investment property includes a building which has an estimated Remaining Economic Life of 28 years. What is the annual Capital Recovery Rate:

a. on a straight-line basis?
b. on a sinking fund basis at a "safe" rate of 5%?
c. on a level annuity basis if the discount rate (r) is 10.45%?

a. Straight-Line Capital Recovery

Enter: CLEAR

6 **27** n **0**

i **1** FV PMT **CRR = .037037**

or CRR $= \frac{1}{n}$

Enter: CLEAR

6 **1** SAVE↑ **27** ÷

CRR = .037037

```
                CLEAR

     27.000000    → N
       .000000    → I
      1.000000    → F
                  PMT
       .037037

                CLEAR

      1.000000      ↑
     27.000000      ÷
       .037037   ◊
```

b. Safe-Rate Capital Recovery

Enter: CLEAR

6 **27** n **5**

i **1** FV PMT

CRR = .018292

```
           CLEAR

 27.000000   → N
  5.000000   → I
  1.000000   → F
             PMT
   .018292
```

c. Level-Annuity Capital Recovery

Enter: CLEAR

6 **27** n **10.45**

i **1** FV PMT

CRR = .007663

```
           CLEAR

 27.000000   → N
 10.450000   → I
  1.000000   → F
             PMT
   .007663
```

CAPITALIZATION RATE

Capitalization Rates are applied to the amount of property investment (typically buildings) to be fully recovered via periodic income payments over the income projection period. They are calculated according to the method of providing for capital recovery: straight line, sinking fund at the "safe" rate, or level annuity.

As noted in an earlier section, Capitalization Rates are equal to the sum of a Discount Rate plus the appropriate Capital Recovery Rate.

Example - An income-producing property contains buildings with an estimated remaining economic life of 45 years. The indicated Discount Rate (r) applicable to this investment is 9.85%. What is the annual Capitalization Rate:

a. with straight-line capital recovery?
b. with sinking-fund capital recovery at a 5.25% "safe" rate?
c. with level-annuity capital recovery?

NOTE:

For arithmetic functions, enter the Discount Rate as a decimal figure.

a. Straight-Line Capital Recovery

Enter: CLEAR

6 **45** n **0** i

1 FV PMT **.0985** + ◊

CR = .120722

```
                CLEAR

45.000000    → N
  .000000    → I
 1.000000    → F
               PMT
  .022222
  .098500      +
  .120722      ◊
```

b. Sinking-Fund Capital Recovery at "Safe" Rate

Enter: CLEAR

6 **45** n **5.25** i

1 FV PMT **.0985** + ◊

CR = .104333

```
                CLEAR

45.000000    → N
 5.250000    → I
 1.000000    → F
               PMT
  .005833
  .098500      +
  .104333      ◊
```

c. Level-Annuity Capital Recovery

Enter: CLEAR

6 **45** n **9.85** i

1 FV PMT **.0985** + ◊

CR = .099958

```
                CLEAR

45.000000    → N
 9.850000    → I
 1.000000    → F
               PMT
  .001458
  .098500      +
  .099958      ◊
```

CLEAR

45 n **9.85** i **1** PV PMT

CR = .099958

```
                CLEAR

45.000000    → N
 9.850000    → I
 1.000000    →P
               PMT
  .099958
```

RATE OF RETURN ON FULLY AMORTIZED INVESTMENT - END-OF-PERIOD PAYMENTS

This procedure uses Installment to Amortize One routine illustrated in Chapter 2, solving for i . The amount of the Capital Outlay for the investment is entered as PV . The rate of return is expressed as an annual rate.

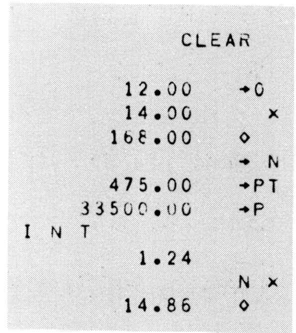
Example - An investor has an opportunity to buy a sandwich lease for $33,500. The net rental income is $475 per month, with rental payments at the end of the month. The remaining lease term is 14 years. What is the indicated annual rate of return (r) on the investment if he buys at the offering price?

Enter: CLEAR

12 STO 0 **14** × n **475**

PMT **33500** PV i ×

r = 14.86%



RATE OF RETURN ON FULLY AMORTIZED INVESTMENT - BEGINNING-OF-PERIOD PAYMENTS

For this procedure, the first payment amount is deducted from the amount of Capital Outlay, and the number of payment periods is reduced by one.

The keystrokes are:

1. Press CLEAR.
2. Key in number of payments per year, press STO 0 ; key in number of years, press ×; key in 1, press − n .
3. Key in payment amount per period, press PMT.
4. Key in Capital Outlay, press SAVE↑; key in payment amount per period, press − PV .
5. Press i ×.

Example - What is the indicated annual yield (r) on the investment in the Preceding section if monthly rental payments are receivable at the beginning of each month?

Enter: CLEAR

12 STO 0 **14** × **1** − n **475** PMT

33500 RCL PMT − PV i ×

r = 15.13%

```
                CLEAR

          12.00     →0
          14.00      ×
         168.00      ◇
           1.00      -
                    → N
         475.00     →PT
         475.00     ←PT
                     -
                    →P
I  N  T
           1.26
                    N ×
          15.13      ◇
```

RATE OF RETURN ON LEVEL ANNUITY PLUS REVERSION

This procedure uses the routine illustrated in Chapter 5 as Effective Interest Rate - No Fees for a balloon-payment mortgage. It is also the Bond Yield-to-Call routine illustrated in the HP-81 Operating Guide.

Example - An investment property has just been purchased for $60,000. NOI is forecast to be $8,500 per year for 15 years. Resale proceeds are forecast at $75,000 in 15 years. If the investor holds the property for the full 15 years, and expectations about NOI and Resale Proceeds are fully realized, what rate of return will he earn on the investment?

Enter: CLEAR

3 EXT( ) 8

EXT( ) 2 **75000** FV **15** SAVE ↑

365 × STO DAY **8500** PMT

60000 PV i

The annual yield rate will be 14.705%

```
                  CLEAR

BON                   8

                   AN2

    75000.000      → F
       15.000        ↑
      365.000        ×
     5475.000        ◇
                   →4*
     8500.000      →PT
    60000.000      →P
     YIELD
     TO          CALL
       14.705
```

RATE OF RETURN ON VARIABLE ANNUITY - 9 PAYMENTS OR LESS - END-OF-PERIOD PAYMENTS

If a variable annuity has 9 payments or less, it is possible to calculate the yield rate over the entire income projection period automatically on the HP-81. The yield is calculated as an Internal Rate of Return using the

Rate of Return for Uneven Cash Flow routine provided in the HP-81 Operating Guide.

NOTE

This routine will not accept negative cash flows (payments). If there are negative cash flows or more than 9 payments, IRR for a variable annuity must be calculated through interpolation via the NPV or Alternative Method routines illustrated in the succeeding sections.

The keystrokes are:

1. Press [CLEAR].
2. Key in amount of Capital Outlay, press [CHS] [PV].
3. Key in total number of cash flows, press [n].
4. Key in cash flow 1, press [STO] [1] .
5. Key in cash flow 2, press [STO] [2] . Repeat for each successive cash flow, storing each in the register corresponding with its number (up to 9 cash flows).
6. Press [EXT( )] [1] to calculate and print out IRR. (The calculation may take up to two minutes.)

NOTE:

If there is a reversion in addition to the final payment, these two values must first be summed and entered as a single figure in this routine.

Example - A property investment requires $50,000. The forecast annual NOI (including reversion in the last year) is as follows:

Year	NOI
1	$9,000
2	$8,000
3	$6,500
4	$7,500
5	$8,500
6	$6,000
7	$5,000
8	$34,000 (Including Reversion)

What is the expected rate of return (IRR) if the investment is held for 8 years and the cash flow forecasts are realized?

Enter: [CLEAR]

50000 [CHS] [PV] **8** [n] **9000** [STO] [1]

8000 [STO] [2] **6500** [STO] [3] **7500** [STO]

[4] **8500** [STO] [5] **6000** [STO] [6] **5000**

[STO] [7] **34000** [STO] [8] [EXT( )] [1]

IRR = 10.86%

```
              CLEAR
-   50000.00    →P
        8.00    → N
     9000.00    →1
     8000.00    →2
     6500.00    →3
     7500.00    →4
     8500.00    →5
     6000.00    →6
     5000.00    →7
    34000.00    →8
        DISC ROR
       10.86
```

RATE OF RETURN ON VARIABLE ANNUITY - 9 PAYMENTS OR LESS - BEGINNING-OF-PERIOD PAYMENTS

To find the yield (IRR) on a variable annuity with cash flows receivable at the beginning of each period, the preceding routine is used, with some minor changes:

1. Capital Outlay less the first periodic payment is entered as a negative figure in [PV] .
2. The number of payments less one is entered in [n].
3. Each successive cash flow is stored in the register number equal to one less than its payment number.

Example 1 - What is the annual yield to maturity (IRR) for the property in the preceding example if each cash flow is receivable at the beginning of each year and the reversion occurs at the beginning of the final year?

NOTE:
If there is a reversion in this case, it is entered separately for the last payment period, and includes the period of the reversion.

Enter: [CLEAR]

50000 [CHS] [SAVE ↑] **9000** [+] [PV] [7]

[n] **8000** [STO] [1] **6500** [STO] [2]

7500 [STO] [3] **8500** [STO] [4] **6000**

[STO] [5] **5000** [STO] [6] **34000**

```
              CLEAR
-   50000.00    ↑
     9000.00    +
                →P
        7.00    → N
     8000.00    →1
     6500.00    →2
     7500.00    →3
     8500.00    →4
     6000.00    →5
```

[STO] [7] [EXT( )] [1]

IRR = 14.06%

```
 5000.00    →6
34000.00    →7
   DISC ROR
   14.06
```

Example 2 - What is the IRR in Example 1 if there is a lump-sum reversion of $30,000 at the end of year 8? (The last cash flow is thus $4,000.)

Enter: [CLEAR]

50000 [CHS] [SAVE ↑] **9000** [+] [PV] **8** [n]

8000 [STO] [1] **6500** [STO] [2] **7500**

[STO] [3] **8500** [STO] [4] **6000** [STO] [5]

5000 [STO] [6] **4000** [STO] [7] **30000**

[STO] [8] [EXT( )] [1]

IRR = 13.14%

```
- 50000.00    ↑
   9000.00    +
              →P
      8.00    → N
   8000.00    →1
   6500.00    →2
   7500.00    →3
   8500.00    →4
   6000.00    →5
   5000.00    →6
   4000.00    →7
  30000.00    →8
    DISC ROR
     13.14
```

RATE OF RETURN ON VARIABLE ANNUITY - 10 OR MORE PAYMENTS

When a variable annuity (stream of variable cash flows) has 10 or more payments, IRR can be calculated with successive iterations of either the Net Present Value or the Alternative Method for Present Worth of a Variable Annuity routines illustrated in Chapter 8.

The objective is to find the yield rate at which NPV equals zero. IRR is equal to i when CO = PW. The equation for this is

$$PW = \sum_{t=1}^{n} \left(\frac{CF_t}{(1+i)^t} \right)$$

The NPV or Alternative Method routine is worked at a test rate. If NPV is positive, the test rate is too low and the routine should be worked again at a higher test rate. If NPV is negative, the test rate is too high and the routine should be worked again at a lower test rate. When one positive and one negative NPV is obtained, the IRR is found by interpolation.

The process for interpolation is:

1. Subtract negative NPV from positive NPV (this is the same as adding two positive numbers).
2. Divide the figure derived in Step 1 into the positive NPV.

3. Multiply the figure derived in Step 2 by the difference between the two test rates.

4. Add the figure derived in Step 3 to the lower test rate. The result is in the indicated IRR. (Note: This method gives an approximation of the true IRR, but it is more than sufficiently precise for nearly all real estate problems.)

NOTE:
It is possible to use the storage registers to store cash flows for successive use, and both save entry time and avoid entry errors. In the NPV routine, 13 registers are available: 1-9, 14, 15, 17 and 19. In the Alternative NPV Method routine, 15 registers are available: 1-9 and 14-19.

Example - An income property is for sale for $50,000. The forecast NOI plus reversion over a 12-year income projection period (all payments receivable at the end of the year) is as follows:

Year	NOI	Year	NOI
1	$5,000	8	$5,000
2	$6,000	9	$6,000
3	$7,000	10	$7,000
4	$5,000	11	$5,000
5	$4,000	12	$4,000
6	$6,000	12	$16,000 (Resale Proceeds-Reversion)
7	$7,000		

What is the indicated Rate of Return (r) on this investment, calculated on an Internal Rate of Return over the full 12-year income projection period?

a. Finding a Test Rate

Although it is possible to start the trial-and-error process of calculating IRR with the NPV routine using virtually any test rate, a reasonable first approximation can be found by calculating the average rate of return. This is average NOI less average capital loss, divided by capital outlay.

$$\text{ARR} = \frac{\dfrac{\Sigma\ \text{NOI}}{n} - \left(\dfrac{\text{CO - Reversion}}{n}\right)}{\text{CO}}$$

Enter: [CLEAR]

5000 [Σ+] **6000** [Σ+] **7000** [Σ+] **5000** [Σ+]

4000 [Σ+] **6000** [Σ+] **7000** [Σ+] **5000** [Σ+]

6000 [Σ+] **7000** [Σ+] **5000** [Σ+] **4000** [Σ+]

```
              CLEAR

 5000.00        Σ+
 6000.00        Σ+
 7000.00        Σ+
 5000.00        Σ+
 4000.00        Σ+
 6000.00        Σ+
```

[x̄] [STO] [0] 50000 [SAVE ↑] 16000 [−] 12 [÷] [CHS] [RCL] [0] [+] [◊] 50000 [÷] [■] [6] [◊]

arr = 5.5%

This gives the lower limit of IRR, so first test at 7%.

```
   7000.00     Σ+
   5000.00     Σ+
   6000.00     Σ+
   7000.00     Σ+
   5000.00     Σ+
   4000.00     Σ+
σ
   1083.62
N
     12.00
x̄
   5583.33
              →0
  50000.00     ↑
  16000.00     −
     12.00     ÷
   2833.33    ◊
              S ⇄
               ↑
   5583.33    ←0
               +
   2750.00    ◊

  50000.00     ÷
       .06    ◊
   .055000    ◊
```

b. NPV Routine to Solve for IRR

1. Testing at 7%:

Enter: [CLEAR]

7 [i] 50000 [CHS] [PV] 5000 [STO] [1] [Σ+] 6000 [STO] [2] [Σ+] 7000 [STO] [3] [Σ+] 5000 [STO] [4] [Σ+] 4000 [STO] [5] [Σ+] 6000 [STO] [6] [Σ+] 7000 [STO] [7] [Σ+] 5000 [STO] [8] [Σ+] 6000 [STO] [9] [Σ+] 7000 [STO] [•] [4] [Σ+] 5000 [STO] [•] [5] [Σ+] 20000 [STO] [•] [7] [Σ+] [STO] [0]

NPV = +$1639.13, so 7% is too low.

```
              CLEAR
      7.00    → I
-  50000.00    →P
   5000.00     →1
   5000.00     Σ+
-  45327.10    ◊
   6000.00     →2
   6000.00     Σ+
-  40086.47    ◊
   7000.00     →3
   7000.00     Σ+
-  34372.39    ◊
   5000.00     →4
   5000.00     Σ+
-  30557.91    ◊
   4000.00     →5
   4000.00     Σ+
-  27705.96    ◊
   6000.00     →6
   6000.00     Σ+
-  23707.91    ◊
   7000.00     →7
   7000.00     Σ+
-  19348.66    ◊
   5000.00     →8
   5000.00     Σ+
```

```
-   16438.62   ◇
     6000.00  →9
     6000.00  Σ+
-   13175.02   ◇
     7000.00  →4*
     7000.00  Σ+
-    9616.57   ◇
     5000.00  →5*
     5000.00  Σ+
-    7241.11   ◇
    20000.00  →7*
    20000.00  Σ+
     1639.13   ◇
              →0
```

NOTE:
Storing Cash Flows allows them to be recalled without manual re-entry in successive test at other rates.

2. Testing at 8%:

Enter: [CLEAR]

8 [i] **50000** [CHS] [PV] [RCL] [1] [Σ+]
[RCL] [2] [Σ+] [RCL] [3] [Σ+] [RCL] [4]
[Σ+] [RCL] [5] [Σ+] [RCL] [6] [Σ+] [RCL]
[7] [Σ+] [RCL] [8] [Σ+] [RCL] [9] [Σ+]
[RCL] [•] [4] [Σ+] [RCL] [•] [5] [Σ+]
[RCL] [•] [7] [Σ+]

NPV = -$1374.70, so 8% is too high.

IRR is therefore between 7% and 8%.

```
               CLEAR
        8.00  → I
-   50000.00  →P
                ↑
     5000.00  ←1
     5000.00  Σ+
-   45370.37   ◇
                ↑
     6000.00  ←2
     6000.00  Σ+
-   40226.34   ◇
                ↑
     7000.00  ←3
     7000.00  Σ+
-   34669.51   ◇
                ↑
     5000.00  ←4
     5000.00  Σ+
-   30994.36   ◇
                ↑
     4000.00  ←5
     4000.00  Σ+
-   28272.03   ◇
                ↑
     6000.00  ←6
     6000.00  Σ+
-   24491.01   ◇
                ↑
     7000.00  ←7
     7000.00  Σ+
-   20406.58   ◇
                ↑
     5000.00  ←8
     5000.00  Σ+
-   17705.23   ◇
                ↑
     6000.00  ←9
     6000.00  Σ+
```

```
-    14703.74    ◇
                 ↑
      7000.00   ←4*
      7000.00    Σ+
-    11461.39    ◇
                 ↑
      5000.00   ←5*
      5000.00    Σ+
-     9316.97    ◇
                 ↑
     20000.00   ←7*
     20000.00    Σ+
-     1374.70    ◇
```

3. Interpolation: (Do Not Press CLEAR

Enter:

RCL 0 x⇄y − ◇ RCL 0

x⇄y ÷

(IRR is .54 from 7% to 8%.)

6 **.01** × **.07** + ◇

IRR is .075439 or 7.5439%.

(At 7.5439%, NPV = -$33.68.)

```
                 ↑
      1639.13   ←0
-     1374.70    ⇄
                 -
      3013.83    ◇
                 ↑
      1639.13   ←0
      3013.83    ⇄
                 ÷
          .54    ◇
      .010000    ×
      .005439    ◇
      .070000    +
      .075439    ◇
```

1. Testing at 7%:

NOTE:
Enter Cash flows in reverse order, and store in sequence entered, using Storage Registers 1-9 and 14-19. This makes it possible to recall and use Cash Flows in proper sequence for successive test calculations, without manual re-entry of data.

Enter: CLEAR

1 SAVE↑ **7** % + STO 0 **20000**

STO 1 ÷ **5000** STO 2 +

÷ **7000** STO 3 + ÷

6000 STO 4 + ÷ **5000** STO

```
          CLEAR
         1.00    ↑
         7.00    %
          .07    +
                →0
     20000.00   →1
               N ÷
     18691.59    ◇
      5000.00   →2
```

[5] [+] [░] [÷] **7000** [STO] [6] [+] [░] [÷] **6000** [STO] [7] [+] [░] [÷] **4000** [STO] [8] [+] [░] [÷] **5000** [STO] [9] [+] [░] [÷] **7000** [STO] [•] [4] [+] [░] [÷] **6000** [STO] [•] [5] [+] [░] [÷] **5000** [STO] [•] [6] [+] [░] [÷] **50000** [−] [◊]

NPV = +$1639.13.

(This is identical with the result from the NPV method.) A 7% rate is too low.

```
                +
              N ÷
   22141.67   ◇
    7000.00   →3
                +
              N ÷
   27235.21   ◇
    6000.00   →4
                +
              N ÷
   31060.94   ◇
    5000.00   →5
                +
              N ÷
   33701.81   ◇
    7000.00   →6
                +
              N ÷
   38039.08   ◇
    6000.00   →7
                +
              N ÷
   41158.02   ◇
    4000.00   →8
                +
              N ÷
   42203.75   ◇
    5000.00   →9
                +
              N ÷
   44115.66   ◇
    7000.00   →4*
                +
              N ÷
   47771.64   ◇
    6000.00   →5*
                +
              N ÷
   50253.37   ◇
    5000.00   →6*
                +
              N ÷
   51639.13   ◇
   50000.00   -
    1639.13   ◇
```

2. Testing at 8%:

Enter: [CLEAR]

1 [SAVE↑] **8** [%] [+] [STO] [0] [RCL] [1] [░] [÷] [RCL] [2] [+] [░] [÷] [RCL] [3] [+] [░] [÷] [RCL] [4] [+] [░]

```
          CLEAR
       1.00   ↑
       8.00   %
        .08   +
              →0
              ↑
   20000.00   →1
```

[÷] [RCL] [5] [+] [ ] [÷] [RCL] [6]

[+] [ ] [÷] [RCL] [7] [+] [ ] [÷]

[RCL] [8] [+] [ ] [÷] [RCL] [9] [+]

[ ] [÷] [RCL] [•] [4] [+] [ ] [÷]

[RCL] [•] [5] [+] [ ] [÷] [RCL] [•]

[6] [+] [ ] [÷] **50000** [−] [◊]

NPV = -$1374.70
(This is identical with the result from the NPV method.) An 8% rate is too high.

```
                N  ÷
   18518.52     ◊
                   ↑
    5000.00     ←2
                   +
                N  ÷
   21776.41     ◊
                   ↑
    7000.00     ←3
                   +
                N  ÷
   26644.82     ◊
                   ↑
    6000.00     ←4
                   +
                N  ÷
   30226.69     ◊
                   ↑
    5000.00     ←5
                   +
                N  ÷
   32617.30     ◊
                   ↑
    7000.00     ←6
                   +
                N  ÷
   36682.69     ◊
                   ↑
    6000.00     ←7
                   +
                N  ÷
   39521.01     ◊
                   ↑
    4000.00     ←8
                   +
                N  ÷
   40297.23     ◊
                   ↑
    5000.00     ←9
                   +
                N  ÷
   41941.88     ◊
                   ↑
    7000.00     ←4*
                   +
                N  ÷
   45316.55     ◊
                   ↑
    6000.00     ←5*
                   +
                N  ÷
   47515.33     ◊
                   ↑
    5000.00     ←6*
                   +
                N  ÷
   48625.30     ◊
   50000.00        -
-   1374.70     ◊
```

3. Interpolation to Find IRR

Enter: (Do Not Press [CLEAR])

[CHS] **1639.13** [STO] [0] [+] [◊] [RCL]

[0] [$x \rightleftarrows y$] [÷] [ ] [6] [7] [+] [◊]

IRR = 7.543870%.

	S ⇄
1639.13	→0
	+
3013.83	◊
	↑
1639.13	←0
3013.83	⇄
	÷
.54	◊
7.000000	+
7.543870	◊

MODIFIED IRR - VARYING REINVESTMENT RATE

The traditional IRR technique assumes that all positive cash flows are reinvested at the IRR to earn compound interest over the income projection period. It also assumes that all negative cash flows are to be discounted at the IRR. This means that cash can be invested today to earn compound interest at the IRR until it is needed to cover the forecast negative cash flows.

Neither of these assumptions is necessarily realistic or valid. It is possible to compensate for either or both by using real-market rates to discount all negative flows (including Capital Outlay) to the present at a "safe" rate that will ensure liquidity when funds are needed; and to compound all positive flows at a realistic reinvestment rate to the end of the income projection period.

This procedure results in a single (negative) Present Worth figure, and a single Future Worth figure as well. IRR is then found by applying the Future Worth of One routine illustrated in Chapter 2, solving for [i].

Example 1 - Negative Cash Flows, Reinvestment of Positive Flows at IRR.

A development project requires a total capital investment (development costs) of $600,000, staged as follows: $150,000 immediately, plus $150,000 in each of year 1-3. Net sales proceeds over a total 10-year sellout period are projected as: Year 1 - $0, Year 2 - $50,000, Years 3-5 - $125,000, Year 6 - $140,000, Year 7 - $150,000, Year 8 - $175,000, Year 9 - $100,000, Year 10 - $50,000.

What is the indicated IRR for the developer, assuming he can earn 5.5% on the money required to cover future cash outlays (negative cash flows)?

The net cash flows projected are:

Year	Cash Flow	Year	Cash Flow
0	-$150,000	7	+$150,000
1	-$150,000	8	+$175,000
2	-$100,000	9	+$100,000
3	-$25,000	10	+$50,000
4	+$125,000		
5	+$125,000		
6	+$140,000		

The steps in the procedure are:

1. Calculate Present Worth of the negative cash flows at the "safe" rate.
2. Store the figure obtained in Step 1 as a negative number for use as [PV] in the NPV routine.
3. Find IRR with the NPV routine and interpolation, entering 0 as the cash flow for years with a negative cash flow.

Enter: [CLEAR]

5.5 [i] 150000 [PV] 150000 [Σ+] 100000 [Σ+] 25000 [Σ+] [CHS] [STO] [0]

PV is -$403,315.68

```
              CLEAR

     5.50     → I
150000.00     →P
150000.00      Σ+
292180.09     ◇
100000.00      Σ+
382025.34     ◇
 25000.00      Σ+
403315.68     ◇
              S ⇄
              →0
```

1. Testing at 12%:

Enter: [CLEAR]

12 [i] [RCL] [0] [PV] 0 [STO] [1] [Σ+] 0 [STO] [2] [Σ+] 0 [STO] [3] [Σ+] 125000 [STO] [4] [Σ+] 125000 [STO] [5] [Σ+] 140000 [STO] [6] [Σ+] 150000 [STO] [7] [Σ+] 175000 [STO] [8] [Σ+] 100000 [STO] [9] [Σ+] 50000 [STO] [•] [4] [Σ+]

```
                CLEAR

       12.00     → I
                   ↑
-  403315.68     →0
                 →P
         .00     →1
         .00      Σ+
-  403315.68     ◇
         .00     →2
         .00      Σ+
-  403315.68     ◇
         .00     →3
         .00      Σ+
-  403315.68     ◇
   125000.00     →4
```

NPV is +$8672.41, so 12% is too low.

```
     125000.00    Σ+
-    323875.92    ◇
     125000.00    →5
     125000.00    Σ+
-    252947.56    ◇
     140000.00    →6
     140000.00    Σ+
-    182019.20    ◇
     150000.00    →7
     150000.00    Σ+
-    114166.82    ◇
     175000.00    →8
     175000.00    Σ+
-     43487.26    ◇
     100000.00    →9
     100000.00    Σ+
-      7426.25    ◇
      50000.00    →4*
      50000.00    Σ+
       8672.41    ◇
```

2. Testing at 13%:

Enter: CLEAR

13 i RCL 0 PV RCL 1 Σ+ RCL 2 Σ+ RCL 3 Σ+ RCL 4 Σ+ RCL 5 Σ+ RCL 6 Σ+ RCL 7 Σ+ RCL 8 Σ+ RCL 9 Σ+ RCL • 4 Σ+

NPV is -$13956.28, so 13% is too high.

```
                      CLEAR

         13.00    → I
                    ↑
-    403315.68    ←0
                  →P
                    ↑
           .00    ←1
           .00    Σ+
-    403315.68    ◇
                    ↑
           .00    ←2
           .00    Σ+
-    403315.68    ◇
                    ↑
           .00    ←3
           .00    Σ+
-    403315.68    ◇
                    ↑
     125000.00    ←4
     125000.00    Σ+
-    326650.84    ◇
                    ↑
     125000.00    ←5
     125000.00    Σ+
-    258805.85    ◇
                    ↑
     140000.00    ←6
     140000.00    Σ+
-    191561.25    ◇
                    ↑
     150000.00    ←7
     150000.00    Σ+
-    127802.15    ◇
```

```
                      ↑
   175000.00         →8
   175000.00         Σ+
-   61974.18          ◊
                      ↑
   100000.00         →9
   100000.00         Σ+
-   28685.70          ◊
                      ↑
    50000.00        →4*
    50000.00         Σ+
-   13956.28          ◊
```

3. Interpolation to Find IRR:

(Do Not Press [CLEAR])

[CHS] **8672.41** [+] [◊] **8672.41** [$x \rightleftarrows y$]

[÷] **12** [+] [ ] [6] [◊]

IRR is 12.383248% (12.38%)

```
                     S ⇄
     8672.41           +
    22628.69           ◊

     8672.41
    22628.69           ⇄
                       +
         .38           ◊
       12.00           +
   12.383248           ◊
```

Example 2 - Using the cash flow figures in Example 1, what is IRR if the "safe" rate for negative cash flows is 5.5%, and the reinvestment rate for positive cash flows is 10%?

Enter: [CLEAR]

5.5 [i] **150000** [PV] **150000** [Σ+] **100000** [Σ+]

25000 [Σ+] **(Do Not Press [CHS])** [STO] [1]

Present Worth of negative cash flows at 5.5% is $403,315.68

```
                 CLEAR
        5.50       → I
   150000.00       →P
   150000.00       Σ+
   292180.09        ◊
   100000.00       Σ+
   382025.34        ◊
    25000.00       Σ+
   403315.68        ◊
                   →1
```

Enter: [CLEAR]

6 [n] **10** [i] **125000** [PV] [FV] [STO] [2] **5**

[n] [FV] [RCL] [2] [+] [STO] [2] **4** [n]

140000 [PV] [FV] [RCL] [2] [+] [STO] [2]

```
                CLEAR
         6.00      → N
        10.00      → I
    125000.00      →P
F   V
    221445.13
                   →2
         5.00      → N
F   V
```

3 [n] 150000 [PV] [FV] [RCL] [2] [+] [STO] [2] 2 [n] 175000 [PV] [FV] [RCL] [2] [+] [STO] [2] 1 [n] 100000 [PV] [FV] [RCL] [2] [+] 50000 [+] [◊] [STO] [2]

Future Worth of positive cash flows at 10% is $1,199,132.88.

```
     201313.75
                  ↑
     221445.13   ←2
                  +
                 →2
          4.00   → N
     140000.00   →P
F  V
     204974.00
                  ↑
     422758.88   ←2
                  +
                 →2
          3.00   → N
     150000.00   →P
F  V
     199650.00
                  ↑
     627732.88   ←2
                  +
                 →2
          2.00   → N
     175000.00   →P
F  V
     211750.00
                  ↑
     827382.88   ←2
                  +
                 →2
          1.00   → N
     100000.00   →P
F  V
     110000.00
                  ↑
    1039132.88   ←2
                  +
      50000.00    +
    1199132.88   ◊

                 →2
```

[CLEAR] 10 [n] [RCL] [1] [PV] [RCL] [2] [FV] [i]

IRR is 11.51%

```
             CLEAR
         10.00   → N
                  ↑
     403315.68   ←1
                 →P
                  ↑
    1199132.88   ←2
                 → F
I  N  T
         11.51
```

NOTE:

After-Tax Internal Rate of Return is calculated exactly the same as any other IRR except that After-Tax Cash Flows are used.

CHAPTER 10: MORTGAGE-EQUITY (ELLWOOD) ANALYSIS

Analyzing and appraising real estate investment properties in terms of their mortgage and equity investment components constitutes Mortgage-Equity Analysis. It was formalized and popularized by the late L. W. Ellwood. This is why it is frequently referred to as "Ellwood Analysis."

This framework of analysis is used to estimate the Present Worth (Market Value or Investment Value) of the total property investment and of the equity investment position. Property value is estimated by capitalizing Net Operating Income at the Overall Rate:

$$V = \frac{NOI}{R}$$

Present Worth of the equity investment position is estimated by capitalizing Cash Throw-off to equity at the Equity Dividend Rate:

$$Ve = \frac{CTO}{e}$$

The Mortgage-Equity framework is also used to estimate the dollar amount of resale proceeds (PR), or the percentage of increase (app.) or decrease (dep.) in resale proceeds over initial investment (Capital Outlay), required to achieve a given Basic Rate (r) or Equity Yield Rate (y).

Finally, the analysis can be used to calculate the Basic Rate (r) on the total property investment, or the Equity Yield Rate (y) on the equity investment. Both are calculated as an Internal Rate of Return.

If all figures were available in dollar amounts, it would be unnecessary to have a separate Mortgage-Equity framework. However, often the dollar value of Present Worth, Resale Proceeds (reversion) and Mortgage Principal are unknown. Only NOI is given as a dollar figure, with mortgage loan terms and capital gain (app.) or loss (dep.) on resale given as percentages. Thus it is necessary to calculate the Basic Rate (r) and the Overall Rate (R) to apply to NOI to estimate value.

NOTE:

Specifications for the income stream are that NOI be a level annuity. The total income stream is thus a level annuity plus a reversion receivable at the end of the payment period. Also, all cash flows (NOI, ADS and CTO) are before-tax cash flows, and all rates of return (r, i and y), as well as all capitalization rates (R, R and e) are before-tax annual rates.

CALCULATION OF BASIC RATE AND OVERALL RATE

The basic formulas are (see the appendix for definitions of all symbols):

Basic Rate: $r = y - mc \qquad r = mF + (1\text{-}m)y - mp1/s_n$

Overall Rate: $R = r + \text{dep.}1/s_n$

$R = r - \text{app.}1/s_n$

$R = y - mc \begin{array}{l} + \text{dep.}1/s_n \\ - \text{app.}1/s_n \end{array} \qquad R = mF + (1\text{-}m)y - mp1/s_n \begin{array}{l} + \text{dep.}1/s_n \\ - \text{app.}1/s_n \end{array}$

$c = y + p1/s_n - F$

The given values required are:

i = mortgage interest rate

m = loan-to-value ratio of mortgage

n_t = total number of mortgage payments to full amortization

y = equity yield rate

n_p = income projection period (investment holding period)

dep./app. = capital loss or gain on resale as a percentage of present worth or value of property

With these values, it is then possible to calculate:

f = mortgage constant

$1/s_n$ = sinking fund factor at the equity yield rate over the income projection period

p = percentage of mortgage principal paid off over the income projection period.

After these values are calculated, c, r and R can be calculated.

Continuing Example - To illustrate all the required calculations to derive the Basic Rate (r) and the Overall Rate (R), the following conditions are assumed: An investor plans to purchase an income property, hold it for 10 years, and then resell it. It is estimated that the proceeds of resale will result in a 15% capital loss. A 25-year mortgage loan with level monthly payments at 8.75% interest can be obtained, with a loan-to-value ratio of 70%. The investor is seeking a 14% yield on his equity investment.

Thus, i = 8.75% or .087

m = 70% or .70

n_t = 25 years or 300 months

y = 14% or .14

n = 10 years

dep. = 15% or .15

NOTE:

In the following illustrations and examples of procedures for calculating the values for components in the formulas for the Basic Rate (r) and the Overall Rate (R), the keystrokes include the entry of intermediate values into storage for future use. In this way, the individual steps for calculation of component values can be combined to calculate r and R in one continuous series of routines. This combined procedure is illustrated at the end of the illustrations for calculating the individual steps.

To accomplish this, it is recommended that the intermediate calculation of component values follow the sequence presented here.

The intermediate values stored and their storage register locations are:

Component Value	Storage Register
Mortgage Payments per Year	0
Total Number of Mortgage Payments to Full Amortization (n_t)	1
Mortgage Interest Rate Per Period (i)	2
Mortgage Constant (F)	3
Sinking Fund Factor at Equity Yield Rate over Income Projection Period ($1/s_n$)	4
Percentage of Mortgage Principal Paid over Income Projection Period (p)	5
Mortgage Coefficient (c)	6
Basic Rate (r)	7

Storage Register 9 is used to store numbers within calculation procedures.

a. Calculation of Mortgage Constant (F)
This calculation uses the routine illustrated in Chapter 3.

Enter:

[CLEAR] [ ] [6] **12** [STO] [0] **25** [×]

[STO] [1] [n] **8.75** [ ] [÷] [STO] [2]

[i] **1** [PV] [PMT] [ ] [×] [STO] [3]

F = .098657

```
                CLEAR
 12.000000      →0
 25.000000      ×
300.000000      ◇
                →1
                → N
  8.750000      N ÷
   .729167      ◇
                →2
                → I
  1.000000      →P
                PMT
   .008221
                N ×
   .098657      ◇
                →3
```

b. Calculation of Sinking Fund Factor ($1/s_n$)

This calculation uses the routine illustrated in Chapter 3. The Equity Yield Rate (y) is entered as [i]. The Income Projection Period is entered as [n].

Enter:

Clear ([ ] [6] has already been entered.)

10 [n] **14** [i] **1** [FV] [PMT] [STO] [4]

$1/s_n$ = .051714

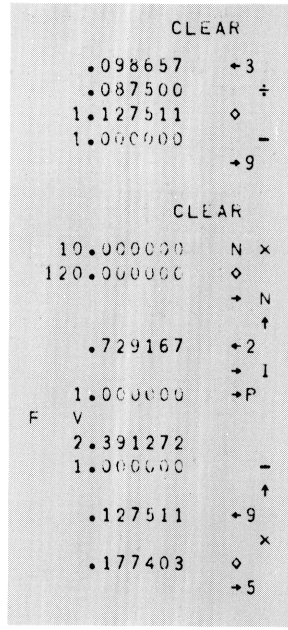
```
                CLEAR
 10.000000      → N
 14.000000      → I
  1.000000      → F
                PMT
   .051714
                →4
```

c. Calculation of Percent of Mortgage Paid Off over the Income Projection Period (p)

There are many different ways to calculate p. Each gives the same result when numbers are entered and set to 6 decimal places. Seven methods are illustrated here, in part because different analysts have individual preferences, and in part to demonstrate that the identical results will be obtained.

1. $p = (\frac{F}{i} - 1)(Sp - 1)$, where Sp is the Future Worth of One factor at the mortgage interest rate (i) over the income projection period (np). i is entered as a decimal figure for arithmetic calculations.

Enter:


CLEAR
.098657 ←3
.087500 ÷
1.127511 ◇
1.000000 -
→9
CLEAR
10.000000 N ×
120.000000 ◇
→ N
↑
.729167 ←2
→ I
1.000000 →P
F V
2.391272
1.000000 -
↑
.127511 ←9
×
.177403 ◇
→5

[CLEAR] ([ ] [6] **is already entered)**

[RCL] [3] **.0875** [÷] **1** [−] [STO] [9]

[CLEAR] **10** [ ] [×] [n] [RCL] [2]

[i] **1** [PV] [FV] **1** [−] [RCL] [9] [×]

[STO] [5]

p = .177403

2. $b = \dfrac{{}^{a}n_{t-p}}{{}^{a}n_{t}}$, where ${}^{a}n_{t-p}$ is the Present Worth of One Per Period factor for the mortgage over its remaining maturity at the end of the income projection period, and ${}^{a}n_{t}$ is the Present Worth of One Per Period factor for the mortgage over its full maturity. b is the mortgage balance outstanding (as a percentage of principal) at the end of the income projection period.

p = 1-b

Enter:

CLEAR (▒ 6 **is already entered)**

RCL 1 n RCL 2 i **1** PMT

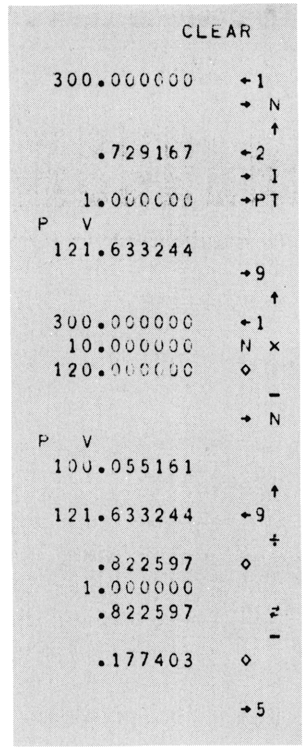
PV STO 9 RCL 1 **10** ▒ ×

− n PV RCL 9 ÷

b = .822597

1 $x \rightleftarrows y$ − ◊ STO 5

p = .177403



3. $b = \dfrac{1/a_{n_t}}{1/a_{n_{t-p}}}$, where $1/a_{n_t}$ is the Installment to Amortize One factor for the mortgage over its full maturity, and $1/a_{n_{t-p}}$ is the Installment to Amortize One factor over the term remaining at the end of the income projection period. b is the percentage of mortgage principal outstanding at the end of the income projection period.

p = 1-b

Enter:

Clear([ ] [6] **is already entered.)**

[RCL] [1] **10** [ ] [×] [−] [n] [RCL]

[2] [i] **1** [PV] [PMT] [STO] [9] [RCL]

[3] [ ] [÷] [RCL] [9] [÷]

b = .822597

1 [$x \rightleftarrows y$] [−] [◊] [STO] [5]

p = .177403

```
              CLEAR

 300.000000    ←1
  10.000000    N  x
 120.000000    ◇
                  -
               →  N
                  ↑
    .729167    ←2
               →  I
   1.000000    →P
              PMT
    .009994
               →9
                  ↑
    .098657    ←3
               N  ÷
    .008221    ◇
                  ↑
    .009994    ←9
                  ÷
    .822597    ◇
   1.000000
    .822597       ⇄
                  -
    .177403    ◇

               →5
```

4. $p = {}^{s}n_{p} \times {}^{1/s}n_{t}$, where ${}^{s}n_{p}$ is the Future Worth of One Per Period factor for the mortgage over the income projection period, and ${}^{1/s}n_{t}$ is the Sinking Fund factor for the mortgage over its full maturity.

Enter:

[CLEAR] **(** [ ] [6] **is already entered)**

10 [ ] [×] [n] [RCL] [2] [i]

1 [PMT] [FV] [STO] [9]

```
              CLEAR

  10.000000    N  x
 120.000000    ◇
               →  N
                  ↑
    .729167    ←2
               →  I
   1.000000    →PT
F  V
 190.802950
               →9
```

[CLEAR] [RCL] [1] [n] [RCL] [2] [i]

1 [FV] [PMT] [RCL] [9] [×] [STO] [5]

p = .177403

```
                CLEAR
300.000000        ←1
                  →  N
                     ↑
   .729167        ←2
                  →  I
  1.000000        →  F
                  PMT
   .000930
                     ↑
190.802950        ←9
                     ×
   .177403        ◇
                  →5
```

5. $p = \frac{sp - 1}{s_t - 1}$, where sp is the Future Worth of One factor at the mortgage interest rate over the income projection period, and s_t is the Future Worth of One factor at the mortgage interest rate over the full maturity of the mortgage.

Enter:

[CLEAR] **([ ] [6] is already entered)**

[RCL] [1] [n] [RCL] [2] [i] **1** [PV]

[FV] [1] [−] [STO] [9] **10** [ ] [×]

[n] [FV] [1] [−] [RCL] [9] [÷] [STO] [5]

p = .177403

```
                CLEAR
300.000000        ←1
                  →  N
                     ↑
   .729167        ←2
                  →  I
  1.000000        →P
F    V
  8.842442
  1.000000           −
                  →9
 10.000000        N  ×
120.000000        ◇
                  →  N
F    V
  2.391272
  1.000000           −
                     ↑
  7.842442        ←9
                     ÷
   .177403        ◇
                  →5
```

6. $p = \dfrac{{}^{s}np}{{}^{s}n_t}$, where ${}^{s}np$ is the Future Worth of One per Period factor at the mortgage interest rate over the income projection period, and ${}^{s}n_t$ is the Future Worth of One per Period factor at the mortgage interest rate over the full maturity of the mortgage.

Enter:

CLEAR **(▒ 6 is already entered)**

RCL 1 n RCL 2 i **1** PMT

FV STO 9 **10** ▒ × n FV

RCL **9** ÷ STO 5

p = .177403

```
                 CLEAR
 300.000000    ←1
               → N
                 ↑
    .729167    ←2
               → I
   1.000000    →PT
F  V
1075.534911
               →9
  10.000000    N ×
 120.000000    ◇
               → N
F  V
 190.802950
                 ↑
1075.534911    ←9
                 ÷
    .177403    ◇
               →5
```

7. $p = \dfrac{1/s_{\,n_t}}{1/s_{\,n\,p}}$, where $1/s_{\,n\,t}$ is the Sinking Fund Factor for the mortgage over its full maturity, and $1/s_{\,n\,p}$ is the Sinking Fund Factor for the mortgage over the income projection period.

Enter:

CLEAR (▒ 6 is already entered)

10 ▒ × n RCL 2 i

1 FV PMT STO 9 RCL 1 n

PMT RCL 9 ÷ STO 5

p = .177403

	CLEAR
10.000000	N ×
120.000000	◇
	→ N
	↑
.729167	←2
	→ I
1.000000	→ F
	PMT
.005241	
	→9
	↑
300.000000	←1
	→ N
	PMT
.000930	
	↑
.005241	←9
	÷
.177403	◇
	→5

NOTE:
In subsequent examples and illustrations in this manual Procedure 3 is used, primarily because F is already calculated and stored.

NOTE:
The remaining balance on a mortgage (b), and therefore its complement (p), can also be computed using either the Accumulated Interest Paid between Periods and Remaining Balance or Amortized Loan Schedule routines illustrated in Chapters 6 and 7. These are lengthy procedures, however, and are not recommended for use in Mortgage-Equity Analysis.

Simply to illustrate their applicability and consistency with seven procedures demonstrated above, the routines and results in this case would be:

Accumulated Interest Paid Between Periods and Remaining Balance

Enter:

[CLEAR] **12** [STO] [0] **25** [×] [n] **8.75**

[ ] [÷] [i] **1** [PV] [PMT] [STO] [PMT]

[CLEAR] **1** [STO] [1] **10** [ ] [×]

[STO] [2] [RCL] [i] [i] [RCL] [PMT]

[PMT] **1** [PV] [Σ+]

b = .822598

1 [SAVE↑] **.822598** [−] [◊]

p = .177402

```
                CLEAR

  12.000000     →0
  25.000000      ×
 300.000000     ◊
                → N
   8.750000     N ÷
    .729167     ◊
                → I
   1.000000     →P
                PMT
    .008221
                →7*
                CLEAR
   1.000000     →1
  10.000000     N ×
 120.000000     ◊
                →2
    .729167     ← I
                → I
    .008221     →PT
                →PT
   1.000000     →P
                Σ+
P V
    .822598
I N T
    .809170
   1.000000      ↑
    .822598      −
    .177402     ◊
```

Amortized Loan Schedule

Enter:

[CLEAR] [ ] [6] **12** [STO] [0] **25** [×]

[n] **8.75** [ ] [÷] [i] **1** [PV] [PMT]

[STO] [PMT] [CLEAR] **10** [ ] [×] [STO]

[1] **10** [ ] [×] [STO] [2] [RCL] [PV]

[PV] [RCL] [i] [i] [RCL] [PMT] [PMT]

[EXT( )] [6]

b = .822598

1 [SAVE↑] **.822598** [−] [◊]

p = .177402

```
                CLEAR

   12.000000        →0
   25.000000         ×
  300.000000        ◇
                    → N
    8.750000        N ÷
     .729167        ◇
                    → I
    1.000000        →P
                    PMT
     .008221
                    →7*

                CLEAR

   10.000000        N ×
  120.000000        ◇
                    →1
   10.000000        N ×
  120.000000        ◇
                    →2
    1.000000        ←P
                    →P
     .729167        ← I
                    → I
     .008221        ←PT
                    →PT
120
     .006014
     .002207
     .822598
I N T
     .006014
    1.000000         ↑
     .822598         −
     .177402        ◇
```

Calculation of Mortgage Coefficient (c)

$$c = y + p\ 1/s_n\ \text{-F}$$

The Equity Yield Rate (y) is entered as a decimal figure. The other values are already calculated and available in storage.

Enter:

Clear (6 is already entered)

.14 SAVE ↑ RCL 5 RCL 4 ×

\+ RCL 3 − ◊

c = .050517 STO 6

```
      CLEAR

.140000     ↑
.177403    ←5
            ↑
.051714    ←4
            ×
.009174     ◊
            +
            ↑
.098657    ←3
            −
.050517     ◊

           →6
```

Calculation of Basic Rate or Discount Rate (r)

$$r = y - mc$$

y and m are entered as decimal figures. c is already calculated and available in storage register 6.

Enter:

CLEAR **.14** SAVE ↑ **.70** RCL **6** ×

− ◊ STO 7

r = .104638 or 10.46%

```
      CLEAR

.140000     ↑
.700000     ↑
.050517    ←6
            ×
.035362     ◊
            −
.104638     ◊

           →7
```

Calculation of Overall Rate (R) - Ellwood Format

$$R = r + \text{dep.}\ 1/s_n$$

dep. is entered as a decimal figure. r and $1/s_n$ are already calculated and available in storage.

Enter:

CLEAR RCL 7 **.15** RCL 4 ×

\+ ◊

R = .112395 or 11.24%

```
      CLEAR

.104638    ←7
.150000     ↑
.051714    ←4
            ×
.007757     ◊
            +
.112395     ◊
```

Calculation of Overall Rate (R) - Built-up Method

$R = mF + (1\text{-}m)y - mp\ 1/s_n + \text{dep.}\ 1/s_n$

m, y and dep. are entered as decimal figures. F, p and $1/s_n$ are already calculated and available in storage.

Enter:

CLEAR .70 RCL 3 × 1 SAVE↑

70 − .14 × + .70 RCL 5 ×

RCL 4 × − ◇

(This calculates and prints out r.)

.15 RCL 4 × + ◇

R = .112395 or 11.24%

```
        CLEAR

 .700000      ↑
 .098657     ←3
              ×
 .069060      ◇
1.000000      ↑
 .700000      −
 .140000      ×
 .042000      ◇
              +
 .700000      ↑
 .177403     ←5
              ×
 .124182      ◇
              ↑
 .051714     ←4
              ×
 .006422      ◇
              −
 .104638      ◇

 .150000      ↑
 .051714     ←4
              ×
 .007757      ◇
              +
 .112395      ◇
```

CALCULATION OF VALUE (PRESENT WORTH) WITH R, GIVEN ONLY NOI

When R is calculated, as above, Value is estimated by the formula:

$$V = \frac{NOI}{R}\ .$$

Example 1 - In the preceding examples R = .112395. If NOI is forecast at $33,500, what is the estimated value or present worth of the property?

Enter:

CLEAR 33500 SAVE↑ .112395 ÷

```
        CLEAR

 33500.00     ↑
  .112395     ÷
298055.96     ◇
```

V or PW = $298,055.96 (probably rounded to $300,000)

Example 2 - A property is forecast to produce NOI of $24,550 annually. The most probable mortgage loan terms are an 82% loan with level monthly payments at 9.25% interest over a maturity of 22 years 8 months. The investor expects to hold the property for 12 years and then sell it at 20% above its present value. If the investor is looking for a 15.35% rate of return on equity investment, what is the value (present worth) of the property? What is the indicated present worth of the equity investment position?

NOTE:
The given values are:

m = .82
y = .1535
i = .0925 or 9.25%
n = 12 (years)
app. = .20 or 20%
NOI = $24,550

Enter:

CLEAR 6

1. To calculate and store p:

12 STO 0 **22** × **8** + n **9.25**
÷ i **1** PV PMT STO 3
12 × n PMT RCL 3
$x \rightleftarrows y$ ÷ **1** $x \rightleftarrows y$ − ◇ STO 5

p = .236386

CLEAR	
12.000000	→0
22.000000	×
264.000000	◇
8.000000	+
	→ N
9.250000	N ÷
.770833	◇
	→ I
1.000000	→P
	PMT
.008798	
	→3
12.000000	N ×
144.000000	◇
	→ N
	PMT
.011522	
	↑
.008798	←3
.011522	⇄
	÷
.763614	◇
1.000000	
.763614	⇄
	−
.236386	◇
	→5

2. To calculate and store F:

[RCL] [3] [ ] [×] [STO] [3]

F = .105576

```
                ↑
  .008798     →3
              N ×
  .105576      ◊
              →3
```

3. To calculate and store $1/s_n$:

[CLEAR] **12** [n] **15.35** [i] **1** [FV] [PMT]

[STO] [4]

$1/s_n$ = .033744

```
            CLEAR

12.000000    → N
15.350000    → I
 1.000000    → F
             PMT
  .033744
             →4
```

4. To calculate and store c:

[CLEAR] **.1535** [SAVE↑] [RCL] [5]

[RCL] [4] [×] [+] [RCL] [3] [−]. [◊]

[STO] [6]

c = .055900

```
            CLEAR

  .153500      ↑
  .236386     →5
               ↑
  .033744     →4
               ×
  .007977    ◊
               +
               ↑
  .105576     →3
               −
  .055900    ◊

              →6
```

5. To calculate R (and r):

[CLEAR] **.1535** [SAVE↑] [RCL] [6] **.82**

[×] [−] [◊]

r = .107662

[RCL] [4] **.20** [×] [−] [◊]

R = .100913

```
          CLEAR

  .153500      ↑
  .055900     →6
  .820000      ×
  .045838    ◊
               −
  .107662    ◊

               ↑
  .033744     →4
  .200000      ×
  .006749    ◊
               −
  .100913    ◊
```

6. To calculate V:

24550 [2] [x⇄y] [÷]

V = $243,278.82

(probably rounded to $245,000)

```
 24550.00
      .10   ⇄
            ÷
243278.82  ◊
    82.00   %
199488.63  ◊
            -
 43790.19  ◊
```

7. To calculate V_e :

82 [%] [◊] [−] [◊]

Mortgage Principal = $199,488.63

Present Worth of Equity = $43,790.19

NOTE:

The following procedure can be substituted for Steps 4 and 5 if the built-up method for calculating r and R is used:

[CLEAR] **6** [RCL] [3] **.82** [×] **1**

[SAVE ↑] **.82** [−] **.1535** [×] [+] [RCL] [5]

[RCL] [4] [×] **.82** [×] [−] [◊]

r = .107662 (the same as before)

[RCL] [4] **.20** [×] [−] [◊]

R = .100913 (the same as before)

```
           CLEAR
 .105576  ←3
 .820000     ×
 .086573  ◊
1.000000     ↑
 .820000     -
 .153500     ×
 .027630  ◊
             +
             ↑
 .236386  ←5
             ↑
 .033744  ←4
             ×
 .007977  ◊
 .820000     ×
 .006541  ◊
             -
 .107662  ◊
             ↑
 .033744  ←4
 .200000     ×
 .006749  ◊
             -
 .100913  ◊
```

CALCULATION OF EQUITY DIVIDEND RATE (e)

The Equity Dividend Rate (e) is applied directly to Cash Throw-Off to Equity to find the present worth of the equity investment position:

$$V_e = \frac{CTO}{e}$$

The Equity Dividend Rate is calculated when CTO and the amount of the equity investment are known in dollar amounts by the formula:

$$e = \frac{CTO}{V_e}$$

Example - The equity investment in an income property is $43,790. NOI is forecast at $24,550, while Annual Debt Service is $21,061. What is the indicated Equity Dividend Rate (e)?

Enter:

[CLEAR] $24,550 [SAVE ↑] 21061 [−] [◊]

cto = $3,489

43790 [÷] [ ] [6] [◊]

e = .079676 (7.97%)

```
             CLEAR

24550.00        ↑
21061.00        -
 3489.00    ◊

43790.00        +
     .08    ◊
 .079676    ◊
```

When dollar amounts are not available, the Equity Dividend Rate can be calculated with all the data used to calculate R, as illustrated in the preceding examples. The formula is:

$$e = y - \frac{mp\ 1/s_n}{(1\text{-}m)} - \frac{app.1/s_n}{(1\text{-}m)} \quad \text{or} \quad e = y - \frac{mp\ 1/s_n}{1 - m} + \frac{dep.1/s_n}{1 - m}$$

Example - An income property has an 82% mortgage with level monthly payments at 9.25% interest fully amortized in 22 years 8 months. The equity investor is seeking a 15.35% Equity Yield Rate over the income projection of 12 years. What is the indicated Equity Dividend Rate, if the proceeds of resale are forecast to be 20% above present value of the property?

NOTE:

From the data above and preceding examples using the same data, the following values apply:

$$
\begin{aligned}
y &= .14 \\
m &= .82 \\
p &= .236386 \\
\text{app.} &= .20 \\
(1\text{-}m) &= .18 \\
1/s_n &= .033744
\end{aligned}
$$

Enter:

[CLEAR] [ ] [6] 1 [SAVE ↑] .82 [−]

[STO] [0] [This stores (1-m)] .1535

[SAVE ↑] .82 [SAVE ↑] .236386 [×]

.033744 [STO] [1] (This stores $1/s_n$) [×]

[RCL] [0] [÷] [−] [RCL] [1]

.20 [×] [RCL] [0] [÷] [−] [◊]

e = .079669 (7.97%)

```
           CLEAR
1.000000       ↑
 .820000       −
              →0
 .153500       ↑
 .820000       ↑
 .236386       ×
 .193837     ◊
 .033744      →1
               ×
 .006541     ◊
               ↑
 .180000      ←0
               +
 .036338     ◊
               −
               ↑
 .033744      ←1
 .200000       ×
 .006749     ◊
               ↑
 .180000      ←0
               ÷
 .037493     ◊
               −
 .079669     ◊
```

CALCULATION OF VALUE (PRESENT WORTH) WITH DOLLAR AMOUNTS GIVEN

This procedure involves calculating the present worth of the future income stream and the present worth of the reversion, and then adding the two together to derive the present worth of the investment. It can be used to estimate property value using NOI, the discount or basic rate (r), and the Proceeds of Resale. It can also be used to estimate the present worth of the equity investment position using CTO, y, and the Net Cash Proceeds of Resale.

The routines employed are the Present Worth of One per Period and the Present Worth of One, both illustrated in Chapter 2. The procedure is the same as that illustrated for the Present Worth of a Balloon-Payment Mortgage in Chapter 5.

Example 1 - An income property is forecast to produce NOI of $20,575 per year. It has just been financed with a $160,000 mortgage, to be fully amortized in level monthly payments at 8.75% interest over 25 years. The anticipated proceeds of resale of the property in 10 years is $191,250. The equity investor expects an Equity Yield Rate of 14.75%.

What is the present worth of the equity investment position? What is the present worth (value) of the property?

Enter:

[CLEAR]

a. Calculate Monthly Mortgage Payment

12 [STO] [0] 25 [×] [n] 8.75 [ ]

[÷] [i] 160000 [PV] [PMT] [STO] [PMT]

Monthly payment is $1315.43

b. Calculate Annual Debt Service

[ ] [×]

Annual Debt Service is $15,785.16

c. Calculate and Store CTO

20575 [x⇄y] [−] [◊] [STO] [1]

CTO is $4789.84

```
              CLEAR

      12.00    →0
      25.00     ×
     300.00    ◊
               →  N
       8.75    N  +
        .73    ◊
               →  I
  160000.00    →P
               PMT
    1315.43
               →7*
               N  ×

   15785.16    ◊
   20575.00
   15785.16       ⇄
                  −
    4789.84    ◊

               →1
```

d. Calculate Mortgage Balance in 10 Years

CLEAR RCL i i RCL PMT

PMT 160000 PV n

True n is 300.00 months.

10 × − n

Remaining n

CLEAR RCL n n RCL i i

RCL PMT PMT PV

Balance is $131,615.55

e. Calculate Net Cash Proceeds of Resale

191250 $x \rightleftarrows y$ − ◊ STO 2

NCPR is $59,634.45

f. Calculate Present Worth of CTO

CLEAR 10 n 14.75 i RCL 1

PMT PV STO 0

PW of CTO is $24,269.95

g. Calculate Present Worth of NCPR

CLEAR RCL n n RCL i i

RCL 2 FV PV

PW of NCPR is $15,065.04

```
                 CLEAR
            .73    ← I
                   → I
        1315.43    ←PT
                   →PT
      160000.00    →P
   N
         300.00
          10.00    N  ×
         120.00    ◊
                      −
                   →  N
                 CLEAR
         180.00    ←  N
                   →  N
            .73    ←  I
                   →  I
        1315.43    ←PT
                   →PT
 P    V
      131615.55

      191250.00
      131615.55       ⇄
                      −
       59634.45    ◊

                   →2

                 CLEAR
          10.00    →  N
          14.75    →  I
                      ↑
        4789.84    ←1
                   →PT
 P    V
       24269.95
                   →0

                 CLEAR
          10.00    ←  N
                   →  N
          14.75    ←  I
                   →  I
                      ↑
       59634.45    ←2
                   →  F
```

h. Calculate Present Worth of Equity

[RCL] [0] [+] [◊]

PW Equity is $39,334.99

i. Calculate Present Worth (Value) of Property

160000 [+] [◊]

PW is $199,334.99

```
P  V
   15065.04
                 ↑
   24269.95    ←0
                 +
   39334.99     ◊

  160000.00      +
  199334.99     ◊
```

CALCULATION OF CAPITAL APPRECIATION OR DEPRECIATION ON RESALE, PLUS RESALE PRICE REQUIRED TO ACHIEVE A GIVEN EQUITY YIELD RATE

The percent of capital appreciation or depreciation on resale required to achieve a given equity yield rate can be calculated using either rates or dollar amounts. In addition, when dollar amounts are available, it is possible to calculate the dollar amount of resale proceeds required. The calculations can be applied to either NOI or CTO cash flows.

a. Calculation of dep. or app. Using Rates

The formula for dep. or app. is:

$$\text{App.} = \frac{r - R}{1/s_n} \qquad \text{Dep.} = \frac{R - r}{1/s_n}$$

Example - An investment is producing NOI at an Overall Rate (R) of 10.25%. It has just been financed with a 70% mortgage at 9% interest, fully amortized in level monthly payments over 20 years. What must resale proceeds be at the end of 12 years for the investor to earn a 13% rate of return on the equity investment? What if the desired Equity Yield Rate is 16%?

NOTE:
Both r and $1/s_n$ vary with the value of y.

Enter:

CLEAR [ ] 6

a. Calculate p and F .

12 STO 0 20 × n 9 [ ] ÷

i 1 PV PMT STO 1 RCL n

10 [ ] × − n PMT RCL 1

$x \rightleftarrows y$ ÷ 1 $x \rightleftarrows y$ − ◊ STO 2

p = .289741

RCL 1 [ ] × STO 3

F = .107967

b. Calculate $1/s_n$ at 13%.

CLEAR 10 n 13 i 1 FV PMT

STO 4

$1/s_n$ at 13% = .054290

```
              CLEAR

  12.000000    →0
  20.000000      ×
 240.000000    ◊
               → N
   9.000000    N ÷
    .750000    ◊
               → I
   1.000000    →P
               PMT
    .008997
               →1
 240.000000    ← N
  10.000000    N ×
 120.000000    ◊
                 −
               → N
               PMT
    .012668
                 ↑
    .008997    ←1
    .012668      ⇄
                 +
    .710259    ◊
   1.000000
    .710259      ⇄
                 −
    .289741    ◊

               →2
                 ↑
    .008997    ←1
               N ×
    .107967    ◊
               →3

              CLEAR

  10.000000    → N
  13.000000    → I
   1.000000    → F
               PMT
    .054290
               →4
```

c. Calculate $1/s_n$ at 16%.

16 [i] [PMT] [STO] [5]

$1/s_n$ at 16% = .046901

```
16.000000   → I
              PMT
  .046901
  .          →5
```

d. Calculate r at 13%.

[RCL] [3] **.70** [×] **1** [SAVE↑] **.70** [−]

.13 [×] [+] [RCL] [2] [RCL] [4] [×]

.70 [×] [−] [◊]

r = .103566

```
                ↑
  .107967     ←3
  .700000      ×
  .075577      ◊
 1.000000      ↑
  .700000      −
  .130000      ×
  .039000      ◊
               +
               ↑
  .289741     ←2
               ↑
  .054290     ←4
               ×
  .015730      ◊
  .700000      ×
  .011011      ◊
               −
  .103566      ◊
```

e. Calculate app./dep. at 13%.

.1025 [−] [RCL] **4** [÷]

app./dep. = .019636

```
  .102500      −
               ↑
  .054290     ←4
               ÷
  .019636      ◊
```

f. Calculate r at 16%.

[CLEAR] [RCL] [3] **.70** [×] **1** [SAVE↑]

.70 [−] **.16** [×] [+] [RCL] [2] [RCL]

[5] [×] **.70** [×] [−] [◊]

r = .114065

```
         CLEAR
  .107967     ←3
  .700000      ×
  .075577      ◊
 1.000000      ↑
  .700000      −
  .160000      ×
  .048000      ◊
               +
               ↑
  .289741     ←2
               ↑
  .046901     ←5
               ×
  .013589      ◊
  .700000      ×
  .009512      ◊
               −
  .114065      ◊
```

g. Calculate app./dep. at 16%.

.1025 [−] [RCL] [5] [÷]

app. = .246573

.102500	−
	↑
.046901	←5
	+
.246573	◇

To earn an Equity Yield Rate of 13% , Proceeds of Resale must be 1.96% higher than the original purchase price or value.

To earn an Equity Yield Rate of 16%, Proceeds of Resale must be 24.66% higher than the original purchase price or value.

b. Calculation of dep. or app. Using Dollar Figures

These procedures center on the calculation of what the dollar amount of the reversion (PR or NCPR) must be to achieve a given or desired rate of return (r or y).

In one procedure, the net amount of Future Worth is derived as the amount of the reversion. In another procedure, the net amount of Present Worth of the investment position not covered by periodic income is derived, and the amount of reversion required to cover that net amount of Present Worth is then calculated.

Example 1 - Future Sales Price, Amount of Equity Reversion and app./dep. Required to Achieve a Given Equity Yield Rate.

An investment property is for sale for $100,000. It is expected to produce NOI of $11,000 per year. It can be financed with a $70,000 mortgage at 9% interest, fully amortized in level monthly payments over 20 years. What must the property sell for in 10 years for the investor to earn a 13% rate of return (y) on the equity investment? What must the equity reversion be? What percentage of dep. or app. is involved?

a. Future Worth Method

The keystrokes and steps in this procedure are:

1. Calculate mortgage balance at end of income projection period, press [STO] [1].

2. Calculate Cash Throw-Off to Equity, press [STO] [2] .

3. Calculate Future Worth of CTO: Key in income projection period, press [n] ; key in desired equity yield rate, press [i] ; press [RCL] [2] [PMT] [FV] [STO] [0] [CLEAR] .

4. Calculate Future Worth of Equity Investment: press [RCL] [n] [n] [RCL] [i] [i] ; key in amount of equity investment, press [PV] [FV] .

5. Obtain required equity reversion by subtracting Future Worth of CTO: press [RCL] [0] [−] [◊] .

6. Obtain required proceeds of resale by adding mortgage balance: press [RCL] [1] [+] [◊] .

7. Obtain percentage app. or dep. by: key in original property investment, press [$x \rightleftarrows y$] [ ] [%] [◊] .

(app. is a positive number; dep. is negative.)

Enter:

[CLEAR] **12** [STO] [0] **20** [×] [n] **9** [ ] [÷] [i] **70000** [PV] [PMT] [STO] [PMT] [CLEAR]

Monthly Payment is $629.81.

[RCL] [n] **10** [ ] [×] [−] [n] [RCL] [i] [i] [RCL] [PMT] [PMT] [PV] [STO] [1]

Mortgage balance is $49,718.12.

11000 [SAVE↑] **629.81** [ ] [×] [−] [◊]

CTO is $3,442.28.

```
           CLEAR

   12.00    →0
   20.00     ×
  240.00    ◊
            → N
    9.00    N ÷
     .75    ◊
            → I
70000.00    →P
            PMT
  629.81
            →7*
           CLEAR

  240.00    ← N
   10.00    N ×
  120.00    ◊
              −
            → N
     .75    ← I
            → I
  629.81    ←PT
            →PT
```

[STO] [2] [CLEAR] 10 [n] 13 [i] [RCL]

[2] [PMT] [FV] [STO] [0] [CLEAR]

Future Worth of CTO is $63,405.93.

[RCL] [n] [n] [RCL] [i] [i] 30000

[PV] [FV]

Future Worth of Equity Investment is $101,887.02.

[RCL] [0] [−] [◊]

Required NCPR is $38,431.09.

[RCL] [1] [+] [◊]

Required Resale Proceeds are $88,149.21

100000 [$x \rightleftarrows y$] [ ] [%] [◊]

dep. allowable is 11.85%.

```
P    V
      49718.12
                  →1
      11000.00     ↑
        629.81    N ×
       7557.72    ◊
                   -
       3442.28    ◊

                  →2

              CLEAR
         10.00    → N
         13.00    → I
                   ↑
       3442.28    ←2
                  →PT
F    V
      63405.93
                  →0
              CLEAR
         10.00    ← N
                  → N
         13.00    ← I
                  → I
      30000.00    →P
F    V
     101837.02
                   ↑
      63405.93    ←0
                   -
      38431.09    ◊

                   ↑
      49718.12    ←1
                   +
      88149.21    ◊

     100000.00
      88149.21     ⇄
                  Δ %
-        11.85    ◊
```

b. Present Worth Method

The keystrokes and steps involved are:

1. Calculate mortgage balance at end of income projection period, press [STO] [1].

2. Calculate Cash Throw-Off, press [STO] [2].

3. Calculate Present Worth of CTO: press [CLEAR]; key in income projection period, press [n]; key in desired Equity Yield Rate, press [i]; enter CTO by pressing [RCL] [2] [PMT]; press [PV].

4. Find equity investment not covered by Present Worth of CTO: key in equity investment, press [$x \rightleftarrows y$] [−] [◊] [PV] [CLEAR]

5. Find required equity reversion: press [RCL] [n] [n] [RCL] [i] [i] [RCL] [PV] [PV] [FV].

6. Find required proceeds of resale by adding mortgage balance: press [RCL] [1] [+] [◊].

7. Find percentage dep. or app.: key in original property investment, press [$x \rightleftarrows y$] [ ] [%] [◊].

(app. is a positive figure; dep. is negative.)

Enter:

[CLEAR] **12** [STO] [0] **20** [×] [n] **9** [ ] [÷] [i] **70000** [PV] [PMT] [STO] [PMT] [CLEAR] [RCL] [n] **10** [ ] [×] [−] [n] [RCL] [i] [i] [RCL] [PMT] [PMT] [PV] [STO] [1]

b = $49,718.12

11000 [SAVE↑] **629.81** [ ] [×] [−] [◊] [STO] [2]

CTO = $3442.28

```
          CLEAR
    12.00    →0
    20.00     ×
   240.00    ◊
             → N
     9.00    N +
      .75    ◊
             → I
 70000.00    →P
             PMT
   629.81
             →7*
          CLEAR
   240.00    ← N
    10.00    N ×
   120.00    ◊
              -
             → N
      .75    ← I
             → I
   629.81    ←PT
             →PT
P  V
 49718.12
             →1
 11000.00     ↑
   629.81    N ×
  7557.72    ◊
              -
  3442.28    ◊
             →2
```

[CLEAR] **10** [n] **13** [i] [RCL] [2]

[PMT] [PV]

PW of CTO = $18,678.65

30000 [$x \rightleftarrows y$] [−] [◊] [PV] **= $11,321.35**

(Equity investment not covered by PW CTO)

[CLEAR] [RCL] [n] [n] [RCL] [i] [i]

[RCL] [PV] [PV] [FV]

Required NCPR = $38,431.09.

[RCL] [1] [+] [◊]

Required PR = $88,149.21.

100000 [$x \rightleftarrows y$] [ ] [%] [◊]

dep. = 11.85%

```
                CLEAR

           10.00    → N
           13.00    → I
                      ↑
          3442.28   ←2
                    →PT
   P    V
        18678.65
        30000.00
        18678.65      ⇄
                      −
        11321.35    ◊

                    →P
                CLEAR

           10.00    ← N
                    → N
           13.00    ← I
                    → I
        11321.35    ←P
                    →P
   F    V
        38431.09
                      ↑
        49718.12    ←1
                      +
        88149.21    ◊

       100000.00
        88149.21      ⇄
                    Δ %
−          11.85    ◊
```

Example 2 - Future Sales Price (Resale Proceeds) and Percentage app. or dep. Required to Achieve aGiven Discount Rate (r).

An investment property was recently acquired for $65,800. NOI is forecast to be $6,350 per year. What must it resell for (net) in 12 years to produce a rate of return (r) of 10.45% on the total property investment? What percentage app. or dep. over the original purchase price does this represent?

a. Future Worth Method

Enter:

[CLEAR] 12 [n] 10.45 [i] 6350 [PMT]

[FV] [STO] [0] [CLEAR] [RCL] [n] [n]

[RCL] [i] [i] 65800 [PV] [FV] [RCL]

[0] [−] [◊]

Required resale proceeds = $77,359.15

65800 [$x \rightleftarrows y$] [ ] [%] [◊]

Required app. is 17.57%

```
           CLEAR
      12.00    → N
      10.45    → I
    6350.00    →PT
F   V
 139518.37
               →0
           CLEAR
      12.00    ← N
               → N
      10.45    ← I
               → I
   65800.00    →P
F   V
 216877.52
                 ↑
 139518.37     ←0
                 -
   77359.15    ◇
   65800.00
   77359.15      ⇄
               Δ %
      17.57    ◇
```

b. Present Worth Method

Enter:

[CLEAR] 12 [n] 10.45 [i] 6350 [PMT] [PV]

65800 [$x \rightleftarrows y$] [−] [◊] [PV] [CLEAR]

[RCL] [n] [n] [RCL] [i] [i] [RCL] [PV]

[PV] [FV]

Required resale proceeds = $77,359.15

65800 [$x \rightleftarrows y$] [ ] [%] [◊]

Required app. is 17.57%

```
           CLEAR
      12.00    → N
      10.45    → I
    6350.00    →PT
P   V
   42329.46
   65800.00
   42329.46      ⇄
                 -
   23470.54    ◇
               →P
           CLEAR
      12.00    ← N
               → N
      10.45    ← I
               → I
   23470.54    ←P
               →P
F   V
   77359.15
   65800.00
   77359.15      ⇄
               Δ %
      17.57    ◇
```

CALCULATION OF EQUITY YIELD RATE (y) FROM DOLLAR FIGURES AS AN INTERNAL RATE OF RETURN

The income stream conventionally forecast in Mortgage-Equity or Ellwood Analysis is a level annuity plus a reversion. It is either a level NOI flow plus Proceeds of Resale, or a level CTO flow plus NCPR.

The routine to find rate of return on investment as an Internal Rate of Return is the same as those illustrated for a level annuity plus reversion (or balloon payment) illustrated in Chapters 5 and 9.

The equity yield rate (y) can be calculated on the equity investment using CTO and NCPR; the discount rate or basic rate (r) can be calculated on the property investment using NOI and PR.

Example 1 - An investor has just purchased an income property for $123,750. A mortgage of $95,000 was obtained, with level monthly payments of $819.93. NOI is forecast at $13,200. The investor plans to hold the property for 12 years and then resell it. Anticipated resale proceeds are $135,000, at which time the mortgage balance will be $67,315.

What is the indicated equity yield rate?

What is the indicated discount rate?

a. Equity Yield Rate

Enter:

[CLEAR] [ ] [EXT( )] [2] **135000**

[SAVE ↑] **67315** [−] [FV] **12** [SAVE ↑] **365**

[×] [STO] [DAY] **13200** [SAVE ↑] **819.93**

[SAVE ↑] **12** [×] [−] [PMT] **123750**

[SAVE ↑] **95000** [−] [PV] [ ] [i]

y = 16.06%

```
                 CLEAR

                   AN2

 135000.00           ↑
  67315.00           -
                   → F
     12.00           ↑
    365.00           ×
   4380.00         ◇
                   →4*
  13200.00           ↑
    819.93           ↑
     12.00           ×
   9839.16         ◇
                     -
                   →PT
 123750.00           ↑
  95000.00           -
                    →P
  YIELD
  TO            CALL
     16.06
```

b. Discount Rate

Enter:

[CLEAR] [ ] [EXT( )] [2] **135000** [FV]

12 [SAVE ↑] **365** [×] [STO] [DAY] **13200**

[PMT] **123750** [PV] [ ] [i]

r = 11.07%

```
                  CLEAR

                    AN2
 135000.00        → F
     12.00          ↑
    365.00          ×
   4380.00        ◇
                  →4*
  13200.00        →PT
 123750.00        →P
  YIELD
  TO           CALL
     11.07
```

CALCULATION OF EQUITY YIELD RATE FROM EQUITY DIVIDEND RATE

This procedure utilizes the formula presented earlier for the relationship between the Equity Yield Rate (y) and the Equity Dividend Rate.

The formula is:

$$y = e + \frac{mp\ 1/s_n}{1 - m} - \frac{dep.\ 1/s_n}{1 - m} \qquad y = e + \frac{mp\ 1/s_n}{1 - m} - \frac{app.\ 1/s_n}{1 - m}$$

Example - Using the property income and price information in Example 1 in the preceding section, what is the Equity Yield Rate, given that CTO is $3361? The values of components are:

e = .1169	p = .2914
m = .7677	1 - m = .2323
app. = .0909	$1/s_n$ = .0323

Enter:

[CLEAR] [ ] **4** **.1169** [SAVE ↑] **.7677**

[SAVE ↑] **.2914** [×] **.0323** [×] **.2323** [÷] [+]

.0909 [SAVE ↑] **.0323** [×] **.2323** [÷] [+] [◇]

y = .1606 or 16.06%

```
          CLEAR

   .1169        ↑
   .7677        ↑
   .2914        ×
   .2237     ◇
   .0323        ×
   .0072     ◇
   .2323        ÷
   .0311     ◇
                +
   .0909        ↑
   .0323        ×
   .0029     ◇
   .2323        ÷
   .0126     ◇
                +
   .1606     ◇
```

NOTE:

If Cash Flows (NOI or CTO) are uneven, then the procedure to calculate r or y as Internal Rate of Return is one of the Variable annuity routines presented and illustrated in Chapter 9.

CHAPTER 11 - DEPRECIATION

Depreciation is a method of allocating the cost of an asset over its useful life. Depreciation is an annual deduction from Net Operating Income before taxable income and income tax liability are figured. It is an accounting expense charged against cash income.

The three most common accounting methods are Straight Line, Declining Balance, and Sum-of-the Year's Digits (SOYD) depreciation. Declining balance and SOYD are methods of "accelerated" depreciation whereby higher depreciation amounts are charged in the early years of an asset's life than with straight line depreciation.

It is important to note that land is not depreciable for income tax purposes. Therefore, the acquisition cost of improvements (buildings, fixtures, site improvements) must be separated from land value at time of property acquisition before a depreciation schedule can be calculated.

If there is an expected salvage value at the end of the useful life of the depreciable asset, the salvage value must be deducted from the asset's acquisition to derive the amount to be depreciated (tax basis), if the straight line or SOYD methods are to be used. Salvage value need not be subtracted from acquisition cost under the declining-balance method, but the asset may not be depreciated below salvage value. However, for most real estate depreciation problems, salvage value is not a consideration or a legal requirement.

If accelerated depreciation is used, the difference between total depreciation charged over a given period of time and the total amount that would have been charged under straight line depreciation is called "excess depreciation". With some minor exceptions, the amount of "excess depreciation" is "recaptured" and taxed as ordinary income when the property is resold.

The HP-81 routines on depreciation allow the analyst to:

1) Calculate and print the schedule of annual depreciation and remaining balance at the end of each year, using either straight line, declining balance or SOYD depreciation.
2) Calculate total depreciation charged over a given period.
3) Calculate "excess" depreciation at the end of any given period.
4) Calculate and identify the "crossover" period when striaght line depreciation equals or exceeds accelerated depreciation, at which point it pays to switch to straight line depreciation.
5) Calculate partial-year depreciation at the beginning and end of the depreciation period.

Continuing Example: To illustrate the several routines and procedures on the HP-81 that provide depreciation calculations, and to show the differences resulting from the application of different depreciation methods, a continuing example will be used.

A property has just been acquired for $150,000. The purchase price is allocated between $25,000 for land and $125,000 for improvements (building). The remaining useful life of the building is agreed to be 25 years. There is no salvage value forecast at the end of the useful life of the building.

Thus, the depreciable cost is $125,000. This is also the tax basis of the investment in the building.

STRAIGHT LINE DEPRECIATION

In straight-line depreciation, an equal amount of depreciation is deducted from the balance of depreciable cost each year.

a. Calculation of Depreciation Schedule and Annual Balance

The keystrokes are:

1) Key in depreciable amount, press [SAVE ↑] [SAVE ↑]

2) Key in useful life in years, press [÷] (this prints the amount of annual depreciation)

3) To find and print the first year's end-of-year balance, press [STO] [1] [−] [◊]

4) To find and print each successive year's end-of-year balance press [RCL] [1] [−] [◊]

Example: What is the schedule of straight-line depreciation and remaining depreciable balance for 10 years?

Enter: [CLEAR]

125000 [SAVE ↑] [SAVE ↑] **25** [÷]

Annual depreciation is $5,000

[STO] [1] [−] [◊] **Year 1 balance = $120,000**

[RCL] [1] [−] [◊] **Year 2 balance = $115,000**

[RCL] [1] [−] [◊] **Year 3 balance = $110,000**

[RCL] [1] [−] [◊] **Year 4 balance = $105,000**

```
             CLEAR
125000.00         ↑
                  ↑
    25.00         ÷
  5000.00      ◊
                 →1
                  −
120000.00      ◊
                  ↑
  5000.00        →1
                  −
115000.00      ◊
                  ↑
```

[RCL] [1] [−] [◊] **Year 5 balance = $100,000**

[RCL] [1] [−] [◊] **Year 6 balance = $95,000**

[RCL] [1] [−] [◊] **Year 7 balance = $90,000**

[RCL] [1] [−] [◊] **Year 8 balance = $85,000**

[RCL] [1] [−] [◊] **Year 9 balance = $80,000**

[RCL] [1] [−] [◊] **Year 10 balance = $75,000**

```
  5000.00   ←1
             -
110000.00   ◇
             ↑
  5000.00   ←1
             -
105000.00   ◇
             ↑
  5000.00   ←1
             -
100000.00   ◇
             ↑
  5000.00   ←1
             -
 95000.00   ◇
             ↑
  5000.00   ←1
             -
 90000.00   ◇
             ↑
  5000.00   ←1
             -
 85000.00   ◇
             ↑
  5000.00   ←1
             -
 80000.00   ◇
             ↑
  5000.00   ←1
             -
 75000.00   ◇
```

b. Accumulated Depreciation Charged

To find total depreciation charged over a given period, subtract the remaining balance at the end of that period from the original investment amount to be depreciated.

Example: In the example above, how much depreciation has been charged over 10 years?

Enter: (Do not press [CLEAR])

125000 [$x \rightleftarrows y$] [−] [◊]

Total depreciation charged is $50,000

```
125000.00
 75000.00    ⇄
              -
 50000.00    ◆
```

c. Partial-Year Depreciation, Beginning and End of Period

For income tax purposes expecially, calendar-year or Fiscal-year depreciation charges must be calculated. When the acquisition date (the start of the depreciation period) does not coincide with the start of the accounting year - which is usual - the amount of depreciation in the first and last accounting years must be taken as fractions of full-year depreciation.

The keystrokes and steps are:

1) Calculate the amount of annual straight-line depreciation according to the procedure in Section a. above, press [STO] [1]

2) Calculate the percentage of a year from acquisition date to start of the next accounting year.

3) Multiply the Step 1 Figure by the Step 2 Figure to find Year 1 depreciation.

4) Press [−] [◊] to find balance at beginning of first full accounting year.

5) Press [RCL] [1] [−] [◊] to find balance at end of each successive full accounting year.

6) For last (partial) year, press [RCL] [1]; calculate percent of year represented by last partial year's depreciation; press [×] [−] [◊] to find end-of-period balance.

Example: If the example property were bought on September 1, what is the 10-year schedule of depreciation?

Enter: [CLEAR]

125000 [SAVE ↑] [SAVE ↑] **25** [÷] [STO] [1]

Annual depreciation is $5,000.

4 [SAVE ↑] **12** [÷] [×]

Partial-year (4 months) depreciation is $1666.67 at beginning

[−] [◊] **Balance at end of 4 months is $123,333.33**

[RCL] [1] [−] [◊] **Balance end of 1st full year is $118,333.33**

```
              CLEAR
 125000.00         ↑
                   ↑
     25.00         ÷
   5000.00    ◊
             →1
      4.00         ↑
     12.00         ÷
       .33    ◊
                   ×
   1666.67    ◊
                   −
 123333.33    ◊
                   ↑
   5000.00    ←1
                   −
 118333.33    ◊
                   ↑
```

[RCL] [1] [−] [◊] Balance end of 2nd full year is $113,333.33

[RCL] [1] [−] [◊] Balance end of 3rd full year is $108,333.33

[RCL] [1] [−] [◊] Balance end of 4th full year is $103,333.33

[RCL] [1] [−] [◊] Balance end of 5th full year is $98,333.33

[RCL] [1] [−] [◊] Balance end of 6th full year is 93,333.33

[RCL] [1] [−] [◊] Balance end of 7th full year is 88,333.33

[RCL] [1] [−] [◊] Balance end of 8th full year is 83,333.33

[RCL] [1] [−] [◊] Balance end of 9th full year is 78,333.33

[RCL] [1] 8 [SAVE ↑] 12 [÷] [×]

Partial-year (8 months) depreciation is $3333.33 at end

[−] [◊] Balance at end of 10 years is $75,000

```
  5000.00   ←1
             -
113333.33   ◊
             ↑
  5000.00   ←1
             -
108333.33   ◊
             ↑
  5000.00   ←1
             -
103333.33   ◊
             ↑
  5000.00   ←1
             -
 98333.33   ◊
             ↑
  5000.00   ←1
             -
 93333.33   ◊
             ↑
  5000.00   ←1
             -
 88333.33   ◊
             ↑
  5000.00   ←1
             -
 83333.33   ◊
             ↑
  5000.00   ←1
             -
 78333.33   ◊
             ↑
  5000.00   ←1
     8.00    ↑
    12.00    +
      .67   ◊
             ×
 .3333.33   ◊
             -
 75000.00   ◊
```

NOTE:

There is no "excess depreciation" or "crossover" calculation for straight line depreciation. These apply only to accelerated depreciation methods.

DECLINING BALANCE DEPRECIATION

In declining balance depreciation, a constant percentage of the remaining balance is applied each year to find the amount of depreciation. This is then subtracted from the previous year's balance to find the next balance.

The constant percentage is a multiple of the straight-line depreciation percentage. The applicable multiples under income tax law are 125%, 150% and 200%; operationally they are 1.25, 1.50 and 2.00. These are termed the "declining factors".

a. Calculation of Depreciation Schedule and Remaining Balance (Book Value) at End of Each Year

This routine calculates and prints each year's declining-balance depreciation and end-of-year balance. The routine will print the values for the entire useful life of the asset unless it is stopped. To stop the routine at the end of a given period, press any action key.

The steps and keystrokes are:

1) Key in number of first year in schedule, press [STO] [1]

2) Key in number of years of useful life, press [STO] [2]

3) Key in depreciable amount, press [PV]

4) Key in declining factor, press [i]

5) Press [EXT( )] [2]

Example: What is the 10-year depreciation schedule for the example property on a 150% declining-balance basis?

Enter: [CLEAR]

1 [STO] [1] **25** [STO] [2] **125000** [PV]

1.5 [i] [EXT( )] [2]

```
              CLEAR

          1.00    →1
         25.00    →2
     125000.00    →P
          1.50    → I
            DCL
FACTR
          1.50
1
       7500.00
     117500.00
2
       7050.00
     110450.00
3
       6627.00
     103823.00
4
```

```
        6229.38
       97593.62
5
        5855.62
       91738.00
6
        5504.28
       86233.72
7
        5174.02
       81059.70
8
        4863.58
       76196.12
9
        4571.77
       71624.35
10
        4297.46
       67326.89
```

NOTE:
To stop the routine after 10 years, press any action key.

b. Accumulated Depreciation Charged

To find total depreciation charged over a given period, subtract the remaining balance at the end of that period from the original depreciable amount.

Example: In the example in Section a) above, how much depreciation has been charged over 10 years?

Enter: (Do not press CLEAR)

R↓ RCL PV $x \rightleftarrows y$ − ◊

Total depreciation charged is $57,673.11

```
 67326.89   ↓
125000.00   ←P
 67326.89   ⇄
            −
 57673.11   ◊
```

c. Partial-Year Depreciation, Beginning and End of Period

To find the amount of depreciation in the partial year at the beginning of the period, when the acquisition date is different from the beginning of the accounting year, first calculate the amount of first-year declining-balance depreciation and multiply by the percentage of a year from the acquisition date to the beginning of the first full accounting year. Then subtract that from the depreciable amount to obtain the balance at the beginning of the first full accounting year.

Then apply the declining-balance routine over the income projection period, less the initial fractional-year period.

For the fractional year's depreciation at the end of the period, calculate the applicable percentage of a year and multiply it by the last full year's depreciation of the income projection period. Subtract that amount of depreciation from the previous year's balance to find the balance at the end of the income projection period.

Example: What is the 150% declining-balance depreciation schedule for the example property over 10 years, if the property was purchased on September 1 and depreciation is charged on a calendar-year basis?

Enter: [CLEAR]

125000 [SAVE↑] [SAVE↑] **25** [÷] **1.5** [×]

4 [SAVE↑] **12** [÷] [×]

4 months' depreciation is $2500

[−] [◊] **Balance after 4 months is $122,500**

[PV] **1** [STO] [1] **25** [STO] [2]

1.50 [i] [EXT( )] [2]

NOTE:
Stop the routine after year 10 by pressing any action key.

```
              CLEAR
     125000.00        ↑
                      ↑
         25.00        ÷
       5000.00     ◇
          1.50        ×
       7500.00     ◇
          4.00        ↑
         12.00        ÷
           .33     ◇
                      ×
       2500.00     ◇
                      -
     122500.00     ◇
                     →P
          1.00       →1
         25.00       →2
          1.50       → I
            DCL
FACTR
          1.50
1
       7350.00
     115150.00
2
       6909.00
     108241.00
3
       6494.46
     101746.54
4
       6104.79
      95641.75
5
       5738.51
      89903.24
6
       5394.19
      84509.05
7
       5070.54
      79438.51
8
       4766.31
```

```
   74672.20
9
    4480.33
   70191.87
10
    4211.51
   65980.36

    4211.51      ↑
       8.00      ↑
      12.00      ÷
        .67   ◊
                 ×
    2807.67   ◊
   70191.87
    2807.67      ⇄
                 -
   67384.20   ◊
```

4211.51 [SAVE ↑] **8** [SAVE ↑] **12** [÷] [×]

Last 8 months' depreciation is $2807.67

70191.87 [$x \rightleftarrows y$] [−] [◊]

Final balance is $67,384.20

d. Calculation of Excess Depreciation

The steps and keystrokes are:

1) Key in total depreciation charged as calculated in Section b), press [SAVE ↑]

2) Key in depreciable amount, press [SAVE ↑]; key in useful life of asset in years, press [÷] ; key in number of years in income projection period, press [×] .

This is total straight-line depreciation over the income projection period.

3) Press [−] [◊] . This is "excess depreciation".

Example: What is the excess depreciation charged on a 150% declining-balance basis for the example property over 10 years?

Enter: [CLEAR]

57673.11 [SAVE ↑] **125000** [SAVE ↑]

25 [÷] **10** [×]

[−] [◊] **Excess Depreciation is $7673.11.**

```
                 CLEAR

   57673.11      ↑
  125000.00      ↑
      25.00      ÷
    5000.00   ◊
      10.00      ×
   50000.00   ◊
                 -
    7673.11   ◊
```

e. Crossover Point

To identify the "crossover point" at which the straight-line depreciation charge on the remaining balance exceeds the declining-balance depreciation charge, first calculate the schedule of declining-balance depreciation. Then start comparing the straight-line depreciation for the remaining useful life on the remaining balance at the beginning of each year with the declining-balance charge for that year. A good starting point is the year in which declining-balance depreciation first falls below annual straight-line depreciation on the original depreciable amount over the entire useful life.

The cross-over point is the end of the year in which declining-balance depreciation last exceeds or equals the straight-line depreciation amount on the remaining balance less salvage value at the beginning of the year.

Example: What is the cross-over point with 150% declining balance depreciation for the example property?

Start testing for year 8, where 150% declining-balance depreciation is $4863.58. This is the first year in which 150% declining-balance depreciation is less than original straight-line depreciation of $5000.

Enter: [CLEAR]

1) Key in remaining balance at beginning of year, press [SAVE ↑]

2) Key in remaining years of useful life at beginning of year, press [÷]

This gives annual straight-line depreciation charge on the remaining balance over the remaining useful life.

Repeat steps 1 and 2 above until the result for a given year exceeds the declining-balance depreciation charge for that year in the printed schedule of declining-balance depreciation.

Enter: [CLEAR]

Year 8: 81059.70 [SAVE ↑] 18 [÷]

($4503.32 is less than $4863.58)

Year 9: 76196.12 [SAVE ↑] 17 [÷]

($4482.12 is less than $4571.77)

Year 10: 71624.35 [SAVE ↑] 16 [÷]

($4476.52 is more than $4297.46)

```
          CLEAR

 81059.70      ↑
    18.00      ÷
  4503.32   ◇
 76196.12      ↑
    17.00      ÷
  4482.12   ◇
 71624.35      ↑
    16.00      ÷
  4476.52   ◇
```

The "crossover" point is at the end of year 9. Depreciation in year 10 is $4297.46 which is less than $4476.52.

This procedure can be applied to any declining-balance depreciation method (125%, 150%, 200%) over any useful life. As an aid to analysts, the following table shows the "crossover" point for each of the three declining-balance methods, over the most common useful lives used for real estate. The "crossover" point is shown as a year number at the end of which a switch to straight-line depreciation is advantageous.

TABLE OF CROSSOVER OR SWITCH POINTS WITH NO SALVAGE VALUE

Useful Life	Declining-Balance Depreciation Method		
(Years)	125%	150%	200%
10	2	4	5
15	3	5	8
20	4	7	10
25	5	9	13
30	6	10	15
33 1/3	7	12	17
40	8	14	20
50	10	17	25

SUM-OF-THE-YEARS' DIGITS DEPRECIATION

SOYD is a method of accelerated depreciation, whereby larger depreciation amounts are charged in the early years of the asset's useful life than occurs in straight-line depreciation. For each year, a fraction of the depreciable amount is deducted as depreciation. The numerator of that fraction for each year is the number of years of remaining useful life at the beginning of that year. The denominator is the sum of the numbers of the years of useful life.

a. Calculation of Depreciation Schedule and Remaining Balance at End of Each Year

The HP-81 routine calculates and prints the depreciation amount and the end-of-year balance for each year. To stop the routine at any point, press any action key.

The keystrokes and steps are:

1) Key in the number of the first year of the schedule, press [STO] [1]

2) Key in the number of years of useful life, press [STO] [2]

3) Key in depreciable amount, press [PV]

4) Press EXT( ) 4

Example: Using the figures for the example property, what is the 10-year schedule of depreciation charges and end-of-year balances?

Enter: CLEAR

1 STO 1 **25** STO 2

125000 PV EXT( ) 4

```
                       CLEAR

               1.00       →1
              25.00       →2
          125000.00       →P
S      Y     D
 1
            9615.38
          115384.62
 2
            9230.77
          106153.85
 3
            8846.15
           97307.70
 4
            8461.54
           88846.16
 5
            8076.92
           80769.24
 6
            7692.31
           73076.93
 7
            7307.69
           65769.24
 8
            6923.08
           58846.16
 9
            6538.46
           52307.70
10
            6153.85
           46153.85
```

b. Accumulated Depreciation Charged

To find total depreciation charged over a given period, subtract the remaining balance at the end of that period from the original depreciable amount.

Example: In the example in Section a) above, how much depreciation has been charged over 10 years?

Enter: (Do not press CLEAR **)**

R↓ RCL PV $x \rightleftarrows y$ − ◊

Total accumulated depreciation is $78,846.15.

```
           46153.85       ↓
          125000.00      ←P
           46153.85       ⇄
                          −
           78846.15       ◊
```

c. Partial-Year Depreciation Beginning and End of Period

To find the amount of depreciation in the partial year at the beginning of the period, when the acquisition date is different from the beginning of the accounting year, first calculate the normal SOYD depreciation schedule. Then calculate the fraction of a year from the acquisition date to the beginning of the accounting year, then calculate the fraction of a year remaining from the beginning of the accounting year to the first full year of holding. After these factors have been obtained, apply them to the normal SOYD schedule as shown below:

Normal Year 1 scheduled depreciation [×] factor 1 = Year 1 value

Normal Year 1 scheduled depreciation [×] factor 2 +

Normal year 2 scheduled depreciation [×] factor 1 = Year 2 value

Normal year 2 scheduled depreciation [×] factor 2 +

Normal year 3 scheduled depreciation [×] factor 1 = Year 3 value

Continue this procedure for each year. In the final year which is one year beyond the useful life due to the partial years at the beginning and end of the schedule, the depreciation is simply:

Normal year N scheduled depreciation [×] factor 2 = Year N + 1 value

Example: What is the SOYD depreciation schedule for the example property in the first 4 years if the property was purchased on Sept. 1 and depreciation is charged on a calendar year basis.

ENTER: [CLEAR]

4 [SAVE ↑] **12** [÷] [STO] **1** **factor 1 is .33**

1 [$x \rightleftarrows y$] [−] [STO] **2** **factor 2 is .67**

9615.38 [STO] **0** [RCL] **1** [×]

YEAR 1 depreciation is 3205.13

[RCL] **2** [ ] [×]

```
              CLEAR
       4.00       ↑
      12.00       +
        .33    ◇
              →1
       1.00
        .33       ⇄
                  −
              →2
    9615.38   →0
                  ↑
        .33   ←1
                  ×
    3205.13    ◇
                  ↑
        .67   ←2
               N  ×
    6410.25    ◇
```

9230.77 [STO] **0** [RCL] **1** [×] [+] [◊]

Year 2 depreciation is 9487.18

[RCL] **2** [ ] [×]

8846.15 [STO] **0** [RCL] **1** [×] [+] [◊]

Year 3 depreciation is 9102.56

[RCL] **2** [ ] [×]

8461.54 [STO] **0** [RCL] **1** [×] [+] [◊]

Year 4 depreciation is 8717.95

```
9230.77    →0
            ↑
    .33    ←1
            ×
3076.92    ◊
            +
9487.18    ◊

            ↑
    .67    ←2
           N ×
6153.85    ◊
8846.15    →0
            ↑
    .33    ←1
            ×
2948.72    ◊
            +
9102.56    ◊

            ↑
    .67    ←2
           N ×
5897.43    ◊
8461.54    →0
            ↑
    .33    ←1
            ×
2820.51    ◊
            +
8717.95    ◊
```

d. Calculation of Excess Depreciation

The steps and keystrokes are:

1) Key in total depreciation charged as calculated in Section b), press [SAVE ↑]

2) Key in depreciable amount, press [SAVE ↑] ; key in useful life of asset in years, press [÷] ; key in number of years in income projection period, press [×]. This is total straight-line depreciation over the income projection period.

3) Press [−] [◊] . This is "excess depreciation".

Example: What is the excess depreciation charged on the example property, using SOYD depreciation over 10 years?

Enter: CLEAR

78846.15 SAVE↑ **125000** SAVE↑

25 ÷ **10** × − ◊

Excess depreciation is $28,846.15

```
         CLEAR

 78846.15      ↑
125000.00      ↑
    25.00      ÷
  5000.00    ◊
    10.00      ×
 50000.00    ◊
               −
 28846.15    ◊
```

e. Crossover Point

There is no "crossover" point for SOYD depreciation schedules. At every point on the schedule, the SOYD depreciation charge exceeds the straight-line depreciation charge on the remaining balance over the remaining useful life.

While it is legal to switch from SOYD to straight-line depreciation at any time, it is not advantageous for the investor to do so.

CHAPTER 12 - INCOME PROJECTION AND ESTIMATION

In most real estate investment and valuation problems (among others), it is necessary to calculate the future income and expense flows that are utilized in appraisal, financing and investment analysis.

The HP-81 has an advantage in calculating Before-Tax Cash Flows in that sequential or chain calculation capabilities can be utilized to work from Potential Gross Income to Cash Throw-Off to Equity in one continuous operation. With After-Tax Cash Flows, however, the ability of the HP-81 to store values and to calculate schedules of depreciation and annual interest payments considerably shortens calculating time, as well as reducing the possibilities of manual entry error.

BEFORE - TAX CASH FLOWS

The several before - tax cash flows applicable to real estate analysis and problems are:

PGI : Potential Gross Income

EGI : Effective Gross Income

NOI Net Operating Income (also called Net Income Before Recapture)

CTO: Cash Throw-Off to Equity (also called Gross Spendable Cash)

All are annual flows in real estate analysis.

These terms and symbols are further explained in the Appendix.

The derivation of these cash flows follows a set sequence:

1. Potential Gross Income is calculated by multiplying the rental per unit times the number of units, and that product times the number of rental payment periods per year. This gives what the property would generate in rental income if it were fully occupied.
2. Deduct Allowance for Vacancy and Rental Loss. The result is Rent Collections, which is also Effective Gross Income if there is no "Other Income".
3. Add "Other Income", such as receipts from concessions (laundry equipment, etc.), which is produced from sources other than the rental of space. This produces Effective Gross Income.
4. Deduct Operating Expenses. These are expenditures the landlord-investor must make, by contract or custom, to preserve the property and keep it capable of producing the forecast gross income. The result is Net Operating Income.
5. Deduct Annual Debt Service on the mortgage. This produces Cash Throw-Off to Equity.

Thus: PGI - Vac + Other = EGI

EGI - OE = NOI

NOI - ADS = CTO

Example: A 60-unit apartment building has rentals of $250 per unit per month. Three units are currently vacant, which is a typical vacancy ratio for competitive properties. Concession income from coin-operated laundry equipment averages $6 per occupied unit per month.

Management fees are 3.5% of rent collections. Other operating expenses are: Property Taxes $27,350; Insurance $3,255; Repairs and Maintenance $14,285 plus a free apartment for the building superintendent; Utilities (sewer and water) $7850; Heat and Air Conditioning $11,450; Replacements $3975; Other (Miscellaneous) $3125.

The property has just been financed with a $700,000 mortgage, fully amortized in level monthly payments at 9.5% interest over 20 years.

a. What is Effective Gross Income?

b. What is Net Operating Income?

c. What is Cash Throw-Off to Equity?

d. What is the Operating Expense Ratio? $\left(\text{OER} = \dfrac{\text{OE}}{\text{EGI}}\right)$

e. What is the Debt Service Coverage Ratio? $\left(\text{DS COV} = \dfrac{\text{NOI}}{\text{ADS}}\right)$

Enter: CLEAR

0 **(Round to nearest dollar)**

60 SAVE↑ **250** × **12** × **PGI = $180,000**

SAVE↑ SAVE↑ **3** SAVE↑ **60** ÷ ×

Vac. Allow. = $9,000.

− ◇ STO 1 **Rent Collections =**

$171,000

57 SAVE↑ **6** × **12** ×

```
        CLEAR

        60.     ↑
       250.     ×
     15000.   ◇
        12.     ×
    180000.   ◇
                ↑
                ↑
         3.     ↑
        60.     ÷
          .   ◇
                ×
      9000.   ◇
                −
    171000.   ◇
               →1
        57.     ↑
         6.     ×
       342.   ◇
        12.     ×
```

"Other Income" = $4,104

[+] [◊] [STO] [2] **EGI = $175,104**

[RCL] [1] **3.5** [%] [◊] **Management = $5,985**

27350 [+] **3255** [+] **14285** [+]

250 [SAVE ↑] **12** [×] [+]

7850 [+] **11450** [+] **3975** [+]

3125 [+] [◊] [STO] [3]

[RCL] [2] [x⇄y] **OE = $80,275**

[−] [◊] [STO] [4] **NOI = $94,829**

[CLEAR] **12** [STO] [0] **20** [×] [n]

9.5 [ ] [÷] [i]

700000 [PV] [PMT] [ ] [×] [STO] [5]

ADS = $78,299

[RCL] [4] [x⇄y] [−] [◊] [STO] [6]

CTO* = $16,530

[RCL] [3] [RCL] [2] [ ] [4] [÷]

Operating Expense Ratio = .4584

*CTO is stored for further use in calculating After-Tax Cash Flow.

```
    4104.   ◊
             +
  175104.   ◊
           →2
             ↑
  171000.  ←1
     3.5     %
    5985.   ◊
   27350.    +
    3255.    +
   14285.    +
     250.    ↑
      12.    ×
    3000.   ◊
             +
    7850.    +
   11450.    +
    3975.    +
    3125.    +
   80275.   ◊
           →3
             ↑
  175104.  ←2
   80275.    ⇄
             −
   94829.   ◊
           →4
        CLEAR
      12.  →0
      20.    ×
     240.   ◊
           → N
     9.5   N ÷
       1.   ◊
           → I
  700000.  →P
           PMT
    6525.
           N ×
   78299.   ◊
           →5
             ↑
   94829.  ←4
   78299.    ⇄
             −
   16530.   ◊
           →6
             ↑
   80275.  ←3
             ↑
  175104.  ←2
```

[0] [RCL] [4] [RCL] [5]

[4] [÷]

Debt Service Coverage Ratio = 1.2111

```
                 ÷
      .4584      ◊
                 ↑
     94829.     ←4
                 ↑
     78299.     ←5
                 ÷
     1.2111      ◊
```

BEFORE - TAX REVERSIONS (RESALE PROCEEDS)

The reversion receivable at the end of the income projection period is usually based on forecast or anticipated resale of the property at that time. The several before-tax reversion amounts applicable to real estate analysis and problems are:

SP : Resale Price

PR : Proceeds of Resale

b : Outstanding Mortgage Balance

NCPR : Net Cash Proceeds of Resale to Equity

These terms and symbols are further explained in the Appendix.

The derivation of these reversions is as follows:

1. Forecast or estimated Resale Price. Deduct sales and disposition expenses (brokerage commission, legal fees, etc.). The result is Proceeds of Resale.
2. Calculate Outstanding Balance of the Mortgage at the end of the Income Projection Period and subtract it from Proceeds of Resale. The result is Net Cash Proceeds of Resale.

Thus : SP - Disp. Exp. = PR

PR - b = NCPR

Example: The apartment property in the preceding example is expected to be resold in 10 years. The forecast resale price is $800,000. The broker's commission is expected to be 6% and other selling or disposition expenses are 2.5%. The mortgage is the same as that indicated in the preceding example.

a. What are the Forecast Proceeds of Resale?

b. What will the Mortgage Balance be in 10 years?

c. What are the Forecast Net Cash Proceeds of Resale?

Enter: [CLEAR]

[ ] [0] **(Round to nearest dollar)**

800000 [SAVE↑] **6** [SAVE↑] **2.5** [+] [%] [◊]

Sales Expense is $68,000

[−] [◊] [STO] [3] **PR = $732,000**

[CLEAR] **12** [STO] [0] **20** [×] [n] **9.5**

[ ] [÷] [i] **700000** [PV] [ ] [2]

[PMT] [STO] [PMT] [ ] [0]

Monthly Mortgage Payment = $6524.92

[CLEAR] **1** [STO] [1] **10** [ ] [×] [STO]

[2] [RCL] [i] [i] [RCL] [PMT] [PMT]

[RCL] [PV] [PV] [Σ+] **b = $504,258**

[R↓] [RCL] [3] [$x \rightleftarrows y$] [−] [◊]

NCPR = $227,742

```
                CLEAR

      800000.       ↑
          6.        ↑
        2.5         +
                    %
       68000.   ◊
                    -
      732000.   ◊

               →3
                CLEAR
         12.   →0
         20.        ×
        240.   ◊
               →  N
        9.5    N    ÷
         1.    ◊
               →  I
      700000.  →P
               PMT
      6524.92
               →7*
                CLEAR
          1.   →1
         10.   N    ×
        120.   ◊
               →2
          1.   ←  I
               →  I
       6525.   ←PT
               →PT
      700000.  ←P
               →P
                Σ+
P  V
      504258.
I  N  T
      587248.
      504258.       ↓
                    ↑
      732000.  ←3
      504258.       ⇄
                    -
      227742.  ◊
```

AFTER - TAX CASH FLOW

After-tax cash flow is found for each year by deducting Income Tax Liability for that year from CTO. (ATCF = CTO - Tax Liability.)

To derive Income Tax Liability for each year, it is necessary first to calculate Taxable Income. Then ATCF can be found:

1. Calculate mortgage interest payable in the year.
2. Calculate depreciation chargeable during the year.
3. Deduct the figures derived in Steps 2 and 3 from NOI. This is Taxable Income.
4. Multiply Taxable Income by the appropriate tax rate to find Income Tax Liability.
5. Deduct Income Tax Liability from CTO to find ATCF

Thus : Taxable Income = NOI - Int. - Depr.
Tax Liability = Taxable Income X Tax Rate
ATCF = CTO - Tax Liability

Example: The property used in the example in the preceding section on Before - Tax Cash Flows was purchased for $900,000, of which $150,000 was allocated to land. Therefore the "depreciable amount" of investment in the buildings is $750,000. The buildings have an estimated remaining useful life of 25 years, and are to be depreciated on a 125% declining-balance basis.

The mortgage loan terms are those stipulated in the earlier example: Principal of $700,000; Interest rate of 9.5%; Full amortization in level monthly payments over 25 years. The applicable income tax rate is 48%.

What is the schedule of ATCF for 10 years?

NOTE:

The values already stored from the preceding example, and their storage locations, are:

12 - Storage Register 0 ADS - Storage Register 5

NOI - Storage Register 4 CTO - Storage Register 6

Interest per Period - Storage Register 16 (i)

Mortgage Principal - Storage Register 18 (PV)

To clear the storage registers for future use, and to retain these stored values for further use, relocate them as follows:

Enter:

[RCL] [5] [▒] [÷] [PMT] **Monthly**

Mortgage Payment is in Register 17 ([PMT])

[RCL] [4] [STO] [•] [9] **NOI is in Register 19**

[RCL] [6] [STO] [3] **CTO is in Register 3.**

[CLEAR] [▒] [0] **(Round to the nearest dollar.)**

Calculate and store interest payments for years 1-10

1 [STO] [1] **12** [STO] [2] [RCL] [i] [i]

[RCL] [PMT] [PMT] [RCL] [PV] [PV] [Σ+]

[STO] [4] **Year 1 interest is: $65,973**

13 [STO] [1] **24** [STO] [2] [Σ+] [STO] [5]

Year 2 interest is: $64,750

25 [STO] [1] **36** [STO] [2] [Σ+] [STO] [6]

Year 3 interest is: $63,402

37 [STO] [1] **48** [STO] [2] [Σ+] [STO] [7]

Year 4 interest is: $61,928

49 [STO] [1] **60** [STO] [2] [Σ+] [STO] [8]

Year 5 interest is: $60,301

61 [STO] [1] **72** [STO] [2] [Σ+] [STO] [9]

Year 6 interest is: $58,514

73 [STO] [1] **84** [STO] [2] [Σ+] [STO]

[•] [0] **Year 7 interest is: $56,552**

85 [STO] [1] **96** [STO] [2] [Σ+] [STO]

[•] [1] **Year 8 interest is: $54,394**

```
            CLEAR

     78299.   ←5
              N ÷
      6525.   ◇
              →PT
                ↑
     94829.   ←4
              →9*
                ↑
     16530.   ←6
              →3

            CLEAR

         1.   →1
        12.   →2
         1.   ← I
              → I
      6525.   ←PT
              →PT
    700000.   ←P
              →P
               Σ+
 P   V
    687674.
I  N  T
     65973.
              →4
        13.   →1
        24.   →2
               Σ+
 P   V
    674125.
I  N  T
     64750.
              →5
        25.   →1
        36.   →2
               Σ+
 P   V
    659228.
I  N  T
     63402.
              →6
        37.   →1
        48.   →2
               Σ+
 P   V
    642857.
I  N  T
     61928.
              →7
        49.   →1
        60.   →2
               Σ+
```

97 STO 1 **108** STO 2 Σ+ STO

• 2 **Year 9 interest is: $52,021**

109 STO 1 **120** STO 2 Σ+ STO

• 4 **Year 10 interest is: $49,413**

NOTE:

Do not use Storage Register 13. It is needed for the next step.) *(Do not press* CLEAR *)*

```
P  V
      624859.
I N T
       60301.
                 →8
          61.    →1
          72.    →2
                 Σ+
P  V
      605074.
I N T
       58514.
                 →9
          73.    →1
          84.    →2
                 Σ+
P  V
      583327.
I N T
       56552.
                 →0*
          85.    →1
          96.    →2
                 Σ+
P  V
      559422.
I N T
       54394.
                 →1*

          97.    →1
         108.    →2
                 Σ+
P  V
      533144.
I N T
       52021.
                 →2*
         109.    →1
         120.    →2
                 Σ+
P  V
      504258.
I N T
       49413.
                 →4*
```

Calculate 125% declining-balance depreciation schedule for 10 years:

1 [STO] [1] 25 [STO] [2] 750000 [PV]

1.25 [i] [EXT( )] [2]

(Press any action key as year 10 calculation starts, to stop the routine.)

NOTE:
Each year's depreciation charge must be entered manually later in calculating Taxable Income.

(Do not press [CLEAR])

.48 [STO] [0] The tax rate is in Storage Register 0

[RCL] [•] [9] [STO] [1]

NOI is transferred to Storage Register 1

[RCL] [1] [RCL] [4] [−] 37500 [−] [◊]

Year 1 Taxable Income

[ ] [×] Year 1 Tax Liability

[RCL] [3] [$x \rightleftarrows y$] [−] [◊]

Year 1 ATCF = $20,679*

[RCL] [1] [RCL] [5] [−] 35625 [−] [◊]

[ ] [×] [RCL] [3] [$x \rightleftarrows y$] [−] [◊]

Year 2 ATCF = $19,192*

[RCL] [1] [RCL] [6] [−] 33844 [−] [◊]

[ ] [×] [RCL] [3] [$x \rightleftarrows y$] [−] [◊]

Year 3 ATCF = $17,690*

[RCL] [1] [RCL] [7] [−] 32152 [−] [◊]

[ ] [×] [RCL] [3] [$x \rightleftarrows y$] [−] [◊]

```
              1.     →1
             25.     →2
         750000.     →P
           1.25      → I
            DCL
FACTR
              1.
1
          37500.
         712500.
2
          35625.
         676875.
3
          33844.
         643031.
4
          32152.
         610879.
5
          30544.
         580335.
6
          29017.
         551318.
7
          27566.
         523752.
8
          26188.
         497564.
9
          24878.
         472686.
10
          23634.
         449052.
            .48      →0
                      ↑
          94829.     ←9*
                     →1
                      ↑
          94829.     ←1
                      ↑
          65973.     ←4
                      −
          37500.      −
-          8644.      ◊
                    N ×
-          4149.      ◊
                      ↑
          16530.     ←3
-          4149.      ⇄
                      −
          20679.      ◊
```

Year 4 ATCF = $16,170

[RCL] [1] [RCL] [8] [−] **30544** [−] [◊]

[ ] [×] [RCL] [3] [$x \rightleftarrows y$] [−] [◊]

Year 5 ATCF = $14,618

[RCL] [1] [RCL] [9] [−] **29017** [−] [◊]

[ ] [×] [RCL] [3] [$x \rightleftarrows y$] [−] [◊]

Year 6 ATCF = $13,027

[RCL] [1] [RCL] [•] [0] [−] **27566** [−]

[◊] [ ] [×] [RCL] [3] [$x \rightleftarrows y$] [−]

[◊] **Year 7 ATCF = $11,389**

[RCL] [1] [RCL] [•] [1] [−] **26188** [−]

[◊] [ ] [×] [RCL] [3] [$x \rightleftarrows y$] [−]

[◊] **Year 8 ATCF = $9,691**

[RCL] [1] [RCL] [•] [2] [−] **24878** [−]

[◊] [ ] [×] [RCL] [3] [$x \rightleftarrows y$] [−]

[◊] **Year 9 ATCF = $7,924**

[RCL] [1] [RCL] [•] [4] [−] **23634** [−]

[◊] [ ] [×] [RCL] [3] [$x \rightleftarrows y$] [−]

[◊] **Year 10 ATCF = $6,075**

```
                 ↑
   94829.   ←1
                 ↑
   64750.   ←5
                 -
   35625.        -
-   5546.   ◊
            N    ×
-   2662.   ◊
                 ↑
   16530.   ←3
-   2662.        ⇄
                 -
   19192.   ◊
                 ↑
   94829.   ←1
                 ↑
   63402.   ←6
                 -
   33844.        -
-   2417.   ◊
            N    ×
-   1160.   ◊
                 ↑
   16530.   ←3
-   1160.        ⇄
                 -
   17690.   ◊
                 ↑
   94829.   ←1
                 ↑
   61928.   ←7
                 -
   32152.        -
     749.   ◊
            N    ×
     360.   ◊
                 ↑
   16530.   ←3
     360.        ⇄
                 -
   16170.   ◊
                 ↑
   94829.   ←1
                 ↑
   60301.   ←8
                 -
   30544.        -
    3984.   ◊
            N    ×
    1912.   ◊
```

```
                  ↑
 16530.     ←3
  1912.           ⇄
                  −
 14618.     ◇
                  ↑
 94829.     ←1
                  ↑
 58514.     ←9
                  −
 29017.           −
  7298.     ◇
            N     ×
  3503.     ◇
                  ↑
 16530.     ←3
  3503.           ⇄
                  −
 13027.     ◇
                  ↑
 94829.     ←1
                  ↑
 56552.     ←0*
                  −
 27566.           −
 10711.     ◇
            N     ×
  5141.     ◇
                  ↑
 16530.     ←3
  5141.           ⇄
                  −
 11389.     ◇
                  ↑
 94829.     ←1
                  ↑
 54394.     ←1*
                  −
 26188.           −
 14247.     ◇
            N     ×
  6839.     ◇
                  ↑
 16530.     ←3
  6839.           ⇄
                  −
  9691.     ◇
```

```
                  ↑
 94829.     ←1
                  ↑
 52021.     ←2*
                  −
 24878.           −
 17930.     ◇
            N     ×
  8606.     ◇
                  ↑
 16530.     ←3
  8606.           ⇄
                  −
  7924.     ◇
                  ↑
 94829.     ←1
                  ↑
 49413.     ←4*
                  −
 23634.           −
 21782.     ◇
            N     ×
 10455.     ◇
                  ↑
 16530.     ←3
 10455.           ⇄
                  −
  6075.     ◇
```

*Assumes other income available for tax shelter. Otherwise, tax liability is 0, and ATCF is $16,530.

The summary schedule of calculated values to derive the 10-year ATCF flow is:

Year	(1) NOI	(2) Interest	(3) Depreciation	(4) Taxable Income (1) - (2) - (3)	(5) Tax Liability (4) X .48	(6) CTO	(7) ATCF (6) - (5)
1	$94,829	$65,973	$37,500	-$8,644	-$4,149	$16,530	$20,679*
2	94,829	64,750	35,625	- 5,546	- 2,662	16,530	19,192*
3	94,829	63,402	33,844	- 2,417	- 1,160	16,530	17,690*
4	94,829	61,928	32,152	749	360	16,530	16,170
5	94,829	60,301	30,544	3,984	1,912	16,530	14,618
6	94,829	58,514	29,017	7,298	3,503	16,530	13,027
7	94,829	56,552	27,556	10,711	5,141	16,530	11,389
8	94,829	54,394	26,188	14,247	6,839	16,530	9,691
9	94,829	52,021	24,878	17,930	8,606	16,530	7,924
10	94,829	49,413	23,634	21,782	10,455	16,530	6,075

*Assume Other Income to take advantage of tax shelter. Otherwise, ATCF if $16,530.

AFTER - TAX NET CASH PROCEEDS OF RESALE

ATNCPR = NCPR - Tax Liability

To calculate Tax Liability, it is necessary to find the Gain on Resale. This is divided between Excess Depreciation, which is taxed (fully or partially, depending on whether any Excess Depreciation is "forgiven") as ordinary income; and the remainder, which is Capital Gain taxed at the capital gains tax rate.

The steps are:

1. Calculate total depreciation charged. Subtract this from the original purchase price (Capital Outlay) to obtain Tax Basis.
2. Subtract Tax Basis from Proceeds of Resale. The result is Gain on Resale.
3. Subtract total straight-line depreciation over the income projection period from total depreciation charged. This produces Excess Depreciation.
4. Subtract Excess Depreciation from Gain on Resale to obtain Capital Gain.
5. Multiply Excess Depreciation by ordinary income tax rate. This produces ordinary income tax liability on resale.
6. Multiply Capital Gain by capital gains tax rate. This produces capital gains tax liability on resale.
7. Add the figures derived in Steps 5 and 6 to obtain total Tax Liability on resale.
8. Subtract total Tax Liability from Net Cash Proceeds of Resale to obtain After-Tax Net Cash Proceeds of Resale.

Thus: CO - Total Dep. = Tax Basis

PR - Tax Basis = Gain on Resale

Total Dep. - S-L Dep. = Excess Dep.

Gain on Resale - Excess Dep. = Capital Gain

(Excess Dep. X Ord. Tax Rate) + (Cap. Gain X CG Tax Rate) = Tax Lia.

NCPR - Tax Lia. = ATNCPR

Example: A new, first-owner non-residential property cost $60,000 for the site and $240,000 for the building (improvements). The estimated useful life of the building is 40 years, and it is depreciated on a 150% declining-balance basis.

The property has been financed with a 75% mortgage at 10.25% interest, fully amortized in monthly payments over 23 years 7 months.

The property is forecast to resell at $275,000 in 12 years, with disposition expenses of 8%. The applicable ordinary income tax rate is 48%, and the capital gains tax rate is 30%.

What is the indicated ATNCPR?

Enter: CLEAR

0 **(Round to nearest dollar.)**

(a) Calculate and store PR

275000 SAVE↑ **8** % − ◊ STO 3

PR = $253,000

```
          CLEAR

275000.       ↑
     8.       %
 22000.       -
253000.     ◊

            →3
```

(b) Calculate and store NCPR

CLEAR **12** STO 0 **23** × **7** + n

10.25 ÷ i **300000** SAVE↑

75 % PV PMT STO PMT

CLEAR RCL n **12** × − n

RCL i i

RCL PMT PMT PV **b = $171,464**

RCL 3 $x \rightleftarrows y$ − ◊ STO 4

NCPR = $81,536

```
          CLEAR

    12.     →0
    23.       ×
   276.     ◊
     7.       +
            → N
 10.25      N +
     1.     ◊
            → I
300000.       ↑
    75.       %
225000.     →P
            PMT
  2112.
            →7*

          CLEAR

   283.     ← N
    12.     N ×
   144.     ◊
              -
            → N
     1.     ← I
            → I
  2112.     ←PT
            →PT
P  V
171464.
              ↑
253000.     ←3
171464.       ⇄
              -
 81536.     ◊

            →4
```

(c) Calculate and store total depreciation charged

12 STO 1 40 STO 2 240000

PV 1.50 i

EXT( ) 2 (Press CE as soon as calculation starts, to stop routine.)

R↓ RCL PV $x \rightleftarrows y$ − ◊ STO

5 Total Depreciation Charged = $88,288

(d) 300000 $x \rightleftarrows y$ − ◊

Basis = $211,712

(e) RCL 3 $x \rightleftarrows y$ − ◊ STO 6

Gain on Resale = $41,288

(f) RCL 5 RCL PV 40 ÷ 12 ×

− ◊ STO 7

Excess Depreciation = $16,288

(g) RCL 6 $x \rightleftarrows y$ − ◊

Capital Gain = $25,000

(h) .30 × Capital Gains Tax = $7,500

(i) RCL 7 .48 × Ordinary Income Tax = $7,818

(j) + ◊ Total Tax = $15,318

```
         12.    →1
         40.    →2
     240000.    →P
        1.5     → I
         DCL
FACTR
          2.
12
       5911.
     151712.

     151712.     ↓
     240000.    ←P
     151712.     ⇄
                 −
      88288.    ◊

                →5
     300000.
      88288.     ⇄
                 −
     211712.    ◊

                 ↑
     253000.    ←3
     211712.     ⇄
                 −
      41288.    ◊

                →6
                 ↑
      88288.    ←5
     240000.    ←P
         40.     ÷
       6000.    ◊
         12.     ×
      72000.    ◊
                 −
      16288.    ◊

                →7
                 ↑
      41288.    ←6
      16288.     ⇄
                 −
      25000.    ◊

         .3      ×
       7500.    ◊
                 ↑
      16288.    ←7
        .48      ×
       7818.    ◊
                 +
      15318.    ◊

                 ↑
```

(k) [RCL] [4] [$x \rightleftarrows y$] [−] [◊]

ATNCPR = $66,218

```
81536.    ←4
15318.     ≠
           −
66218.     ◊
```

CHAPTER 13: REAL ESTATE DECISION MAKING -- INVESTMENT ANALYSIS AND FEASIBILITY ANALYSIS

The HP-81 can be used effectively in real estate decision making, using both routines and procedures that have been described and illustrated in previous chapters of this manual, as well as other procedures illustrated below.

Decision making involves making a choice from among two or more alternative courses of action. The routines and procedures available on the HP-81 make it possible for the analyst to consider almost any combination of outcomes, and compare them with one another to select the "best" alternative, or to compare them with some standard of acceptability to make an accept-reject decision.

Feasibility Analysis is a process of measuring and testing whether a proposed investment is expected to meet an investor's minimum standard(s) of acceptability. If the investment or project proposal meets the investor's standard(s), then it is "feasible".

Investment Analysis consists essentially of comparing alternative investment or project proposals, and making them according to the results of their feasibility tests. The highest-ranking alternative is the "best" in terms of the investor's standard(s) of acceptability.

In addition, there are decisions about the selection of the "best" or optimum financing alternative, decisions concerning rent-buy and sell-lease alternatives, and measures of financial safety or coverage that enter into real estate problem solving. All these are considered and illustrated here in Chapter 13, along with measures and tests of feasibility and sensitivity analysis.

FEASIBILITY TESTS

A feasibility test measures whether a project or investment is likely to meet an investor's standard of acceptability. These standards of acceptability include:

1. The investment should be worth to the investor at least as much as it will cost the investor to acquire it. This criterion is tested by calculating the Present Worth of the Forecast Future Cash Flows from the investment at a rate of discount reflecting the rate of return minimally acceptable to the investor, and comparing that Present Worth to the Capital Outlay required. This procedure uses Present Worth, Net Present Value and the Profitability Index.

2. The investment should produce a rate of return to the investor at least as high as the rate of return desired or required. This criterion is tested by calculating the Internal Rate of Return or Modified Internal Rate of Return on the investment, and comparing it with the investor's desired or required rate of return.

3. The investment should provide for full recovery of the investor's Capital Outlay within the time period desired or required by the investor. This criterion is tested by calculating the Payback Period and comparing it with the investor's desired or required payback period.

a. Present Worth

The PW of any investment is calculated with the routines and procedures illustrated in Chapters 8 and 10. This involves discounting the Forecast Future Cash Flows at a specified rate. For feasibility analysis, that specified rate is the minimally acceptable rate of return to the investor. It is y for estimating equity investment value, r for estimating total property value, and i for estimating the present worth of a mortgage.

1.) Level Annuity, No Reversion

The routine is the one illustrated in Chapters 2 and 3:

Enter: Number of Payment Periods [n]

Rate of Return per Period [i] **Cash Flow per Period** [PMT] [PV] .

2.) Level Annuity with Reversion or Balloon Payment

The routines are as illustrated in Chapters 5, 8 and 10. The Present Worth of the level cash flows is added to the Present Worth of the reversion, both at the investor's minimally acceptable rate of return. The sum is the Present Worth of the investment.

An alternative method utilizes a modification of the NPV routine illustrated in Chapter 8. The amount entered as [PV] is 0. Since the periodic cash flows are equal, they are stored and recalled successively to avoid errors of manual re-entry. The cash flow amount entered for the last period is the sum of the last payment amount plus the reversion.

Example - An income property is forecast to produce NOI of $7537 per year. The investor expects to hold it for 10 years, and then sell it. The Forecast Proceeds of Resale are $60,000.

The property has just been financed with a $50,000 mortgage at 9% interest, with level monthly payments over a 25-year term.

What is the Present Worth of the property at a Discount Rate of 10.5%?

What is the Present Worth of the equity investment position with an Equity Yield Rate of 14%?

Enter: [CLEAR]

12 [STO] [0] **25** [×] [n] **9**

[ ] [÷] [i] **50000** [PV] [PMT]

Monthly Payment is $419.60

[STO] [PMT] [ ] [×] [STO] [3]

Annual Debt Service is $5035.18

[CLEAR] [ ] [0] **(Round to nearest dollar.)**

1 [STO] [1] **10** [ ] [×] [STO] [2]

[RCL] [i] [i] [RCL] [PMT] [PMT]

[RCL] [PV] [PV] [Σ+]

Mortgage Balance in 10 years is $41,370.

[R↓] **60000** [$x \rightleftarrows y$] [−] [◊] [STO] [4]

NCPR = $18,630

[CLEAR] **7537** [STO] [1] **60000** [+] [◊]

[STO] [2]

10th Year NOI + PR = $67,357.

```
              CLEAR
          12.00    →0
          25.00     ×
         300.00    ◊
                   → N
           9.00    N ÷
            .75    ◊
                   → I
       50000.00    →P
                   PMT
         419.60
                   →7*
                   N ×
        5035.18    ◊
                   →3
              CLEAR
             1.    →1
            10.    N ×
           120.    ◊
                   →2
             1.    ← I
                   → I
           420.    ←PT
                   →PT
         50000.    ←P
                   →P
                   Σ+
P V
         41370.
I N T
         41722.
         41370.     ↓
         60000.
         41370.     ⇄
                    −
         18630.    ◊
                   →4
              CLEAR
          7537.    →1
         60000.     +
         67537.    ◊
                   →2
```

[CLEAR] 10.5 [i] 0 [PV] [RCL] [1] [Σ+] [RCL] [1] [Σ+] [RCL] [1] [Σ+] [RCL] [1] [Σ+] [RCL] [1] [Σ+] [RCL] [1] [Σ+] [RCL] [1] [Σ+] [RCL] [1] [Σ+] [RCL] [1] [Σ+] [RCL] [2] [Σ+]

PW Property = $67,440

```
          CLEAR

   10.5     → I
      .     →P
              ↑
   7537.    ←1
   7537.     Σ+
   6821.    ◊
              ↑
   7537.    ←1
   7537.     Σ+
  12993.    ◊
              ↑
   7537.    ←1
   7537.     Σ+
  18580.    ◊
              ↑
   7537.    ←1
   7537.     Σ+
  23635.    ◊
              ↑
   7537.    ←1
   7537.     Σ+
  28210.    ◊
              ↑
   7537.    ←1
   7537.     Σ+
  32350.    ◊
              ↑
   7537.    ←1
   7537.     Σ+
  36097.    ◊
              ↑
   7537.    ←1
   7537.     Σ+
  39488.    ◊
              ↑
   7537.    ←1
   7537.     Σ+
  42556.    ◊
              ↑
  67537.    ←2
  67537.     Σ+
  67440.    ◊
```

[CLEAR] [RCL] [1] [RCL] [3] [−] [◊]

[STO] [1] **CTO = $2502**

[RCL] [4] [RCL] [1] [+] [STO] [2]

10th Year CTO + NCPR = $21,132.

```
          CLEAR

   7537.    ←1
              ↑
   5035.    ←3
              -
   2502.    ◊
```

CLEAR 14 i 0 PV RCL 1 Σ+ RCL
1 Σ+ RCL 1 Σ+ RCL 1 Σ+
RCL 1 Σ+ RCL 1 Σ+ RCL 1
Σ+ RCL 1 Σ+ RCL 1 Σ+ RCL
2 Σ+

PW Equity = $18,075.

```
               →1
                ↑
18630.         →4
                ↑
 2502.         →1
                +
               →2
        CLEAR
   14.         → I
     .         →P
                ↑
 2502.         →1
 2502.         Σ+
 2195.         ◇
                ↑
 2502.         →1
 2502.         Σ+
 4120.         ◇
                ↑
 2502.         →1
 2502.         Σ+
 5808.         ◇
                ↑
 2502.         →1
 2502.         Σ+
 7290.         ◇
                ↑
 2502.         →1
 2502.         Σ+
 8589.         ◇
                ↑
 2502.         →1
 2502.         Σ+
 9729.         ◇
                ↑
 2502.         →1
 2502.         Σ+
10729.         ◇
                ↑
 2502.         →1
 2502.         Σ+
11606.         ◇
                ↑
 2502.         →1
 2502.         Σ+
12375.         ◇
                ↑
21132.         →2
21132.         Σ+
18075.         ◇
```

3.) Variable Annuity

The routine is the NPV routine illustrated in Chapter 8, except that 0 is entered for CO in [PV] :

Enter:

Rate of return or discount [i]

0 [PV]

Cash Flow for period 1 [Σ+]
Repeat Step 3 for each successive cash flow.

b. Net Present Value

Net Present Value is the difference between Present Worth and Capital Outlay required: NPV = PW - CO.

The test of feasibility is NPV $\geq$ 0.

If Present Worth at the investor's required or desired rate of return is equal to or greater than the Capital Outlay required to acquire the investment position, then the investment is "feasible".

Example 1 - Level Annuity plus Reversion

An investment property has just been purchased for $62,500, including a $50,000 mortgage. NOI is forecast at $7537 annually, while CTO is $2502. The property is expected to be resold in 10 years for $60,000, at which time NCPR would be $18,630.

The desired rate of return on the property investment is 10.5%, while y is 14%.

What is the NPV of the property investment?
What is the NPV of the equity investment?

NPV of property investment using NPV routine:

Enter: [CLEAR]

7537 [STO] [1] 60000 [+] [STO] [2]

10th Year Payment is $67,537.

```
              CLEAR

    7537.00     →1
   60000.00      +
                →2
```

CLEAR 10.5 i 62500 CHS PV RCL 1 Σ+ RCL 1 Σ+ RCL 1 Σ+ RCL 1 Σ+ RCL 1 Σ+ RCL 1 Σ+ RCL 1 Σ+ RCL 1 Σ+ RCL 1 Σ+ RCL 2 Σ+

NPV Property = +$4940.27

```
              CLEAR

          10.50    → I
-      62500.00    →P
                    ↑
        7537.00    ←1
        7537.00    Σ+
-      55679.19    ◇
                    ↑
        7537.00    ←1
        7537.00    Σ+
-      49506.50    ◇
                    ↑
        7537.00    ←1
        7537.00    Σ+
-      43920.36    ◇
                    ↑
        7537.00    ←1
        7537.00    Σ+
-      38865.04    ◇
                    ↑
        7537.00    ←1
        7537.00    Σ+
-      34290.08    ◇
                    ↑
        7537.00    ←1
        7537.00    Σ+
-      30149.84    ◇
                    ↑
        7537.00    ←1
        7537.00    Σ+
-      26403.03    ◇
                    ↑
        7537.00    ←1
        7537.00    Σ+
-      23012.24    ◇
                    ↑
        7537.00    ←1
        7537.00    Σ+
-      19943.66    ◇
                    ↑
       67537.00    ←2
       67537.00    Σ+
        4940.27    ◇
```

NPV of equity investment using PV routines:

Enter: CLEAR 10 n 14 i 2502 PMT PV STO 1

```
              CLEAR

          10.00    → N
          14.00    → I
        2502.00    →PT
P    V
       13050.72
                   →1
```

[CLEAR] [RCL] [n] [n] [RCL] [i] [i]

18630 [FV] [PV] [RCL] [1] [+] [◊]

PW equity = $18076.05

12500 [–] [◊]

NPV equity = $5576.05

```
               CLEAR
       10.00   ← N
               → N
       14.00   ← I
               → I
    18630.00   → F
P   V
     5025.33
               ↑
    13050.72   ←1
               +
    18076.05   ◊

    12500.00   -
     5576.05   ◊
```

Example 2 - NPV of a Variable Annuity
A rental property has 7 years remaining on the lease to the single tenant. The property is for sale for $200,000. A mortgage in the amount of $137,500 can be obtained.

A potential investor seeking an after-tax rate of return on his equity investment of 12% has forecast the after-tax cash flows and reversion, based on lease terms, as follows:

Year	ATCF
1	$11,846
2	$ 9,673
3	$ 8,217
4	$ 6,743
5	$ 5,018
6	$ 3,716
7	$ 2,284
7(Reversion)	$51,883

Should the investor purchase the property?

Enter: [CLEAR]

200000 [SAVE ↑] 137500 [–] [◊] [STO] [1]

Equity Investment Required = $62,500

2284 [SAVE ↑] 51883 [+] [◊] [STO] [2]

7th Year Cash Flow = $54,167

```
               CLEAR

   200000.00   ↑
   137500.00   -
    62500.00   ◊

               →1
     2284.00   ↑
    51883.00   +
    54167.00   ◊

               →2
```

[CLEAR] 12 [i] [RCL] [1] [CHS] [PV]

11846 [Σ+] 9673 [Σ+] 8217 [Σ+] 6743 [Σ+]

5018 [Σ+] 3716 [Σ+] 54167 [Σ+]

No. NPV = -$4845.57

```
            CLEAR
      12.00     → I
                  ↑
   62500.00    ←1
                S ⇄
                →P
   11846.00    Σ+
-  51923.21    ◊
    9673.00    Σ+
-  44211.96    ◊
    8217.00    Σ+
-  38363.26    ◊
    6743.00    Σ+
-  34077.96    ◊
    5018.00    Σ+
-  31230.61    ◊
    3716.00    Σ+
-  29347.97    ◊
   54167.00    Σ+
-   4845.57    ◊
```

c. Profitability Index

The Profitability Index is the Ratio of Present Worth to Capital Outlay:

$$PI = \frac{PW}{CO}$$

The test of feasibility is: $PI \geq 1$

Example 1 - PV Routines

Using the property income and capital outlay figures from Example 1 in the preceding section on Net Present Value, what is the Profitability Index for the property investment at a Discount Rate of 10.5%?

Enter: [CLEAR]

10 [n] 10.5 [i] 7537 [PMT]

[PV] [STO] [1]

[CLEAR] [RCL] [n] [n] [RCL] [i] [i]

60000 [FV] [PV] [RCL] [1] [+] [◊]

PW = $67,440.27

```
            CLEAR
      10.00     → N
      10.50     → I
    7537.00    →PT
P   V
   45333.34
               →1
            CLEAR
      10.00     ← N
                → N
      10.50     ← I
                → I
   60000.00     → F
P   V
   22106.93
```

62500 (÷)

PI = 1.08 (The investment is feasible.)

```
                ↑
45333.34      ←1
                +
67440.27      ◇

62500.00        ÷
    1.08      ◇
```

Example 2 - NPV Routine

Using the after-tax cash flow and reversion figures from Example 2 in the preceding section on Net Present Value, what is the Profitability Index on an equity investment of $62,500 at an after-tax rate of return of 12%?

Enter: (CLEAR)

2284 (SAVE↑) **51883** (+) (◇) (STO) (1)

7th Year Cash Flow

(CLEAR) **12** (i) **0** (PV) **11846** (Σ+)

9673 (Σ+) **8217** (Σ+) **6743** (Σ+)

5018 (Σ+) **3716** (Σ+) (RCL) (1) (Σ+)

PW = $57,654.43

62500 (÷)

PI = 0.92 (The investment is not feasible.)

```
           CLEAR

 2284.00        ↑
51883.00        +
54167.00      ◇

              →1

           CLEAR

   12.00      → I
     .00      →P
11846.00       Σ+
10576.79      ◇
 9673.00       Σ+
18288.04      ◇
 8217.00       Σ+
24136.74      ◇
 6743.00       Σ+
28422.04      ◇
 5018.00       Σ+
31269.39      ◇
 3716.00       Σ+
33152.03      ◇
                ↑
54167.00      ←1
54167.00       Σ+
57654.43      ◇
62500.00        ÷
     .92      ◇
```

d. Internal Rate of Return

As noted previously in Chapters 2, 4, 5, 9 and 11, an Internal Rate of Return is that rate of discount at which the Present Worth of Forecast Future Cash Flows from an investment exactly equals the required Capital Outlay.

The test of feasibility is: IRR $\geq$ Target

The calculated IRR from the property investment and forecast cash flow data must be equal to or greater than the rate of return required or desired by the investor.

Example 1 - Level Annuity Plus Reversion
An investor is considering purchasing a rental property for $62,500. He can finance the purchase with a $50,000 mortgage. The forecast CTO for 10 years is $2502 annually. He expects to resell the property in 10 years and realize an NCPR of $18,630.

Should he make the investment if he is looking for an 18% equity yield rate?

Enter: CLEAR

EXT( ) 2 **18630** FV **10**

SAVE ↑ **365** × STO DAY **2502**

PMT **12500** PV i

Yes. IRR is 21.75%, which is greater than 18%.

```
            CLEAR
              AN2
18630.00       → F
   10.00         ↑
  365.00         ×
 3650.00       ◇
              →4*
 2502.00      →PT
12500.00       →P
YIELD
TO          CALL
   21.75
```

Example 2 - Variable Annuity
An investment property is forecast to produce the following after-tax cash flow over a 10-year income projection period.

Year	ATCF
1	$2,861
2	$2,753
3	$2,631
4	$2,508
5	$2,387
6	$2,162
7	$1,894
8	$1,583
9	$1,212
10	$ 808
10 (Reversion)	$14,765 (At NCPR)

The investor can acquire the equity investment position for $12,500. What is the after-tax rate of return, calculated as an IRR?

Enter: [CLEAR]

808 [SAVE ↑] **14765** [+] [◊] [STO] [•] [4]

Year 10 Cash Flow = $15,573.

Test at 20%:

[CLEAR] **20** [i] **12500** [CHS] [PV] **2861**
[STO] [1] [Σ+] **2753** [STO] [2] [Σ+] **2631**
[STO] [3] [Σ+] **2508** [STO] [4] [Σ+] **2387**
[STO] [5] [Σ+] **2162** [STO] [6] [Σ+] **1894**
[STO] [7] [Σ+] **1583** [STO] [8] [Σ+] **1212**
[STO] [9] [Σ+] [RCL] [•] [4] [Σ+] [STO]
[•] [9]

NPV = – $141.88 (20% is too high).

Test at 19.5%:

[CLEAR] **19.5** [i] **12500** [CHS] [PV] [RCL]
[1] [Σ+] [RCL] [2] [Σ+] [RCL] [3] [Σ+]
[RCL] [4] [Σ+] [RCL] [5] [Σ+] [RCL] [6]
[Σ+] [RCL] [7] [Σ+] [RCL] [8] [Σ+] [RCL]
[9] [Σ+] [RCL] [•] [4] [Σ+] [STO] [0]

```
                   CLEAR
       808.00          ↑
     14765.00          +
     15573.00          ◊
                     →4*
                   CLEAR
        20.00        → I
-    12500.00         →P
      2861.00         →1
      2861.00         Σ+
-    10115.83          ◊
      2753.00         →2
      2753.00         Σ+
-     8204.03          ◊
      2631.00         →3
      2631.00         Σ+
-     6681.46          ◊
      2508.00         →4
      2508.00         Σ+
-     5471.97          ◊
      2387.00         →5
      2387.00         Σ+
-     4512.69          ◊
      2162.00         →6
      2162.00         Σ+
-     3788.64          ◊
      1894.00         →7
      1894.00         Σ+
-     3260.06          ◊
      1583.00         →8
      1583.00         Σ+
-     2891.90          ◊
      1212.00         →9
      1212.00         Σ+
-     2657.01          ◊
                       ↑
     15573.00        ←4*
     15573.00         Σ+
-      141.88          ◊
                     →9*
                   CLEAR
        19.50        → I
-    12500.00         →P
                       ↑
      2861.00         ←1
      2861.00         Σ+
-    10105.86          ◊
                       ↑
      2753.00         ←2
      2753.00         Σ+
-     8178.02          ◊
```

NPV = +$106.71 (19.5% is too low.)

Interpolate to Find IRR:

[RCL] [·] [9] [CHS] [+] [◊] [RCL] [0]

[x⇄y] [÷] **20** [SAVE↑] [RCL] [i] [−]

[◊] [×] **0.21%**

[RCL] [i] [+] [◊]

IRR = 19.71%

```
                     ↑
       2631.00     ←3
       2631.00     Σ+
-      6636.26     ◊
                     ↑
       2508.00     ←4
       2508.00     Σ+
-      5406.40     ◊
                     ↑
       2387.00     ←5
       2387.00     Σ+
-      4426.88     ◊
                     ↑
       2162.00     ←6
       2162.00     Σ+
-      3684.46     ◊
                     ↑
       1894.00     ←7
       1894.00     Σ+
-      3140.20     ◊
                     ↑
       1583.00     ←8
       1583.00     Σ+
-      2759.54     ◊
                     ↑
       1212.00     ←9
       1212.00     Σ+
-      2515.66     ◊
                     ↑
      15573.00     ←4*
      15573.00     Σ+
        106.71     ◊
                   →0
                     ↑
-       141.88     ←9*
                   S ⇄
                     +
        248.59     ◊
                     ↑
        106.71     ←0
        248.59       ⇄
                     ÷
           .43     ◊
         20.00       ↑
         19.50     ← I
                     −
           .50     ◊

                     ×
           .21     ◊
         19.50     ← I
                     +
         19.71     ◊
```

e. Payback Period

The Payback Period is the number of years required to return or "payback" the amount of Capital Outlay, disregarding any rate of discount. It is n when

$$\sum_{t=1}^{n} (CF_t) \geq CO \qquad \text{The test of feasibility is: } n \leq \text{Target}$$

An investment is feasible when the Capital Outlay is forecast to be repaid within the period required or desired by the investor.

The procedure to find the Payback Period is to use the NPV routine, entering 0 as [i]. When NPV becomes posivibe, press [R↓] [R↓] to find n .

If cash flows are level, $n \geq \dfrac{CO}{CTO}$

Example 1 - Level Cash Flows
An investor has just purchased an income property for $62,500, of which $12,500 was equity. CTO is forecast at $2502 annually. What is the Payback Period?

Enter: [CLEAR]

12500 [SAVE ↑] **2502** [÷] **= 4.996**

Therefore n = 5

```
              CLEAR

   12500.00     ↑
    2502.00     ÷
       5.00     ◇
```

Example 2 - Variable Cash Flows
Suppose the equity investor in the property illustrated in Example 2 in the preceding section on Internal Rate of Return is looking for a return of his equity investment from ATCF in 6 years. Is this investment feasible?

Enter: [CLEAR]

0 [i] **12500** [CHS] [PV] **2861** [Σ+]

2753 [Σ+] **2631** [Σ+] **2508** [Σ+] **2387** [Σ+]

NPV is positive

[R↓] [R↓]

n = 5 years

Yes, by this standard the investment is feasible.

```
                  CLEAR

          .00    → I
-    12500.00    →P
      2861.00    Σ+
-     9639.00    ◇
      2753.00    Σ+
-     6886.00    ◇
      2631.00    Σ+
-     4255.00    ◇
      2508.00    Σ+
-     1747.00    ◇
      2387.00    Σ+
       640.00    ◇
          .00    ↓
         5.00    ↓
```

GROSS INCOME MULTIPLIER

The Gross Income Multiplier (or Gross Rent Multiplier) is the ratio of Sales Price to Gross Income. The Gross Income figure used can be either Potential Gross Income or Effective Gross Income, depending on which is appropriate or applicable in the particular case.

$$\text{GIM} = \frac{\text{SP}}{\text{PGI}} \qquad \text{or} \qquad \text{GIM} = \frac{\text{SP}}{\text{EGI}}$$

Example - An apartment property recently sold for $885,700. It contains 63 units, renting for $247.50 per month each. Five units were vacant at the time of sale, which is a normal vacancy ratio in this market. What is the indicated Gross Income Multiplier, using both PGI and EGI?

Enter: [CLEAR]

885700 [SAVE↑] 63 [SAVE↑] 247.50

[×] 12 [×]

PGI = $187,110

[÷]

$$\text{GIM} = \frac{\text{SP}}{\text{PGI}} = 4.73$$

```
           CLEAR
885700.00        ↑
    63.00        ↑
   247.50        ×
 15592.50     ◊
    12.00        ×
187110.00     ◊
                 ÷
     4.73     ◊
```

[CLEAR] 63 [SAVE↑] 247.50 [×] 12 [×]

[SAVE↑] [SAVE↑] 5 [SAVE↑] 63 [÷] [×]

[−] [◊]

EGI = $172,260

885700 [$x \rightleftarrows y$] [÷]

$$\text{GIM} = \frac{\text{SP}}{\text{EGI}} = 5.14$$

```
           CLEAR
    63.00        ↑
   247.50        ×
 15592.50     ◊
    12.00        ×
187110.00     ◊
                 ↑
                 ↑
     5.00        ↑
    63.00        ÷
      .08     ◊
                 ×
 14850.00     ◊
                 −
172260.00     ◊
885700.00
172260.00        ⇄
                 ÷
     5.14     ◊
```

FINANCIAL COVERAGE (SAFETY)RATIOS

In evaluating investment proposals, the safety or ability of forecast income to cover required payments (cash outflows) is often as important a consideration as profitability.

The required payments are Operating Expenses and Debt Service. They are compared with the Cash Flows available to cover them.

a. Operating Expense Ratio.

This is the ratio of Operating Expenses to Effective Gross Income.

$$OER = \frac{OE}{EGI}$$

For safety or coverage purposes, the lower the OER, the better.

Example - An apartment property has forecast Effective Gross Income of $42,866 and annual Operating Expenses of $17,694. What is the indicated Operating Expense Ratio?

Enter: [CLEAR]

[ ] [4] **17694** [SAVE ↑] **42866** [÷]

OER = .4128

```
                 CLEAR
17694.0000           ↑
42866.0000           ÷
     .4128           ◊
```

b. Debt Service Coverage Ratio

This is the ratio of Net Operating Income to Annual Debt Service: $\frac{NOI}{ADS}$

For safety or coverage purposes, the higher this ratio, the better.

Example - An apartment with Forecast Effective Gross Income of $42,866 and Operating Expenses of $17,694 per year also has a mortgage with level monthly payments of $1395.75. What is the indicated Debt Service Coverage Ratio?

Enter: [CLEAR]

42866 [SAVE ↑] **17694** [−] [◊]

NOI = $25,172

1395.75 [SAVE ↑] **12** [×]

ADS = $16,749

```
              CLEAR

42866.00             ↑
17694.00             -
25172.00         ◊

 1395.75             ↑
   12.00             ×
16749.00         ◊
                     ÷
    1.50         ◊
    1.5029       ◊
```

[÷] [▒] [4] [◊]

Debt Service Coverage Ratio = 1.5029

c. Breakeven Cash Throw-Off Ratio

This is the ratio of Effective Gross Income to the sum of Operating Expenses and Annual Debt Service:

$$\frac{\text{EGI}}{\text{OE} + \text{ADS}}$$

The ratio measures the extent to which EGI covers required cash outlays. For safety or coverage purposes, the higher this ratio, the better.

Example - For the apartment property illustrated in the preceding examples in this section on Financial Coverage Ratios, what is the Breakeven Cash Throw-Off Ratio?

Enter: [CLEAR]

42866 [SAVE ↑] **17694** [SAVE ↑] **1395.75**

[SAVE ↑] **12** [×] [+] [◊] [÷] [▒] [4] [◊]

$$\frac{\textbf{EGI}}{\textbf{OE + ADS}} = \textbf{1.2445}$$

```
            CLEAR
42866.00        ↑
17694.00        ↑
 1395.75        ↑
   12.00        ×
16749.00    ◊
                +
34443.00    ◊

       .        ÷
    1.24    ◊
  1.2445    ◊
```

FINANCIAL (MORTGAGE) ANALYSIS

Mortgage Financing terms and conditions can influence real estate investment decisions and profitability. HP-81 procedures and routines are readily adaptable to the analysis of financing considerations in real estate problems and decisions.

a. Impact of Financing Alternatives

Terms of mortgage financing can and do vary. The equity investor (borrower) sometimes has a choice amoung two or more alternative financing packages. To select the alternative which is best for the borrower, the present worth of the equity investment under each alternative financing package is calculated. The alternative which produces the highest PW of the equity investment is the one the equity investor should select.

Example - An investor is considering the purchase of an income property for \$200,000. NOI is forecast at \$22,000 annually. The investor plans to hold the property for 10 years, at which time the proceeds of resale are forecast to be 90% of the purchase price (\$180,000). The investor is seeking a 13% rate of return (y) on the equity investment.

The investor is considering four alternative financing plans which are available to him:

1. A 75% mortgage at 9.5% interest, with full amortization in level monthly payments over 20 years
2. A 70% mortgage at 9.25% interest, with full amortization in level monthly payments over 20 years;
3. A 60% mortgage at 9% interest, with full amortization in level monthly payments over 20 years, plus a 15% second mortgage at 10.5% interest, with full amortization in level monthly payments over 10 years.
4. A 75% mortgage at 9.5% interest with full amortization in level monthly payments over 30 years.

Which is the best financing package for the equity investor?

Enter: [CLEAR]

1. 75%, 20 year loan at 9.5%

12 [STO] [0] **20** [×] [n] **9.5** [ ] [÷] [i]

200000 [SAVE↑] **75** [%] [PV] [PMT] [STO]

[PMT]

[CLEAR] [RCL] [n] **10** [ ] [×] [−] [n]

[RCL] [i] [i] [RCL] [PMT] [PMT] [PV]

b = \$108,054.34

```
          CLEAR

     12.00    →O
     20.00     ×
    240.00    ◇
              → N
      9.50    N ÷
       .79    ◇
              → I
 200000.00     ↑
     75.00     %
 150000.00    →P
              PMT
   1398.20
              →7*
          CLEAR
    240.00    ← N
     10.00    N ×
    120.00    ◇
               −
              → N
       .79    ← I
              → I
   1398.20    ←PT
              →PT
 P  V
 108054.34
```

180000 [x⇄y] [−] [◊] [STO] [1]

NCPR = $71,945.66

22000 [SAVE ↑] [RCL] [PMT] [▒] [×]

[−] [◊] [PMT]

CTO = $5221.64

[CLEAR] **10** [n] **13** [i] [RCL] [PMT]

[PMT] [PV] [STO] [2]

[CLEAR] [RCL] [n] [n] [RCL] [i] [i]

[RCL] [1] [FV] [PV] [RCL] [2] [+] [◊]

PW Equity = $49,528.23

150000 [+] [◊]

PW Property = $199,528.23 (vs. $200,000 cost)

2. 70%, 20-year loan at 9.25%:

[CLEAR] **20** [▒] [×] [n] **9.25** [▒]

[÷] [i] **200000** [SAVE ↑] **70** [%] [PV]

[PMT] [STO] [PMT]

```
 180000.00
 108054.34      ⇄
                -
  71945.66    ◇

             →1
  22000.00      ↑
   1398.20   ←PT
             N  ×
  16778.36    ◇
                -
   5221.64    ◇

             →PT
          CLEAR
     10.00   →  N
     13.00   →  I
   5221.64   ←PT
             →PT
P  V
  28333.88
             →2
          CLEAR
     10.00   ←  N
             →  N
     13.00   ←  I
             →  I
                ↑
  71945.66   ←1
             →  F
P  V
  21194.35
                ↑
  28333.88   ←2
                +
  49528.23    ◇

 150000.00      +
 199528.23    ◇
          CLEAR
     20.00   N  ×
    240.00    ◇
             →  N
      9.25   N  ÷
       .77    ◇
             →  I
 200000.00      ↑
     70.00      %
 140000.00   →P
             PMT
   1282.21
             →7*
```

CLEAR RCL n 10 ▒ × − n

RCL i i RCL PMT PMT PV

b = $100,147.33

180000 x⇄y − ◊ STO 1

NCPR = $79,852.67

22000 SAVE↑ RCL PMT ▒ ×

− ◊ PMT

CTO = $6613.44

CLEAR 10 n 13 i RCL PMT

PMT PV STO 2

CLEAR RCL n n RCL i i

RCL 1 FV PV RCL 2 + ◊

PW Equity = $59,409.78

140000 + ◊

PW Property = $199,409.78

```
                CLEAR

      240.00    ← N
       10.00    N ×
      120.00    ◇
                  −
                → N
         .77    ← I
                → I
     1282.21    ←PT
                →PT
P   V
   100147.33
   180000.00
   100147.33      ⇄
                  −
    79852.67    ◇

                →1
    22000.00      ↑
     1282.21    ←PT
                N ×
    15386.56    ◇
                  −
     6613.44    ◇

                →PT
                CLEAR

       10.00    → N
       13.00    → I
     6613.44    ←PT
                →PT
P   V
    35886.12
                →2
                CLEAR

       10.00    ← N
                → N
       13.00    ← I
                → I
                  ↑
    79852.67    ←1
                → F
P   V
    23523.67
                  ↑
    35886.12    ←2
                  +
    59409.78    ◇

   140000.00      +
   199409.78    ◇
```

3. 60%, 20-year loan at 9%, plus 15%, 10-year second mortgage at 10.5%:

[CLEAR] 20 [ ] [×] [n] 9 [ ] [÷]

[i] 200000 [SAVE ↑] 60 [%] [PV] [PMT]

[STO] [PMT] [ ] [×] [STO] [1]

ADS on 1st mortgage = $12,956.05

```
CLEAR
     20.00   N ×
    240.00   ◇
             → N
      9.00   N ÷
       .75   ◇
             → I
 200000.00      ↑
     60.00      %
 120000.00   →P
             PMT
   1079.67
             →7*
             N ×
  12956.05   ◇
             →1
```

[CLEAR] [RCL] [n] 10 [ ] [×] [−] [n]

[RCL] [i] [i] [RCL] [PMT] [PMT] [PV]

b = $85,231.07

180000 [$x \rightleftarrows y$] [−] [◇] [STO] [2]

NCPR = $94,768.93

```
CLEAR
    240.00   ← N
     10.00   N ×
    120.00   ◇
                −
             → N
       .75   ← I
             → I
   1079.67   ←PT
             →PT
P   V
  85231.07
 180000.00
  85231.07      ⇄
                −
  94768.93   ◇
             →2
```

[CLEAR] 10 [ ] [×] [n] 10.5 [ ]

[÷] [i] 200000 [SAVE ↑] 15 [%] [PV]

[PMT] [ ] [×]

ADS on 2nd Mortgage = $4857.66

[RCL] [1] [+] [◇] **Total ADS = $17,813.71**

```
CLEAR
     10.00   N ×
    120.00   ◇
             → N
     10.50   N ÷
       .88   ◇
             → I
 200000.00      ↑
     15.00      %
  30000.00   →P
             PMT
    404.80
             N ×
   4857.66   ◇
                ↑
  12956.05   ←1
                +
  17813.71   ◇
```

22000 x⇄y − ◊ PMT

CTO = $4186.29

CLEAR **10** n **13** i RCL PMT PMT

PV STO 1

CLEAR RCL n n RCL i i

RCL 2 FV PV RCL 1 + ◊

PW Equity = $50,633.63

150000 + ◊

PW Property = $200,633.63

4. 75%, 30-year loan at 9.5%:

CLEAR **30** × n **9.5** ÷

i **200000** SAVE↑ **75** % PV PMT

STO PMT

```
 22000.00
 17813.71     ⇄
              -
  4186.29     ◊

             →PT
           CLEAR
    10.00    → N
    13.00    → I
  4186.29    ←PT
             →PT
P V
 22715.81
             →1
           CLEAR
    10.00    ← N
             → N
    13.00    ← I
             → I
               ↑
 94768.93    ←2
             → F
P V
 27917.82
               ↑
 22715.81    ←1
               +
 50633.63     ◊

150000.00      +
200633.63     ◊
           CLEAR
    30.00    N ×
   360.00     ◊
             → N
     9.50    N ÷
      .79     ◊
             → I
200000.00      ↑
    75.00      %
150000.00    →P
             PMT
  1261.28
             →7*
```

[CLEAR] [RCL] [n] 10 [ ] [×] [−] [n]

[RCL] [i] [i] [RCL] [PMT] [PMT] [PV]

b = $135,311.56

180000 [$x \rightleftarrows y$] [−] [◊] [STO] [1]

NCPR = $44,688.44

22000 [SAVE ↑] [RCL] [PMT] [ ] [×]

[−] [◊] [PMT]

CTO = $6864.62

[CLEAR] **10** [n] **13** [i] [RCL] [PMT]

[PMT] [PV] [STO] [2]

[CLEAR] [RCL] [n] [n] [RCL] [i] [i]

[RCL] [1] [FV] [PV] [RCL] [2] [+] [◊]

PW Equity = $50,413.81

150000 [+] [◊]

PW Property = $200,413.81

```
            CLEAR

    360.00   ← N
     10.00   N ×
    120.00   ◇
               −
             → N
       .79   ← I
             → I
   1261.28   ←PT
             →PT
P  V
 135311.56
 180000.00
 135311.56     ⇄
               −
  44688.44   ◇

             →1
  22000.00     ↑
   1261.28   ←PT
             N ×
  15135.38   ◇
               −
   6864.62   ◇

             →PT
            CLEAR

     10.00   → N
     13.00   → I
   6864.62   ←PT
             →PT
P  V
  37249.12
             →2
            CLEAR

     10.00   ← N
             → N
     13.00   ← I
             → I
               ↑
  44688.44   ←1
             → F
P  V
  13164.69
               ↑
  37249.12   ←2
               +
  50413.81   ◇

 150000.00     +
 200413.81   ◇
```

The alternatives in order of attractiveness are:

1. 60% 20-year loan at 9%, plus 15%, 10-year second mortgage at 10.5%
2. 75%, 30-year loan at 9.5%
3. 75%, 20-year loan at 9.5%
4. 70%, 20-year loan at 9.25%

The following table summarizes the NPV and profitability of each alternative.

Alternative No.	Equity Investment	Equity Present Worth	Net Present Value	Profitability Index
1	$50000	49,528.23	-471.76	-.009435
2	$60000	59,409.78	-590.20	-.009837
3	$50000	50,633.63	633.65	+.01267
4	$50000	50,413.81	413.79	+.008276

b. Cash Equivalent Sales Price

When a property is purchased subject to an existing mortgage, or a purchase-money mortgage is taken back by the seller, and the mortgage interest rate is below the going market rate, the cash equivalent sales price is different from (less than) the nominal sales price. This adjustment process is useful in adjusting comparable sales data in Direct Sales Comparison Analysis.

The procedure involves calculating the monthly (periodic) payment at the below-market interest rate, and then capitalizing it at the market rate. This gives the present worth of the mortgage, which is added to the amount of equity payment to get the Cash Equivalent Sales Price.

Example 1 - A property was recently purchased for a nominal price of $60,000. The purchaser assumed the existing mortgage, which had a balance of $38,744. It had a remaining term of 20 years, with level monthly payments at 7% interest. The seller took back a $11,256 second purchase-money mortgage at 8% interest, with a maturity of 20 years (level monthly payments).

The going market interest rate is 8.75% for properties of this type.

What is the cash equivalent price of this transaction?

Enter: CLEAR

60000 SAVE ↑ **38744** − **11256** − ◊

STO 1

Equity investment is $10,000.

```
          CLEAR

60000.00      ↑
38744.00      -
11256.00      -
10000.00      ◊

             →1
```

[CLEAR] **12** [STO] [0] **20** [×] [n] **7**

[ ] [÷] [i] **38744** [PV] [PMT] [STO] [2]

Payment on 1st mortgage is $300.38.

```
              CLEAR
    12.00     →0
    20.00     ×
   240.00     ◊
              → N
     7.00     N ÷
      .58     ◊
              → I
 38744.00     →P
              PMT
   300.38
              →2
```

[CLEAR] [RCL] [n] [n] **8** [ ] [÷] [i]

11256 [PV] [PMT]

Payment on 2nd mortgage is $94.15.

[RCL] [2] [+] [◊] [PMT]

Total monthly payment is $394.53.

```
              CLEAR
   240.00     ← N
              → N
     8.00     N ÷
      .67     ◊
              → I
 11256.00     →P
              PMT
    94.15
              ↑
   300.38     ←2
              +
   394.53     ◊

              →PT
```

[CLEAR] [RCL] [n] [n] **8.75** [ ] [÷] [i]

[RCL] [PMT] [PMT] [PV]

Present Worth of mortgage payments is $44,644.87.

[RCL] [1] [+] [◊]

Cash equivalent sales price is $54,644.87.

```
              CLEAR
   240.00     ← N
              → N
     8.75     N ÷
      .73     ◊
              → I
   394.53     ←PT
              →PT
P   V
 44644.87
              ↑
 10000.00     ←1
              +
 54644.87     ◊
```

Example 2 - The trustees of a hospital are trying to negotiate the purchase of an adjoining property. They recently had it appraised at $385,000. They have been promised a mortgage by a local bank for $335,000 at 8.5% interest, fully amortized in level monthly payments over 25 years.

The owner of the property is insisting on a purchase price of $450,000. However, he is willing to take back a $400,000 purchase-money mortgage with a 25-year term at 6% interest. The trustees are holding out for $385,000.

What advice would you give the trustees as to the appropriate course of action?

Enter: CLEAR

12 STO 0 **25** × n **6** ÷

i **400000** PV PMT STO PMT

CLEAR RCL n n **8.5** ÷ i

RCL PMT PMT PV

PW of Mortgage = $320,059.47

50000 + ◊

Cash equivalent Sales Price = $370,059.47

```
          CLEAR

       12.00    →0
       25.00     ×
      300.00    ◊
                → N
        6.00    N ÷
         .50    ◊
                → I
   400000.00    →P
                PMT
     2577.21
                →7*
          CLEAR

      300.00    ← N
                → N
        8.50    N ÷
         .71    ◊
                → I
     2577.21    ←PT
                →PT
P  V
   320059.47
    50000.00    +
   370059.47    ◊
```

Pay the nominal $450,000. The cash equivalent price is less than the appraised value.

c. Refinancing

It can be mutually advantageous to both borrower and lender to refinance an existing mortgage which has an interest rate substantially below the current market rate, with a loan at a below-market rate. The borrower has the immediate use of tax-free cash, while the lender has substantially increased debt service on a relatively small cash outlay.

To find the benefits to both borrower and lender:

1. Calculate the monthly payment on the existing mortgage;
2. Calculate the monthly payment on the new mortgage;
3. Calculate the net monthly payment received by the lender (and paid by the borrower) by subtracting the figure found in Step 1 from the figure found in Stpe 2;
4. Calculate the NPV to the lender of the net cash advanced;
5. Calculate the effective yield to the lender as an IRR;
6. Calculate the NPV to the borrower of the net cash advanced.

Example - An investment property has an existing mortgage which was originated 8 years ago with an original term of 25 years, fully amortized in level monthly payments at 6.5% interest. The current balance is $133,190.

Although the going current market interest rate is 10.5%, the lender has agreed to refinance the property with a $200,000, 17-year, level-monthly-payment loan at 9% interest.

What are the NPV and effective yield to the lender on the net amount of cash actually advanced?

What is the NPV to the borrower on this amount if he can earn a 14% equity yield rate on the net proceeds of the loan?

Enter: [CLEAR]

12 [STO] [0] 17 [×] [n] 6.5 [ ] [÷]

[i] 133190 [PV] [PMT] [STO] [1]

Monthly Payment on existing mortgage is $1080.33.

9 [ ] [÷] [i] 200000 [PV] [PMT]

Monthly payment on new mortgage is 1917.61.

[RCL] [1] [−] [◊] [PMT]

Net monthly payment is $837.28.

200000 [SAVE ↑] 133190 [−] [◊] [STO] [2]

Net amount of cash advanced is $66,810.

[CLEAR] [RCL] [n] [n] 10.5 [ ] [÷]

[i] [RCL] [PMT] [PMT] [PV]

PW of Net Monthly Payment at 10.5% is $79,507.21.

```
           CLEAR
    12.00   →0
    17.00    ×
   204.00    ◇
            → N
     6.50   N ÷
      .54    ◇
            → I
133190.00   →P
            PMT
  1080.33
            →1
     9.00   N ÷
      .75    ◇
            → I
200000.00   →P
            PMT
  1917.61
             ↑
  1080.33   ←1
             -
   837.28    ◇
            →PT
200000.00    ↑
133190.00    -
 66810.00    ◇
            →2
           CLEAR
   204.00   ← N
            → N
    10.50   N ÷
      .88    ◇
            → I
   837.28   ←PT
            →PT
```

[RCL] [2] [−] [◊]

NPV to Lender is +$12,697.21.

```
P V
79507.21
                 ↑
66810.00        ←2
                 −
12697.21         ◊
```

[CLEAR] [RCL] [n] [n] [RCL] [PMT]

[PMT] [RCL] [2] [PV] [i] [ ] [×]

Effective yield (IRR) to Lender is 13.51%.

```
           CLEAR
  204.00       ← N
               → N
  837.28      ←PT
              →PT
                ↑
66810.00       ←2
               →P
I N T
    1.13
              N ×
   13.51        ◊
```

[CLEAR] [RCL] [n] [n] 14 [ ] [÷]

[i] [RCL] [PMT] [PMT] [PV]

PW of Net Monthly Payment at 14% is $65,032.31.

```
           CLEAR
  204.00       ← N
               → N
   14.00      N ÷
    1.17        ◊
               → I
  837.28      ←PT
              →PT
F V
65032.31
```

[RCL] [2] [$x \rightleftarrows y$] [−] [◊]

NPV to Borrower is +$1,777.69.

```
                ↑
66810.00       ←2
65032.31        ⇄
                −
 1777.69        ◊
```

d. Wrap-Around Mortgages

A wrap-around mortgage is essentially the same as a refinancing mortgage, except that the new mortgage is a junior lien mortgage granted by a different lender, who assumes the payments on the existing mortgage, which remains in full force. The new (second) mortgage is thus "wrapped around" the existing mortgage. The "wrap-around" lender advances the net difference between the new(second) mortgage and the existing mortgage in cash to the lender, and receives as net cash flow the difference between debt service on the new (second) mortgage and debt service on the existing mortgage.

The procedures in calculating NPV and IRR to the lender and NPV to the borrower are exactly the same as those presented in the preceding section on Refinancing.

Example - A mortgage loan on an income property has a remaining balance of $200,000. When the loan was originated 8 years ago, it had a 20-year term with full amortization in level monthly payments at 6.75% interest.

The current market interest rate is 10.25%.

A lender has agreed to "wrap" a $300,000 second mortgage at 9.5%, with full amortization in level monthly payments over 12 years.

The equity investor anticipates a rate of return (y) of 13.5% on the reinvested net cash proceeds on the loan.

What is the effective yield (IRR) to the lender on net cash advanced?

What is the NPV to the borrower of net cash received?

Enter: [CLEAR]

12 [STO] [0] **20** [×] **8** [ ] [×] [−]

[n] **6.75** [ ] [÷] [i] **200000** [PV]

[PMT] [STO] [1]

Monthly payment on existing mortgage is $2030.21.

9.5 [ ] [÷] [i] **300000** [PV] [PMT]

Monthly payment on wrap-around mortgage is $3499.12

[RCL] [1] [−] [◊] [PMT]

Net monthly payment is $1468.91.

300000 [SAVE ↑] **200000** [−] [◊] [STO] [2]

Net Cash Advanced is $100,000.

[CLEAR] [RCL] [n] [n] [RCL] [PMT]

[PMT] [RCL] [2] [PV] [i] [ ] [×]

Effective Yield (IRR) to lender is 14.50%.

```
          CLEAR
    12.00     →0
    20.00      ×
   240.00      ◇
     8.00     N ×
    96.00      ◇
               -
             → N
     6.75     N ÷
      .56      ◇
             → I
200000.00     →P
              PMT
  2030.21
              →1
     9.50     N ÷
      .79      ◇
             → I
300000.00     →P
              PMT
  3499.12
               ↑
  2030.21     ←1
               -
  1468.91      ◇
              →PT
300000.00      ↑
200000.00      -
100000.00      ◇
              →2
          CLEAR
   144.00    ← N
             → N
  1468.91    ←PT
             →PT
```

```
                              ↑
     100000.00              ←2
                            →P
I N T
          1.21
                            N ×
         14.50               ◊
                         CLEAR
        144.00             ← N
                           → N
         13.50             N ÷
          1.13               ◊
                           → I
       1468.91             ←PT
                           →PT
P  V
     104495.74
                              ↑
     100000.00              ←2
     104495.74               ⇄
                              −
-      4495.74               ◊
```

CLEAR RCL n n 13.5 ÷

i RCL PMT PMT PV

PW of Net Monthly Payment at 13.5% is $104,495.74.

RCL 2 $x \rightleftarrows y$ − ◊

NPV to borrower is -$4495.74.

Conclusion: While the wrap-around loan is very attractive to the lender, it is not profitable or feasible for the borrower.

e. Mortgagee Participation Loans (Equity Kickers)

Mortgage lenders, especially life insurance companies, sometimes require a share of property income as part of the price of granting a mortgage loan, in addition to contractual debt service. This sharing in property income is called a mortgagee participation or "equity kicker". The participation may be a percentage of Gross Income, of NOI, of CTO, or even forecast NCPR.

The analysis of mortgagee participation loans takes the same general format used in the preceding sections on Refinancing and Wrap-Around Mortgages. The NPV and/or effective yield (IRR) to borrower and lender are calculated and compared to ascertain which alternative is preferable to each participant.

Example - An investor has agreed to purchase an income property for $270,000. A mortgage loan of $210,000 has been arranged with an institutional lender, with full amortization over 25 years in level monthly payments at 9% interest.

The property has a lease with 10 years at $50,000 per year. Stabilized annual operating expenses are forecast at $22,000 per year. The investor plans to sell the property at the end of the lease term. The forecast proceeds of resale are $250,000.

The lender has just offered the investor two alternative financing plans:

1. An 8.6% interest rate, plus a 4% lender participation in gross income (all other loan terms the same);
2. An 8.35% interest rate, plus a 1/3 lender participation in CTO and NCPR (all other loan terms the same).

The lender will discount participation payments received at 12%.

Assuming the investor goes through with resale plans, and all income and resale forecasts are realized, which alternative is preferable to the lender? Which is preferable to the borrower (investor)?

1. Original loan, no participation

Enter: [CLEAR]

12 [STO] [0] **25** [×] [n] **9** [▒] [÷] [i]

210000 [PV] [PMT] [STO] [PMT]

[CLEAR] [RCL] [n] **10** [▒] [×] [−] [n]

[RCL] [i] [i] [RCL] [PMT] [PMT] [PV]

b = $173,752.38

250000 [x⇄y] [−] [◊] [STO] [1]

NCPR = $76,247.62

50000 [SAVE↑] **22000** [−] [◊]

NOI = $28,000

[RCL] [PMT] [▒] [×] [−] [◊] [PMT]

CTO = $6852.25

```
              CLEAR

     12.00    →0
     25.00     ×
    300.00    ◊
              → N
      9.00    N ÷
       .75    ◊
              →I
 210000.00    →P
              PMT
   1762.31
              →7*

              CLEAR
    300.00    ← N
     10.00    N ×
    120.00    ◊
               -
              → N
       .75    ← I
              → I
   1762.31    ←PT
              →PT
P  V
 173752.38
 250000.00
 173752.38     ⇄
               -
  76247.62    ◊
              →1
  50000.00     ↑
  22000.00     -
  28000.00    ◊
   1762.31    ←PT
              N ×
  21147.75    ◊
               -
   6352.25    ◊
              →PT
```

[CLEAR] [ ] [EXT( )] [2] [RCL] [1]

[FV] 10 [SAVE↑] 365 [×] [STO] [DAY]

[RCL] [PMT] [PMT] 60000 [PV] [ ] [i]

y = 12.90% as an IRR

```
                CLEAR
                  AN2
76247.62           ←1
                   → F
   10.00            ↑
  365.00            ×
 3650.00            ◇
                  →4*
 6852.25          ←PT
                  →PT
60000.00           →P
YIELD
TO            CALL
   12.90
```

NOTE:
NPV to lender = 0 and effective yield to lender = 9.0%.

2. Loan with 4% participation in gross income, 10 years:

[CLEAR] 25 [ ] [×] [n] 8.6 [ ] [÷]

[i] 210000 [PV] [PMT] [STO] [1]

[CLEAR] [RCL] [n] 10 [ ] [×] [−] [n]

[RCL] [i] [i] [RCL] [1] [PMT] [PV]

[STO] [2]

b = $172,131.50

250000 [$x \rightleftarrows y$] [−] [◇] [STO] [3]

NCPR = $77,868.50

50000 [SAVE↑] 4 [%] [◇] [STO] [4]

Participation = $2,000

```
                CLEAR
    25.00         N ×
   300.00           ◇
                  → N
     8.60         N ÷
      .72           ◇
                  → I
210000.00          →P
                  PMT
  1705.15
                   →1
                CLEAR
   300.00         ← N
    10.00         N ×
   120.00           ◇
                    -
                  → N
      .72         ← I
                  → I
                    ↑
  1705.15          ←1
                  →PT
P   V
172131.50
                   →2
250000.00
172131.50           ⇄
                    -
 77868.50           ◇
                   →3
 50000.00           ↑
     4.00           %
  2000.00           ◇
                   →4
```

[CLEAR] 10 [ ] [×] [n] 9 [ ] [÷]

[i] [RCL] [1] [PMT] [PV] [STO] [5]

```
            CLEAR
    10.00   N  ×
   120.00   ◊
            →  N
     9.00   N  ÷
      .75   ◊
            →  I
               ↑
  1705.15   ←1
            →PT
P  V
134607.59
            →5
```

[CLEAR] [RCL] [n] [n] [RCL] [i] [i]

[RCL] [2] [FV] [PV] [RCL] [5] [+] [◊]

[STO] [6]

PW Mortgage Receipts @ 9% = $204,826.45

```
            CLEAR
   120.00   ←  N
            →  N
      .75   ←  I
            →  I
               ↑
172131.50   ←2
            →  F
P  V
 70218.86
               ↑
134607.59   ←5
               +
204826.45   ◊

            →6
```

[CLEAR] 10 [n] 12 [i] [RCL] [4]

[PMT] [PV]

PW Participation @ 12% = $11,300.45

```
            CLEAR
    10.00   →  N
    12.00   →  I
               ↑
  2000.00   ←4
            →PT
P  V
 11300.45
```

[RCL] [6] [+] [◊]

PW Mortgagee Position = $216,126.90

```
               ↑
204826.45   ←6
               +
216126.90   ◊
```

210000 [−] [◊]

NPV to Mortgagee = +$6126.90

```
210000.00      −
  6126.90   ◊
```

[RCL] [1] [ ] [×] [RCL] [4] [+] [◊]

[PMT] **Total Annual Cash Flow to Mortgagee = $22,461.82**

```
               ↑
  1705.15   ←1
            N  ×
 20461.82   ◊
               ↑
  2000.00   ←4
               +
 22461.82   ◊
            →PT
```

[CLEAR] [ ] [EXT( )] [2] [RCL] [2]

[FV] 10 [SAVE↑] 365 [×] [STO] [DAY]

[RCL] [PMT] [PMT] 210000 [PV] [ ] [i]

Effective Yield (IRR) to Mortgagee = 9.54%

50000 [SAVE↑] [RCL] [4] [−] 22000 [−] [◊]

NOI = $26,000.00

[RCL] [1] [ ] [×] [−] [◊] [PMT]

CTO = $5538.18

[CLEAR] [RCL] [3] [FV] 10 [SAVE↑] 365

[×] [STO] [DAY] [RCL] [PMT] [PMT]

60000 [PV] [ ] [i]

y(IRR) = 11.01%

```
              CLEAR

                AN2
172131.50     ←2
              → F
    10.00      ↑
   365.00      ×
  3650.00     ◇
              →4*
 22461.82     ←PT
              →PT
210000.00     →P
 YIELD
 TO         CALL
     9.54
 50000.00      ↑
  2000.00     ←4
               -
 22000.00      -
 26000.00     ◇
               ↑
  1705.15     ←1
              N ×
 20461.82     ◇
               -
  5538.18     ◇

              →PT
              CLEAR
 77868.50     ←3
              → F
    10.00      ↑
   365.00      ×
  3650.00     ◇
              →4*
  5538.18     ←PT
              →PT
 60000.00     →P
 YIELD
 TO         CALL
    11.01
```

3. Loan with 1/3 participation in CTO and NCPR, 10 years:

[CLEAR] 25 [ ] [×] [n] 8.35 [ ]

[÷] [i] 210000 [PV] [PMT] [STO] [1]

```
              CLEAR
    25.00     N ×
   300.00     ◇
              → N
     8.35     N ÷
      .70     ◇
              → I
210000.00     →P
              PMT
```

```
                     1669.80
                                  →1
```

[CLEAR] [RCL] [n] 10 [ ] [×] [−] [n]

[RCL] [i] [i] [RCL] [1] [PMT] [PV]

[STO] [2]

b = $171.092.12

```
                              CLEAR

                    300.00   ← N
                     10.00   N ×
                    120.00   ◇
                              −
                             → N
                       .70   ← I
                             → I
                               ↑
                   1669.80   ←1
                             →PT
P   V
              171092.12
                             →2
```

250000 [$x \rightleftarrows y$] [−] [◊] [STO] [3]

NCPR = $78,907.88

```
              250000.00
              171092.12        ⇄
                               −
               78907.88      ◇

                             →3
```

50000 [SAVE ↑] **22000** [−] [◊]

NOI = $28,000.00

```
               50000.00        ↑
               22000.00        −
               28000.00      ◇
```

[RCL] [1] [ ] [×] [−] [◊] [STO] [4]

CTO = $7962.37

```
                               ↑
                1669.80      ←1
                             N ×
               20037.63      ◇
                               −
                7962.37      ◇

                             →4
```

[CLEAR] **10** [ ] [×] [n] **9** [ ] [÷]

[i] [RCL] [1] [PMT] [PV] [STO] [5]

```
                              CLEAR

                  10.00      N ×
                 120.00      ◇
                             → N
                   9.00      N +
                    .75      ◇
                             → I
                               ↑
                1669.80      ←1
                             →PT
P   V
             131917.02
                             →5
```

CLEAR RCL n n RCL i i

RCL 2 FV PV RCL 5 +

◊ STO 6

PW Mortgage Receipts @ 9% = $201,611.88

```
                CLEAR
   120.00     ← N
              → N
      .75     ← I
              → I
                ↑
171092.12     ←2
              → F
P  V
 69794.86
                ↑
131817.02     ←5
                +
201611.88     ◊

              →6
```

CLEAR 10 n 12 i RCL 4 3

÷ PMT PV STO 7

```
                CLEAR
    10.00     → N
    12.00     → I
                ↑
  7962.37     ←4
     3.00       ÷
  2654.12     ◊
              →PT
P  V
 14996.39
              →7
```

CLEAR RCL n n RCL i i

RCL 3 3 ÷ FV PV RCL 7

+ ◊

PW Participation @ 12% = $23,465.14

```
                CLEAR
    10.00     ← N
              → N
    12.00     ← I
              → I
                ↑
 78907.88     ←3
     3.00       ÷
 26302.63     ◊
              → F
P  V
  8468.74
                ↑
 14996.39     ←7
                +
 23465.14     ◊
```

RCL 6 + ◊

PW Mortgagee's Position = $225,077.01

```
                ↑
201611.88     ←6
                +
225077.01     ◊
```

210000 − ◊

```
210000.00       −
 15077.01     ◊
```

NPV to Mortgagee = +$15,077.01

CLEAR RCL FV RCL 2 + FV

10 SAVE↑ 365 × STO DAY RCL

PMT RCL 1 × + PMT

210000 PV i

Effective Yield (IRR) to Mortgagee = 10.44%

```
        CLEAR
  26302.63   ← F
               ↑
 171092.12   ← 2
               +
             → F
     10.00     ↑
    365.00     ×
   3650.00   ◇
             →4*
   2654.12   ←PT
               ↑
   1669.80   ← 1
             N ×
  20037.63   ◇
               +
             →PT
 210000.00   →P
  YIELD
  TO        CALL
     10.44
```

CLEAR RCL 3 2 × 3 ÷ FV 10

SAVE↑ 365 × STO DAY RCL 4

2 × 3 ÷ PMT 60000 PV i

y(IRR) = 8.00%

```
        CLEAR
  78907.88   ← 3
      2.00     ×
 157815.76   ◇
      3.00     +
  52605.25   ◇
             → F
     10.00     ↑
    365.00     ×
   3650.00   ◇
             →4*
   7962.37   ← 4
      2.00     ×
  15924.74   ◇
      3.00     +
   5308.25   ◇
             →PT
  60000.00   →P
  YIELD
  TO        CALL
      8.00
```

Summary	NPV to Lender	IRR to Lender	IRR to Borrower
1. 9% loan	0	9.00%	12.90%
2. 8.6% loan with 4% Gross Participation	+$6,127	9.54%	11.01%
3. 8.35% loan with 1/3 CTO and NCPR Participation	+$15,077	10.44%	8.00%

Clearly, the mortgagee would prefer the 1/3 participation in CTO and NCPR, while the mortgagor would prefer the original loan.

f. Maturity Associated with Specific Mortgage Yield

By varying the nominal maturity, usually by extending it, it is possible to achieve a given effective yield for the lender or maintain a maximum mortgage constant for the borrower, while keeping the periodic payment amount constant.

This procedure utilizes the routines for solving for [n] illustrated in Chapters 2, 4 and 5.

Example - An $80,000 mortgage was recently negotiated, with full amortization over 20 years in level monthly payments at 9% interest. Subsequently, interest rates have risen, and the lender now wants a 9.25% yield to maturity. The borrower wants to keep the same monthly payments.

1. What maturity term would give a 9.25% yield to the lender with the same monthly payments?
2. What maturity term would be required if the borrower insisted on paying no higher than a 10.5% mortgage constant?

Enter: [CLEAR]

Solve for monthly payment:

12 [STO] [0] 20 [×] [n] 9 [ ] [÷] [i] 80000 [PV] [PMT] [STO] [PMT]

Monthly Payment = $719.78

1. Solve for term with 9.25% yield:

[CLEAR] 9.25 [ ] [÷] [i] [RCL] [PMT] [PMT] [RCL] [PV] [PV] [n] [ ] [÷]

253.05 months or 21.09 years (21 years 2 months)

```
          CLEAR

   12.00        →0
   20.00         ×
  240.00        ◊
                → N
    9.00        N ÷
     .75        ◊
                → I
80000.00        →P
                PMT
  719.78
                →7*
          CLEAR

    9.25        N ÷
     .77        ◊
                → I
  719.78        ←PT
                →PT
80000.00        ←P
                →P
N
  253.05
                N ÷
   21.09        ◊
```

2. Solve for term with 9.25% effective yield and 10.5% constant:

CLEAR RCL PV PV 10.5 % ÷ PMT 9.25 ÷ i n ÷

277.16 months or 23.10 years (23 years 2 months)

```
                 CLEAR
       80000.00    ←P
                   →P
          10.50     %
        8400.00    N ÷
         700.00    ◇
                   →PT
           9.25    N ÷
            .77    ◇
                   →  I
N
         277.16
                   N ÷
          23.10    ◇
```

NOTE:
In this case, monthly payment can be no higher than $700.

g. Market Value and Discount or Premium on an Existing Mortgage

This procedure is described and illustrated in detail in Chapter 4 and 5, in the sections titled Present Worth of a Mortgage.

Refinancing Decisions

Selection among alternative financing packages has been covered and illustrated in three preceding sections of Chapter 13:

Impact of Financing Alternatives
Refinancing
Wrap-Around Mortgages

In the examples provided in those sections, it is demonstrated that the NPV and IRR to the mortgagee (lender) can be calculated, and then compared to find the alternative most preferable to the lender.

It is also possible to calculate IRR and / or NPV to the mortgagor (borrower) on the net amount of cash received on refinancing or wrap-around mortgage proposals. A comparison of these measures indicates which alternative is most preferable to the borrower.

One other method of evaluating refinancing or wrap-around mortgage proposals for the borrower is to calculate the periodic income forecast to be receivable by the borrower on the net cash proceeds of the loan, at the rate the borrower expects to earn on the reinvested proceeds. If this income exceeds the additional debt service required to pay off the net cash proceeds of refinancing, it is an attractive or "feasible" transaction for the borrower.

Example - An existing mortgage has a current balance of $133,190. It has a remaining term of 17 years, with monthly payments of $1080.33.

It is proposed to refinance (or "wrap") with a new mortgage of $180,000 at 9% interest, payable in level monthly payments over 17 years.

The equity investor can reinvest the net proceeds at 13.5% on an annual basis. Should he accept the refinancing (or wrap-around) proposal?

Enter: [CLEAR]

12 [STO] [0] **17** [×] [n] **9** [ ] [÷] [i]

180000 [PV] [PMT]

Monthly payment on new mortgage = $1725.85

1080.33 [−] [◊]

Net additional monthly payment = $645.52

[ ] [×] [STO] [1]

Net additional ADS = $7746.20

[RCL] [PV] **133190** [−] [◊] [PV]

Cash proceeds of refinancing = $46,810.00

17 [n] **13.5** [i] [PMT]

Income from $46,810 at 13.5% = $7149.91

[RCL] [1] [−] [◊]

Coverage of net ADS = -$596.29

```
              CLEAR

    12.00      →0
    17.00        ×
   204.00      ◊
               → N
     9.00      N ÷
      .75      ◊
               → I
180000.00      →P
               PMT
  1725.85
  1080.33        -
   645.52      ◊

               N ×
  7746.20      ◊
               →1
180000.00      ←P
133190.00        -
 46810.00      ◊

               →P
    17.00      → N
    13.50      → I
               PMT
  7149.91
                 ↑
  7746.20      ←1
                 -
-   596.29     ◊
```

No. The income produced from reinvesting the cash proceeds at 13.5% does not cover the net annual debt service on the refinancing (or wrap-around).

Selection Among Disparate Alternatives: Size and Time Disparity

Not all investments require the same capital outlay, nor do all produce income streams of the same length. In order to make investment opportunities requiring different capital outlays comparable for selection of the best for the investor, it is necessary to identify what can or must be earned on the portion of available investment funds not committed to the smaller investment.

Example - An investor is considering two alternative investments. The first requires a Capital Outlay of \$88,000. It has forecast CTO of \$10,000 per year plus forecast NCPR of \$100,000 in 10 years. The second requires an equity investment of \$102,500, with forecast CTO of \$11,500 per year and NCPR of \$115,000 in 10 years. The minimum acceptable equity yield rate (y) is 11%.

1. Using NPV, PI and IRR measures, which appears to be the preferable investment?
2. How much (what rate) must be earned on the unused investment funds available to make the smaller investment preferable in all respects?

Enter: CLEAR

NPV and PI of \$88,000 investment:

10 n **11** i **10000** PMT PV STO 1

CLEAR RCL n n RCL i i

100000 FV PV RCL 1 + ◊

STO 2

PW at 11% = \$94,110.77

88000 − ◊

NPV = +\$6110.77

RCL 2 **88000** ÷

PI = 1.07

NPV and PI of \$102,500 investment:

CLEAR RCL n n RCL i i

11500 PMT PV STO 1

```
          CLEAR

     10.00    → N
     11.00    → I
  10000.00    →PT
P    V
  58892.32
              →1

          CLEAR

     10.00    ← N
              → N
     11.00    ← I
              → I
 100000.00    → F
P    V
  35218.45
               ↑
  58892.32    ←1
               +
  94110.77    ◊
              →2
  88000.00     -
   6110.77    ◊
               ↑
  94110.77    ←2
  88000.00     ÷
      1.07    ◊

          CLEAR

     10.00    ← N
              → N
     11.00    ← I
              → I
  11500.00    →PT
P    V
  67726.17
              →1
```

CLEAR RCL n n RCL i i

115000 FV PV RCL 1 + ◊

STO 2

PW at 11% = $108,227.38

102500 − ◊ RCL 2 102500 ÷

PI=1.06

y(IRR) on $88,000 investment:

CLEAR EXT( ) 2 100000 FV

10 SAVE↑ 365 × STO DAY 10000

PMT 88000 PV i

y = 12.14%

y(IRR) on $102,500 investment:

CLEAR 115000 FV 10 SAVE↑ 365

× STO DAY 11500 PMT 102500

PV i

y = 11.92%

```
                CLEAR
      10.00     → N
                → N
      11.00     → I
                → I
  115000.00     → F
P   V
  40501.22
                  ↑
   67726.17     →1
                  +
  108227.38     ◊
                →2
  102500.00       -
    5727.38     ◊
                  ↑
  108227.38     →2
  102500.00       ÷
       1.06     ◊
                CLEAR
                AN2
  100000.00     → F
      10.00       ↑
     365.00       ×
    3650.00     ◊
                →4*
   10000.00     →PT
   88000.00     →P
  YIELD
  TO          CALL
      12.14
                CLEAR
  115000.00     → F
      10.00       ↑
     365.00       ×
    3650.00     ◊
                →4*
   11500.00     →PT
  102500.00     →P
  YIELD
  TO          CALL
      11.92
```

On a direct comparison basis, the $88,000 investment is clearly preferable:

	$88,000	$102,500
NPV	+$6,110.77	+$5,727.38
PI	1.07	1.06
IRR	12.14%	11.92%

However, there remains the question of what rate must be earned on the unused portion of available investment funds.

Enter: [CLEAR]

102500 [SAVE ↑] **88000** [−] [◊] [PV]

Uninvested available funds = $14,500

11500 [SAVE ↑] **10000** [−] [◊] [STO] [PMT]

Required extra payment = $1500 per year

115000 [SAVE ↑] **100000** [−] [◊] [FV]

Required extra reversion = $15,000

[CLEAR] [RCL] [FV] [FV] **10** [SAVE ↑] **365**

[×] [STO] [DAY] [RCL] [PMT] [PMT]

[RCL] [PV] [PV] [ ] [i]

Required y on extra $14,500 = 10.56%

```
                CLEAR

  102500.00          ↑
   88000.00          -
   14500.00        ◊

                   →P
   11500.00          ↑
   10000.00          -
    1500.00        ◊

                   →7*
  115000.00          ↑
  100000.00          -
   15000.00        ◊

                   → F
                CLEAR

   15000.00      ← F
                 → F
      10.00          ↑
     365.00          ×
    3650.00        ◊
                   →4*
    1500.00        ←PT
                   →PT
   14500.00        ←P
                   →P
   YIELD
   TO          CALL
       10.56
```

If the investor can earn 10.56% on the unused $14,500, the two investments are exactly equal. If he can earn more than 10.56% on the unused funds, the $88,000 investment is preferable; if less, the $102,500 investment is preferable.

OTHER REAL ESTATE DECISIONS

There are other types of real estate problems and decisions which do not fall neatly into the foregoing categories. However the same general process of comparative analysis applies in these cases as well.

a. Rent or Buy

Frequently a tenant considers real estate purchase as an alternative course of action. With the HP-81 it is possible to compare the financial or investment implications of purchase vs. rental. While the final decision may rest on personal or even emotional considerations, these can be weighed against the financial realities.

NOTE:

For illustrative purposes here, only first-year financial requirements are shown.

Example - A family is renting an apartment at $285 per month. Tired of paying rent with no financial results, they decide to buy a comparable house.

They have found a house priced at $39,000, on which they can obtain a 90% mortgage loan at 9% interest with a 30-year maturity (level monthly payments). Closing costs are estimated at $650.

Property taxes are $1050; utilities charges (sewer and water) are $200 per year; insurance and maintenance is estimated at $850, and heating expenses at $1450.

The property is expected to follow the current trend of appreciation of 5% per year.

The family's tax bracket is 38%, applicable to tax-deductible interest and property tax payments. The savings from which the down payment and closing charges would be taken are earning 7% interest.

From a financial point-of-view (using first year figures), is it preferable to buy or to continue to rent?

Enter: [CLEAR]

285 [SAVE ↑] **12** [STO] [0] [×]

Annual rent = $3420

39000 [SAVE ↑] **.10** [×] **650** [+] [◊] **.07** [×]

Interest on down payment and closing costs funds = $318.50

[−] [◊] [STO] [9]

Net First-Year Cost of Renting = $3101.50

[CLEAR] **39000** [SAVE ↑] **.90** [×] [PV]

Mortgage Principal = $35,100

```
            CLEAR

   285.00       ↑
    12.00      →0
                ×
  3420.00   ◊
 39000.00       ↑
      .10       ×
  3900.00   ◊
   650.00       +
  4550.00   ◊

      .07       ×
   318.50   ◊
                −
  3101.50   ◊

               →9

            CLEAR

 39000.00       ↑
      .90       ×
 35100.00   ◊
               →P
```

CLEAR 30 × n 9 ÷

i RCL PV PV PMT STO PMT

282.42 × STO 3

Annual Debt Service = $3389.04

CLEAR 1 STO 1 12 STO 2 RCL

i i RCL PMT PMT RCL PV

PV Σ+

First Year Interest = $3149.28

STO 1 1050 + ◊

Tax Deductible Items = $4199.28

.38 × STO 2

Tax Saving = $1595.73

39000 SAVE↑ .05 × STO 5

Appreciation = $1950.00

RCL 3 RCL 1 − ◊ STO 4

First Year Equity Build-Up = $239.76

RCL 3 1050 + 200 + 850 + 1450

+ RCL 4 − RCL 2 − RCL

```
                    CLEAR

            30.00    N ×
           360.00    ◊
                     → N
             9.00    N ÷
              .75    ◊
                     → I
         35100.00    ←P
                     →P
                     PMT
           282.42
                     →7*
           282.42    N ×
          3389.04    ◊
                     →3
                    CLEAR
             1.00    →1
            12.00    →2
              .75    ← I
                     → I
           282.42    ←PT
                     →PT
         35100.00    ←P
                     →P
                      Σ+
P    V
         34860.21
I N T
          3149.28
                     ←1
          1050.00     +
          4199.28    ◊

              .38     ×
          1595.73    ◊
                     →2

         39000.00     ↑
              .05     ×
          1950.00    ◊
                     →5
                      ↑
          3389.04    ←3
                      ↑
          3149.28    ←1
                      −
           239.76    ◊

                     →4
                      ↑
          3389.04    ←3
          1050.00     +
           200.00     +
```

[5] [−] [◊]

First Year Net Cost of Buying = $3153.55

[RCL] [9] [−] [◊]

Net First Year Cost of Buying = $52.05

```
     850.00      +
    1450.00      +
                 ↑
     239.76     ←4
                 −
                 ↑
    1595.73     ←2
                 −
                 ↑
    1950.00     ←5
                 −
    3153.55     ◊
                 ↑
    3101.50     ←9
                 −
      52.05     ◊
```

Conclusion: Financially, it is a toss-up between renting and buying in this case. The procedure is applicable in any such decision.

b. Sell or Lease; Hold or Sell

The decision whether to sell or hold an income property involves the same general procedure as is applied in the preceding section to solve Rent or Buy problems. The Present Worth of the income stream at a specified rate of discount is compared with the net proceeds of sale to identify which is preferable. IRR required to match the rate of return on the reinvested proceeds can also be calculated.

Example - A rental property with a 15-year lease is producing CTO of $12,000 per year. The forecast NCPR in 15 years is $125,000. The owner has an offer to sell the property today for $130,000.

If the owner can earn a 10.25% equity yield rate (before-tax) on the reinvested sale proceeds, should he sell the property for $130,000?

1. Calculation of NPV:

a. PW of holding (leasing) at 10.25%

[CLEAR] **15** [n] **10.25** [i] **12000** [PMT]

[PV] [STO] **1** [CLEAR] **15** [n] **10.25** [i]

125000 [FV] [PV] [RCL] [1] [+] [◊]

PW = $118,907.26

```
              CLEAR

        15.00     → N
        10.25     → I
     12000.00     →PT
  P  V
     89985.08
                  →1
              CLEAR

        15.00     → N
        10.25     → I
    125000.00     → F
  P  V
     28922.18
```

b. NPV of holding at 10.25%

130000 [−] [◊]

NPV = -$11,092.74

```
                   ↑
   39985.08       ←1
                   +
  118907.26        ◊

  130000.00        -
-  11092.74        ◊

        CLEAR

          AN2

  125000.00      → F
      15.00        ↑
     365.00        ×
    5475.00        ◊
                  →4*
   12000.00       →PT
  130000.00       →P
  YIELD
  TO        CALL
       9.10
```

2. Calculation of Required IRR

[CLEAR] [ ] [EXT( )] [2] **125000** [FV]

15 [SAVE ↑] **365** [×] [STO] [DAY] **12000**

[PMT] **130000** [PV] [ ] [i]

Required IRR = 9.10%

c. Leased Fee and Leasehold

Leased Fee (Lessor's Interest) valuation and analysis uses procedures illustrated in Chapters 5, 7, 8, 10, as well as preceding sections of Chapter 13. They involve estimating the Present Worth of the cash flows during the term of the lease, and adding the Present Worth of the reversion (forecast value of the property when the lease expires). Effective yield to the lessor is calculated as an IRR.

However, leasehold valuation and calculation of yield on a leasehold purchase also require attention, especially since it is usually not possible to obtain rates of discount readily from the market. The leasehold valuation procedure is to calculate it as a residual from Market Value minus Leased Fee Value. The effective yield on a leasehold is calculated as an IRR for a fully amortized annuity.

Example - A property is leased at $1,000 per month with a remaining term of 12 years. The property is forecast to be worth $125,000 when the lease expires. The rate of discount for the leased fee is 10%.

The market rental for this type of property is $1100 per month. The rate of discount for market value purposes is 10.5%. What is the present worth of the leasehold?

Enter: [CLEAR]

```
        CLEAR
```

1. Market Value Estimate

12 [n] 10.5 [i] 1100 [SAVE↑] 12 [×]

[PMT] [PV] [STO] [0]

```
    12.00   → N
    10.50   → I
  1100.00     ↑
    12.00     ×
 13200.00     ◊
            →PT
P   V
 87779.53
             →0
```

[CLEAR] [RCL] [n] [n] [RCL] [i] [i]

125000 [FV] [PV] [RCL] [0] [+] [◊] [STO]

[1]

Market Value = $125,498.75

```
        CLEAR
    12.00   ← N
            → N
    10.50   ← I
            → I
125000.00   → F
P   V
 37719.22
              ↑
 87779.53    ←0
              +
125498.75     ◊

             →1
```

2. PW Leased Fee

[CLEAR] [RCL] [n] [n] 10 [i] 1000

[SAVE↑] 12 [×] [PMT] [PV] [STO] [0]

```
        CLEAR
    12.00   ← N
            → N
    10.00   → I
  1000.00     ↑
    12.00     ×
 12000.00     ◊
            →PT
P   V
 81764.30
             →0
```

[CLEAR] [RCL] [n] [n] [RCL] [i] [i]

125000 [FV] [PV] [RCL] [0] [+] [◊]

PW Leased Fee = $121,593.15

```
        CLEAR
    12.00   ← N
            → N
    10.00   ← I
            → I
125000.00   → F
P   V
 39828.65
              ↑
 81764.30    ←0
              +
121593.15     ◊
```

3. PW Leasehold

[RCL] [1] [$x \rightleftarrows y$] [−] [◊]

PW Leasehold = $3905.59

```
              ↑
125498.75    ←1
121593.15     ⇄
              −
  3905.59     ◊
```

APPENDIX

REAL ESTATE SYMBOLS AND TERMINOLOGY USED IN THE HP-81 REAL ESTATE APPLICATIONS HANDBOOK

1. INCOME SYMBOLS

PGI:	Potential Gross Income (Number of rental units times rental per unit, at 100% occupancy, annually)
v:	Allowance for vacancy and income loss (annual)
EGI:	Effective Gross Income: Rent Collections plus "Other Income" (PGI - v + "Other" = EGI, annual)
OE:	Operating Expenses (annual)
NOI:	Net Operating Income (annual: EGI - OE = NOI) Also: NIBR = NOI
ADS:	Annual Debt Service (Monthly mortgage payment x 12)
CTO:	Cash Throw-Off to Equity (annual: NOI - ADS = CTO); Gross Spendable Income
ATCF:	After-Tax Cash Flow (annual: NOI - Income Tax Liability = ATCF); Net Spendable Income

2. VALUE (Present Worth, Reversion) SYMBOLS

V:	Value (Present Worth)
PW:	Present Worth (Value, Present Value)
SP:	Sales Price
V_m:	Value, Principal, Present Worth of Mortgage
V_e:	Value, Present Worth of Equity
P:	Principal of Mortgage
CPR:	Cash Proceeds of Resale; Reversion (forecast; before tax) CPR = SP - Selling or Disposition Expenses
b:	Balance of Mortgage Outstanding
NCPR:	Net Cash Proceeds of Resale (to equity; before tax); Equity Reversion NCPR = CPR - b
ATNCPR:	After-Tax Net Cash Proceeds of Resale (to equity); After-Tax Equity Reversion
FW:	Future Worth (Reversion; Resale Proceeds)

3. COMPOUND INTEREST AND DISCOUNT FACTOR SYMBOLS

FW 1:	Future Worth of One; Compound Amount of One
FW 1/A:	Future Worth of One per Period; Accumulation of One per Period
SFF:	Sinking Fund Factor; $1/s_n$
$1/s_n$:	Sinking Fund Factor; SFF
PW 1:	Present Worth of One; Reversion Factor
PW 1/A:	Present Worth of One per Period; Level Annuity Factor; Inwood Factor; a_n
a_n:	Present Worth of One per Period; Level Annuity Factor; Inwood Factor; PW 1/A
Amort.:	Installment to Amortize One; $1/a_n$
$1/a_n$:	Installment to Amortize One

4. RATE, CAPITALIZATION RATE, RATE OF RETURN SYMBOLS

R:	Overall Rate (on property investment): annual (NOI $\div$ V; NOI $\div$ SP)
r:	Discount Rate; Basic Rate; "Interest" Rate; Rate of Return on Total Property Investment: annual
f:	Mortgage Constant: annual (ADS $\div$ P)
i:	Mortgage Interest Rate (contract): annual
e:	Equity Dividend Rate: annual (CTO $\div$ V_e)
y:	Equity Yield Rate; Rate of Return on Equity Investment: annual
IRR:	Internal Rate of Return: annual
CRR:	Capital Recovery Rate (on improvements): annual
CR:	Capitalization Rate (for investment in improvements): annual (CR = r + CRR)

5. MORTGAGE-EQUITY (Ellwood) ANALYSIS SYMBOLS

n:	Income Projection Period; Investment Holding Period
NOI:	Net Operating Income (annual)
CF:	Cash Flow (annual)
ADS:	Annual Debt Service
CTO:	Cash Throw-Off to Equity; Equity Dividend (CTO = NOI - ADS)

P:	Mortgage Principal (original)
b:	Mortgage Balance Outstanding at End of Income Projection Period (n)
p:	Percent (or Amount) of Mortgage Paid Off during Income Projection Period (n): $p = P - b$
f:	Mortgage Constant (annual)
i:	Mortgage Interest Rate (contract; annual)
m:	Loan-to-Value Ratio; Mortgage Principal as a Percentage of Value
e:	Equity Dividend Rate (annual)
y:	Equity Yield Rate (annual)
(1-m):	Equity as a Percentage of Value
c:	Mortgage Coefficient ($c = y + p\ 1/s_n - f$)
$1/s_n$:	Sinking Fund Factor at Equity Yield Rate (y) over Income Projection Period (n)
r:	Basic Rate; Discount Rate (annual)
R:	Overall Rate (annual)
dep.:	"Depreciation"; Capital Loss on Resale as a Percentage of Value
app.:	"Appreciation"; Capital Gain on Resale as a Percentage of Value
CPR:	Cash Proceeds of Resale; Reversion at End of Income Projection Period (n)
NCPR:	Net Cash Proceeds of Resale to Equity; Equity Reversion at End of Income Projection Period (n): NCPR = CPR - b

6. CASH FLOW AND INVESTMENT ANALYSIS SYMBOLS

CO:	Capital Outlay; Investment in Time Period 0
CF_t:	Cash Flow (positive or negative) in Time Period "t"
PW:	Present Worth; Prevent Value
NPV:	Net Present Value (NPV = PW - CO)
PI:	Profitability Index (PI = PW $\div$ CO)
IRR:	Internal Rate of Return

7. STATISTICAL SYMBOLS

X:	Value of One Observation of a Variable; The Independent Variable in Linear Regression or Trend Line Analysis

Y: The Dependent Variable in Linear Regression or Trend Line Analysis

Y_c: The Calculated or Estimated Value of the Dependent Variable in Linear Regression or Trend Line Analysis

ΣX or ΣY: The Sum of the Values of X or Y

ΣX^2 or ΣY^2: The Sum of the Squares of the Values of X or Y

$(\Sigma X)^2$ or $(\Sigma Y)^2$ The Square of the Sum of the Values of X or Y

ΣXY: The Sum of the Products of Paired Values of X and Y

$\overline{X}$ or $\overline{Y}$: The Mean of the Values of X or Y

s: Standard Deviation

n: The Number of Observations or Items in a Sample

a: The Value of the Constant or Y-Intercept in Linear Regression or Trend Line Analysis; The Value of Y_c when $x = 0$

b: The Regression Coefficient; The Slope of the Regression Line in Linear Regression or Trend Line Analysis

r^2: The Coefficient of Determination; The Percentage of Total Variance in Y Explained by the Regression Line ($Y_c = a + bx$)

s_{yx}: The Standard Error of the Estimate in Linear Regression or Trend Line Analysis

s_b: The Standard Error of the Regression Coefficient (b)

t: Statistical Significance of the Regression Coefficient (b): $t = b \div s_b$

STORAGE REGISTERS AND CONSTANTS

The HP-81 has 20 Storage Registers, numbered 0 through 19.

Register 0 is always available for operator use. It can be used to store a constant for multiplication and division. It can also be used to store any number for future use.

To multiply a series of numbers by a constant (k), the following steps are followed:

[CLEAR]

k [STO] [0]

First Number [ ] [×]

x k

Second Number ×
x k

and so on.

To divide a series of numbers by a constant (k), the following steps are followed:

CLEAR

k STO 0

First Number ÷
÷ k

Second Number ÷
÷ k

and so on.

To divide a series of numbers into a constant (k), the following steps are followed:

CLEAR

k STO 0

RCL 0 **First Number** ÷

RCL 0 **Second Number** ÷

and so on.

(For details, See HP-81 Operating Guide, pages 13-14.)

Registers 1 through 9 are normally available for operator use when more than one number is to be stored for future use, except when certain routines are employed, as noted below. Registers 10 through 19 are frequently taken up in calculating routines. The numbers stored in these registers can be recalled for future use if the operator knows what value is stored in which register. These are summarized below. (See also HP-81 Operating Guide, pages 5-6.)

Register		Item Stored
0		Constant
1	a)	First time-period date in Length-of-Time Calculation
	b)	Personal or corporate tax rate in After-Tax Mode: EXT( ) 1
	c)	Cash Flow 1 in Cash Flow Analysis
2	a)	Second time-period date in Length-of-Time Calculations
	b)	Capital gains tax rate in After-Tax Mode: EXT( ) 1
	c)	Cash Flow 2 in Cash Flow Analysis
3		Cash flow 3 in Cash Flow Analysis
4		Cash Flow 4 in Cash Flow Analysis
5		Cash Flow 5 in Cash Flow Analysis
6		Cash Flow 6 in Cash Flow Analysis
7		Cash Flow 7 in Cash Flow Analysis
8		Cash Flow 8 in Cash Flow Analysis
9		Cash flow 9 in Cash Flow Analysis
*10	a)	ΣX^2 for Summation $\Sigma+$; Mean Calculation $\bar{x}$
	b)	Slope of line (b) in Linear Regression or Trend Line Analysis
*11	a)	Number of Entries (n) for Summation $\Sigma+$ Mean Calculation $\bar{x}$
	b)	Linear Regression or Trend Line Analysis
*12	a)	ΣX for Summation $\Sigma+$; Mean Calculation $\bar{x}$
	b)	Linear Regression or Trend Line Analysis

*13	a)	Constant or Y-Intercept (a) in Linear Regression or Trend Line Analysis
14	a)	Number of Entries (n) in Linear Regression or Trend Line Analysis DAY
15		Calculated or entered value of FV
16		Calculated or entered value of i
17		Calculated or entered value of PMT
18		Calculated or entered value of PV
19		Calculated or entered value of n

**NOTE:*
Numbers in Storage Registers 10-13 are erased by CLEAR. *All other Storage Registers are unaffected by* CLEAR.

SHIFT KEY OPERATIONS

The Green Key [ ] is a Shift Key which alters the operation of many function keys. While details of Shift Key procedures are presented throughout this volume, the following summary should prove helpful to HP-81 users, especially in developing variations on the procedures presented here. Further details are provided in the HP-81 Operating Guide.

Keystroke Sequence	Calculation and Result
[ ] TL (CAL)	Calculates Trend Line Values
[ ] SOD (CAL)	Calculates Sum-of-Digits Values
[ ] DAY (DAT)	Calculates Date and Day of Week
[ ] % (Δ%)	Calculates Percentage Difference between Two Numbers
[ ] EXT( ) (M( ))	Sets Calculating Mode

(See subsequent section in this Appendix for details on Modes)

i / YTM	Yield to Maturity, Yield to Call
PMT / INT	Effective Yield; Total Interest Accrued Between Two Dates
PV / BND	Price to Maturity; Price to Call
÷ / ÷K	Divide by a Constant (in Storage Register 0)
× / ×K	Multiply by a Constant (in Storage Register 0)
− / LOG	Logarithm to Base 10
y^x / $\sqrt{x}$	Calculates Square Root
$\bar{x}$ / Σ	Prints Running Sum of Values of x: Σ+
Σ+ / Σ−	Deletes Value and Item in Σ+
◊ / PRM	Prints List of Modes Currently Set in Calculator
✱ / PRD	Prints List of Data Stored in Storage Registers 1-19.

The Shift Key is also used to set the number of decimal places shown in answers, from 0 to 6. Just press the Shift Key and the number of decimal places desired.

Example: [ ] [4] sets all printed items at 4 decimal places (rounded).

EXTENDED FUNCTIONS

The HP-81 has 10 Extended Functions which are preprogrammed procedures and printouts based on data inserted in proper sequence. The details of the use of each are shown in the appropriate section of this Handbook, as well as in the HP-81 Operating Guide. For user convenience and handy reference, they are summarized below:

Keystrokes	Calculation and Printout
[EXT( )] [0]	Effective Compound Interest Rate per Period, Given the Annual Nominal Interest Rate
[EXT( )] [1]	Internal Rate of Return on an Uneven Cash Flow or Variable Annuity up to 9 Cash-Flow Time Periods
[EXT( )] [2]	Declining-Balance Depreciation Schedule (No Salvage Value)
[EXT( )] [3]	Depreciation Schedule and "Diminishing Factor" with Salvage Value
[EXT( )] [4]	Sum-of-the-Years' - Digits Depreciation Schedule
[EXT( )] [5]	Interest Rebate, Total Amount of Interest Paid, and Total Pay-Off or Balance on a Prepaid Loan (Uses Rule of 78's)
[EXT( )] [6]	Amortized Loan Schedule: Interest Paid and Principal Paid During Each Period, and Balance Remaining at End of Each Period
[EXT( )] [7]	Effective Annual Interest Rate on Add-on Interest Loan, with Odd Number of Days until First Payment

EXT( ) 8	Yield of a Discounted Note as if it were a Bond Selling at Par with Semi-Annual Coupons
EXT( ) 9	Sum of a Series of Numbers up to 19 Entries, and Percent Each is of that Sum

CALCULATING MODES

The HP-81 offers 5 pairs of alternative calculating modes that provide flexibility in calculating financial yields, rates of return and final values or prices. The details and assumptions of these calculating modes are presented on pages 125-127 of the HP-81 Operating Guide.

The calculating modes are numbered 0 through 9. To set any given mode in the calculator, press in sequence the Shift (Green) Key, the Extended Function Key, and the Number Key in the keyboard corresponding to the number of the desired calculating mode.

For example, to set Mode 1 (After-Tax Mode), press: EXT( ) 1

When the HP-81 is turned on, and throughout all calculations unless the operator changes a mode, the following modes are pre-set in the calculator:

Mode Number	Mode
0	Pre-Tax
3	Semiannual Coupon
5	Actual-Day Month
7	365-Day Year (with automatic leap-year adjustment)
8	Bond (periodic coupon payments)

The operator may set the following changes:

From Mode	To Mode
0	1 : After-Tax Calculations
3	2 : Annual Coupon
5	4 : 30-day Month
7	6 : 360-day Year

8 9 : Note (No periodic payments; final payment only)

If Mode 1 (After-Tax Calculations) is used, the applicable Income Tax rate must be stored in Storage Register 1, and the Capital Gains Tax rate must be stored in Storage Register 2. Both must be entered as a percent: e.g., 30% is entered as 30.

To print out a list of which modes are set at any time, press [ ] [◊]
PRM

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