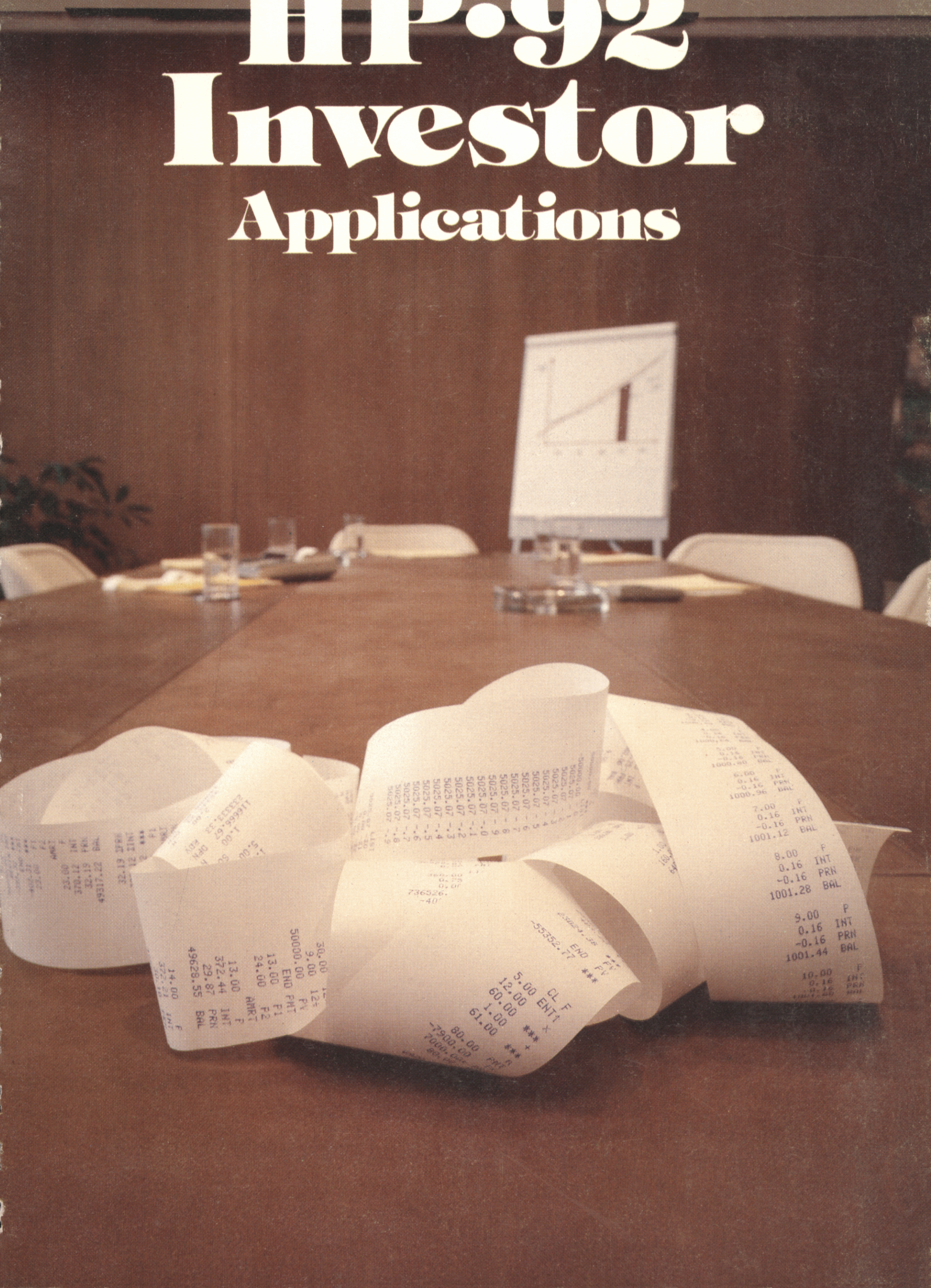
HEWLETT-PACKARD

HP-92 Investor Applications



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Introduction

The HP-92 APPLICATIONS HANDBOOK has been designed to supplement the HP-92 OWNER'S HANDBOOK by providing a collection of key applications in the areas of real estate, investment analysis, savings and leasing, statistics, and securities. Step by step keystroke procedures and examples for over 50 problem types are explained. These procedures may be altered and combined to provide solutions to many more problems.

Whether you are a real estate professional, a securities analyst, or are involved in any field of investment analysis, you will find the HP-92 and this book invaluable for reducing your calculation time and effort in analyzing transactions. No longer are cumbersome financial tables and texts needed—the HP-92 allows you to merely press the appropriate keys and obtain your answer in seconds. In addition, the flexibility and power of the HP-92 will enable you to expand your capabilities to analyze complicated investment alternatives.

To help you in using this Handbook, and in finding the keystroke procedure applicable to your particular problem, the Table of Contents is organized by general topic areas. Each procedure in that topic area is then sub-catagorized and listed individually. For example, if you wish to find the yield to the lender on a wrap-around mortgage, just refer to Wrap-Around Mortgages under Financial Analysis in the REAL ESTATE AND INVESTMENT ANALYSIS section. Or, if you wish to fit a set of data to an appropriate curve, you will find three methods located under Curve Fitting in the STATISTICS section.

The range of applications of the HP-92 is quite large and has not been fully explored. However, we hope that this Handbook will cover the most important recurring situations that confront you.

NOTE

Unless otherwise specified, the keystroke solutions in this book are evaluated with the Print Mode switch in the MAN (manual) position. If NORM (normal) is used, calculated results may be printed on the tape by pressing PRINT x.

TABLE OF CONTENTS

SAVINGS AND LEASING

STATISTICS

SECURITIES

OTHER

APPENDICES

REAL ESTATE AND INVESTMENT ANALYSIS

SIMPLE MORTGAGES

ANNUAL PERCENTAGE RATE CALCULATIONS WITH FEES

Borrowers are sometimes charged fees in connection with the issuance of a mortgage, which effectively raises the interest rate. The actual amount received by the borrower (PV) is reduced, while the periodic payments remain the same. Given the life or term of the mortgage, the interest rate, the mortgage amount, and the basis of the fee charge (how the fee is calculated), the true Annual Percentage Rate may be calculated. Information is entered as follows:

1) Set the Payment Mode switch to END and press CL FIN.

2) Calculate and enter the periodic payment amount of the loan.
   a) Key in the total number of payment periods; press n.
   b) Key in the periodic interest rate; press i.
   c) Key in the mortgage amount; press PV.*
   d) To obtain the periodic payment amount press PMT.*

3) Calculate and key in the actual net amount dispersed.*.
   a) If fees are stated as a percentage of the mortgage amount (points), recall the mortgage amount (RCL PV); key in the fee (percentage) rate; press % − PV.
   b) If fees are stated as a flat charge, recall the mortgage amount (RCL PV); key in the fee amount (flat charge); press − PV.
   c) If fees are stated as a percentage of the mortgage amount plus a flat charge, recall the mortgage amount (RCL PV); key in the fee (percentage) rate, press % −; key in the fee amount (flat charge);press − PV.

4) Press i to obtain the percentage rate per compounding period.

5) To obtain the annual nominal percentage rate, key in the number of periods per year, and press ×.

Example 1:

A borrower is charged 2 points for the issuance of his mortgage. If the mortgage amount is $50,000 for 30 years, and the interest rate is 9% per year, with monthly payments, what annual percentage rate is the borrower paying? (1 point is equal to 1% of the mortgage amount.)

* Positive for cash received; negative for cash paid out.

Keystrokes:	Outputs:	
BEGIN/NOTE ▬ END/BOND (switch to END)		
CL FIN		
30 f 12× →	360.00	Months (into n)
9 f 12÷ →	0.75	% monthly interest rate (into i)
50000 CHS PV →	-50000.00	Loan amount (into PV)
PMT →	402.31	Monthly payment (calculated)
RCL PV 2 % − PV →	-49000.00	Actual amount paid out by lender (into PV)
i →	0.77	% monthly interest rate (calculated)
12 × →	9.23	Annual percentage rate

Example 2:

Using the same information as given in Example 1, calculate the APR if the mortgage fee is $150 instead of a percentage.

Keystrokes:	Outputs:	
BEGIN/NOTE ▬ END/BOND (switch to END)		
CL FIN		
30 f 12× →	360.00	Months (into n)
9 f 12÷ →	0.75	% monthly interest rate (into i)
50000 PV →	50000.00	Loan amount (into PV)
PMT →	-402.31	Monthly payment (calculated)
RCL PV 150 − PV →	49850.00	Effective mortgage amount
i →	0.75	Monthly interest rate (calculated)
12 × →	9.03	Annual percentage rate

Example 3:

Again using the information given in Example 1, what is the APR if the mortgage fee is stated as 2 points plus $150?

Keystrokes:	Outputs:
BEGIN/NOTE — END/BOND (switch set to END)	
CL FIN	
30 f 12× →	360.00 Months (into n)
9 f 12÷ →	0.75 % monthly interest rate (into i)
50000 PV →	50000.00 Loan amount (into PV)
PMT →	−402.31 Monthly payment (calculated)
RCL PV 2 % −	
150 − PV →	48850.00 Effective mortgage amount
i →	0.77 Monthly interest rate (calculated)
RCL f 12÷ →	9.26 Annual percentage rate

PRICE OF A MORTGAGE TRADED AT A DISCOUNT/PREMIUM

Mortgages may be bought and/or sold at prices lower (discounted) or higher (at a premium) than the remaining balance of the loan at the time of purchase. Given the amount of the mortgage, the periodic payment, the timing and amount of the balloon or prepayment, and the *desired* yield rate, the price of the mortgage may be found. It should be noted that the balloon payment amount (if it exists) occurs coincident with, and does not include, the last periodic payment amount.

Information is entered as follows:

1) Set the Payment Mode switch to END and press CL FIN.
2) Key in the total number of periods until the balloon payment or prepayment occurs; press n.
3) Key in the *desired* periodic interest rate (yield) and press i.
4) Key in the periodic payment amount; press PMT.*
5) Key in the balloon payment amount and press FV.*
6) Press PV to obtain the purchase price of the mortgage.

Example 1:

A lender wants to induce the borrower to prepay a low interest rate loan. The interest rate is 5% with 6 years (72 payments) remaining of $137.17 and a

* Positive for cash received; negative for cash paid out.

balloon payment at the end of the sixth year of $2000. If the lender is willing to discount the future payments at 7½%, how much would the borrower need to prepay the note?

Keystrokes:	**Outputs:**	
BEGIN/NOTE ▭ END/BOND		
CL FIN		
72 n 7.5 f 12÷ 137.17		
PMT 2000 FV PV →	-9210.48	Amount necessary to prepay note

Example 2:

A 9½% mortgage with 28 years remaining may be acquired which has a remaining balance of $49,350. Determine the price to pay for this mortgage if the desired annual yield is 12%. (Since the payment amount is not given, it must be calculated.)

Keystrokes:	**Outputs:**	
BEGIN/NOTE ▭ END/BOND		
CL FIN		
28 f 12× →	336.00	Months (into n)
9.5 f 12÷ →	0.79	% monthly interest rate (into i)
49350 CHS PV PMT →	420.40	Monthly payment to be received
12 f 12÷ →	1.00	Desired % monthly interest rate (into i)
PV →	-40555.50	Purchase price to achieve the desired yield (calculated)

YIELD OF A MORTGAGE TRADED AT A DISCOUNT/PREMIUM

The annual yield of a mortgage bought at a discount or premium may be calculated, given the original mortgage amount, interest rate, and periodic payment, as well as the number of payment periods per year, the price paid for the mortgage, and the balloon payment amount (if it exists).

Information is entered as follows:

1) Set the Payment Mode switch to END and press CL FIN.

2) Key in the total number of periods until the balloon payment occurs and press n.
3) Key in the periodic payment amount and press PMT.*
4) Key in the purchase price of the mortgage; press PV.*
5) Key in the balloon payment amount and press FV.*
6) Press i to obtain the yield per period.
7) Key in the number of periods per year and press × to obtain the nominal annual yield.

Example 1:

Find the annual yield of a 7%, 21 year mortgage prepaid in full at the end of the 12th year, if the mortgage amount is $100,000, the purchase price is $86,000, and equal monthly payments of $758.47 are received. The remaining balance at the end of the 12th year is $60,647.67.

Keystrokes:	Outputs:	
BEGIN/NOTE ▬ END/BOND		
CL FIN		
12 f 12× →	144.00	Balloon payment occurs at end of 144th month (into n)
758.47 PMT →	758.47	Monthly payment received (into PMT)
86000 CHS PV →	-86000.00	Amount loaned (into PV)
60647.67 FV i →	0.77	Percent monthly yield (calculated)
RCL f 12÷ →	9.23	Percent annual yield

PRESENT VALUE OF A MORTGAGE WITH BALLOON PAYMENT ONE PERIOD AFTER LAST PAYMENT

Sometimes the balloon payment is due and payable one period after the last periodic payment in the contract maturity. If this is encountered, the present value (or purchase price) of the mortgage loan may be calculated as follows:

1) Set the Payment Mode switch to END and press CL FIN.
2) Key in the total number of payment periods until the balloon payment occurs; press n.

* Positive for cash received; negative for cash paid out.

3) Key in the periodic interest rate; press **i**.
4) Key in the periodic payment amount; press **PMT**.*
5) Key in the balloon amount, less the periodic payment amount, and press **FV**.*
6) Press **PV** to obtain the present value of the mortgage.

Example 1:

A 20-year $60,000 mortgage loan with a balloon payment of $23,507.58 has monthly payments of $483.20. The balloon payment is to be made one month after the last monthly payment. What is the present worth or market value to the purchaser of the mortgage loan if he desires a 9.5% annual yield?

Keystrokes:	Outputs:	
BEGIN/NOTE ▬ END/BOND (switch to END)		
CL FIN		
20 **ENTER↑** 12 **×** 1 **+** **n** ⟶	241.00	The balloon occurs at the end of the 241st month (into n)
9.5 **f** **12÷** ⟶	0.79	Monthly interest rate (into i)
483.20 **PMT** ⟶	483.20	Monthly payment (into PMT)
23507.58 **RCL** **PMT** **−** **FV** ⟶	23024.38	Adjusted balloon amount (into FV)
PV ⟶	-55352.77	Present value (the amount to pay for the opportunity to receive the cash flows)

YIELD OF A MORTGAGE WITH BALLOON PAYMENT ONE PERIOD AFTER LAST PAYMENT

Given the periodic payment amount, total number of periods in mortgage life, mortgage price, and the balloon payment amount which occurs one period after the last payment, the yield is calculated as follows:

1) Set the Payment Mode switch to END and press **CL FIN**.
2) Key in the total number of payment periods until the balloon payment occurs; press **n**.
3) Key in the periodic payment amount; press **PMT**.*

* Positive for cash received; negative for cash paid out.

4) Key in the price of the mortgage; press **PV**.*
5) Key in the balloon amount, less the periodic payment amount, and press **FV**.*
6) Press **i** to obtain the periodic yield.
7) Key in the number of periods per year and press **×** to obtain the annual yield.

Example 1:

What is the annual yield to the lender of a $7900 mortgage which has monthly payments of $80 for 5 years and a balloon payment of $7000 occurring one period after the last periodic payment?

Keystrokes:		Outputs:	
BEGIN/NOTE ▬ END/BOND			
CL FIN			
5 **ENTER↑** 12 **×** 1 **+** **n**	⟶	61.00	The balloon occurs at the end of the 61st payment (into n)
80 **PMT**	⟶	80.00	Monthly payment received (into PMT)
7900 **CHS** **PV**	⟶	-7900.00	Loan amount (into PV)
7000 **RCL** **PMT** **−** **FV**	⟶	6920.00	Adjusted balloon payment amount received (into FV)
i	⟶	0.86	Percent monthly yield (calculated)
12 **×**	⟶	10.28	Percent annual yield

DEFERRED ANNUITIES

Sometimes transactions are established where payments do not begin for a specified number of periods (the payments are deferred). To determine the present value of such an annuity, the following keystrokes may be used:

1) Set the Payment Mode switch and press **CL FIN**.
2) Key in the total number of payments to be made and press **n**.
3) Key in the periodic interest rate and press **i**.
4) Key in the periodic payment amount, press **PMT**.*

* Positive for cash received; negative for cash paid out.

5) To calculate the "deferred" present value of the annuity, press PV.

6) Press STO FV.

7) Key in the total number of periods the payments were deferred and press n.

8) Key in 0 and press PMT.

9) Press PV to determine today's present value of the deferred annuity (or the amount needed today to meet the requirements of the future).

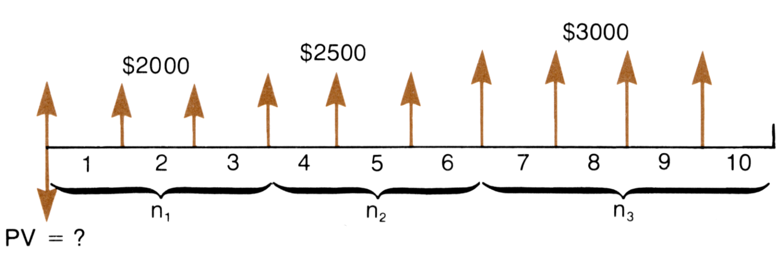
Example 1:

Your firm is responsible for the upkeep of a newly built bridge. No repairs are required until the beginning of the fourth year, when $2000 will be needed for repainting. From then on, it is estimated that $2000 will be needed at the beginning of each year for the next 26 years. You wish to establish a fund which earns 6% annually, specifically for bridge upkeep. How much do you need to deposit in the fund today to meet the upkeep requirements?

Keystrokes:	Outputs:	
BEGIN/NOTE ▮ END/BOND		
CL FIN		
27 n 6 i 2000 CHS PMT PV →	28006.33	Value of the annuity at the end of the 3rd year
STO FV 3 n 0 PMT PV →	–23514.66	Today's value of the annuity

Leases often call for periodic contractual adjustments of rental payments. For example, a 10-year lease may call for rentals of $2000 per year for 3 years, $2500 per year for the next 3 years, and $3000 per year for the last 4 years. This situation illustrates what is called a "step-up" lease. A "step-down" lease is similar, except that rental payments are decreased periodically according to the lease contract. Lease payments are made at the beginning of the year.

In the example cited, the rental payment stream for years 4-6 and years 7-10 are "deferred annuities," as they start at some time in the future. The cash flow diagram from the investor's viewpoint looks like this:



To find today's present value (the amount of the investment) the following keystrokes may be used:

1) Set the Payment Mode switch to BEGIN and press CL FIN.
2) Calculate the present value of the payments in the last time span (here, n_3).
   a) Key in the total number of periods in the last time span and press n.
   b) Key in the periodic discount (interest) rate and press i.
   c) Key in the periodic payment amount; press PMT.*
   d) To calculate the present value press PV.
3) The present value of the last group of cash flows can now be considered as a balloon payment for the next to last group of cash flows (n_2); press FV.*
4) Calculate the present value of the payments in time span n_2 (as in step 2) and press FV.*
5) Calculate the present value of the payments in time span n_1. The display shows the present value of the entire group of cash flows.

Example 2:

A 2-year lease calls for monthly payments (at the beginning of the month) of \$500 per month for the first 6 months, \$600 per month for the next 12 months, and \$750 per month for the last 6 months. If you wish to earn 13.5% annually on these cash flows, how much should you invest?

Keystrokes:	**Outputs:**	
BEGIN/NOTE — END/BOND		
CL FIN		
6 n 13.5 f 12÷ 750 PMT PV →	-4376.69	Present value of last group of payments
CHS FV 12 n 600 PMT PV →	-10602.25	Present value of last 2 groups of cash flows
CHS FV 6 n 500 PMT PV →	-12831.75	Amount to invest to achieve a 13.5% yield

In some instances, the net present value technique may be used to find the present value of a deferred annuity. Refer to the Owner's Handbook for the keystroke procedure.

* Positive for cash received; negative for cash paid out.

CONSUMER LOANS

LOANS WITH A CONSTANT AMOUNT PAID TOWARDS PRINCIPAL

This type of loan is structured such that the principal is repaid in equal installments with the interest paid in addition. Therefore each periodic payment has a constant amount applied toward the principal and a varying amount of interest.

Loan Reduction Schedule

If the constant periodic payment to principal, annual interest rate, and loan amount are known, the total payment, interest portion of each payment, and remaining balance for each successive payment may be calculated as follows:

1) Key in the constant periodic payment to principal; press STO 0.
2) Key in periodic interest rate and press ENTER↑ ENTER↑ ENTER↑.
3) Key in the loan amount.
   If you wish to skip to another time period, press ENTER↑. Then key in the number of payments to be skipped, and press RCL 0 × −.
4) Press x⇄y % to obtain the interest portion of the payment.
5) Press RCL 0 + to obtain the total payment.
6) Press R↓ RCL 0 − to obtain the remaining balance of the loan.
7) Return to step 4 for each successive payment.

Example 1:

A $60,000 land loan at 10% interest calls for equal semi-annual principal payments over a 6-year maturity. What is the loan reduction schedule for the first year? (Constant payment to principal is $5000 semi-annually.) What is the fourth year's schedule (skip 4 payments)?

Keystrokes:	Outputs:	
5000 STO 0		
10 ENTER↑ 2 ÷ ENTER↑		
ENTER↑ ENTER↑ →	5.00	Semi-annual interest rate
60000 x⇄y % →	3000.00	First payment's interest
RCL 0 + →	8000.00	Total first payment
R↓ RCL 0 − →	55000.00	Remaining balance
x⇄y % →	2750.00	Second payment's interest

RCL 0 + →	7750.00	Total second payment
R↓ RCL 0 − →	50000.00	Remaining balance after first year
4 RCL 0 × −		
x⇄y % →	1500.00	Seventh payment's interest
RCL 0 + →	6500.00	Total seventh payment
R↓ RCL 0 − →	25000.00	Remaining balance
x⇄y % →	1250.00	Eighth payment's interest
RCL 0 + →	6250.00	Total eighth payment
R↓ RCL 0 − →	20000.00	Remaining balance after fourth year

ADD-ON INTEREST RATE CONVERTED TO APR

An add-on interest rate determines what portion of the principal will be added on for repayment of a loan. This sum is then divided by the number of months in the loan to determine the monthly payment. For example, a 10% add-on rate for 36 months on $3000 means add one-tenth of $3000 for 3 years (300 × 3)—usually called the "finance charge"—for a total of $3900. The monthly payment is $3900/36.

This keystroke procedure converts an add-on interest rate to an annual percentage rate when the add-on rate and number of months are known.

1) Set the Payment Mode switch to END and press CL FIN.
2) Key in the number of months in loan; press n f 12÷.
3) Key in the add-on rate; press ×.
4) Key in the amount of loan; press PV* x⇄y % +.
5) Press x⇄y ÷ CHS PMT.
6) Press i RCL f 12÷ to obtain the APR.

Example 1:

Calculate the APR and monthly payment of a 5%, $1000 add-on loan which has a life of 18 months.

* Positive for cash received; negative for cash paid out.

Keystrokes:	**Outputs:**
BEGIN/NOTE ▮▮ END/BOND	
CL FIN	
18 n f 12÷	
5 ×	
1000 PV x≷y % + →	1075.00 Amount of loan
x≷y ÷ CHS PMT →	–59.72 Monthly payment
i RCL f 12÷ →	9.27 % annual interest rate

APR CONVERTED TO ADD-ON INTEREST RATE

Given the number of months and annual percentage rate, this procedure calculates the corresponding add-on interest rate.

1) Set the Payment Mode switch to END and press CL FIN.
2) Enter the following information:
   a) Key in number of months of loan, press n.
   b) Key in APR, press f 12÷.
   c) Key in 100, press PV PMT.
3) Press RCL PV RCL n ÷ + CHS 12 × to obtain add-on rate.

Example 1:

What is the equivalent add-on rate for an 18-month loan with an APR of 14%?

Keystrokes:	**Outputs:**
BEGIN/NOTE ▮▮ END/BOND	
CL FIN	
18 n 14 f 12÷ 100 PV	
PMT RCL PV RCL n ÷	
+ CHS 12 × →	7.63 Percent add-on rate

INTEREST REBATE—RULE OF 78's

This procedure finds the unearned interest rebate, as well as the remaining principal balance due for a prepaid consumer loan using the Rule of 78's.

The known values are the current installment number, the total number of installments for which the loan was written, and the total finance charge (amount of interest). The information is entered as follows:

1) Press CL FIN.
2) Key in the number of months in the loan; press f LIFE.
3) Key in the total finance charge; press f BOOK.
4) Key in the payment number when prepayment occurs; press g N1 g N2.
5) Press SOYD to obtain the unearned interest (rebate). The interest portion of the final installment is in the Y-register. (Before going to step 6, be sure that the rebate amount is in the display).
6) Key in the periodic payment amount; press ENTER↑.
7) Key in the number of installments remaining on the loan; press × x⇄y − to obtain the amount of principal outstanding.

Example 1:

A 30-month, $1000 loan having a finance charge of $180, is being repaid at $39.33 per month. What is the rebate and balance due after the 25th regular payment?

Keystrokes:	Outputs:
CL FIN	
30 f LIFE	
180 f BOOK	
25 g N1 g N2	
SOYD ⟶	5.81 Rebate
39.33 ENTER↑ 5 ×	
x⇄y − ⟶	190.84 Outstanding principal

DEPRECIATION

Depreciation is a method of allocating the cost of an asset over its useful life. Depreciation is an annual deduction from Net Operating Income before taxable income and income tax liability are figured. It is an accounting expense charged against cash income.

The three most common accounting methods are Straight Line, Declining Balance, and Sum-of-the-Years'-Digits (SOYD) depreciation. Declining balance and SOYD are methods of "accelerated" depreciation whereby higher depreciation amounts are charged in the early years of an asset's life than with straight line depreciation.

It is important to note that land is not depreciable for income tax purposes. Therefore, the acquisition cost of improvements (buildings, fixtures, site improvements) must be separated from land value at time of property acquisition before a depreciation schedule can be calculated.

If there is an expected salvage value (frequently called the 'reversion') at the end of the useful life of the depreciable asset, the salvage value must be deducted from the asset's acquisition cost to derive the amount to be depreciated (tax basis), if the straight line or SOYD methods are to be used. Salvage value need not be subtracted from acquisition cost under the declining-balance method, but the asset may not be depreciated below the salvage value. However, for most real estate depreciation problems, salvage value is not a consideration or a legal requirement.

If accelerated depreciation is used, the difference between total depreciation charged over a given period of time and the total amount that would have been charged under straight line depreciation is called "excess depreciation." With some minor exceptions, the amount of "excess depreciation" is "recaptured" and taxed as ordinary income when the property is resold.

The following routines allow the analyst to:

1) Calculate partial-year depreciation at the beginning and end of the depreciation period.
2) Calculate "excess" depreciation at the end of any given period.
3) Calculate and identify the "crossover" period when straight line depreciation equals or exceeds accelerated depreciation, at which point it pays to switch to straight line depreciation.

To illustrate the procedures, and to show the differences resulting from the application of different depreciation methods, a continuing example will be used.

Example:

A property has just been acquired for $150,000. The purchase price is allocated between $25,000 for land and $125,000 for improvements (building). The remaining useful life of the building is agreed to be 25 years. There is no salvage value forecast at the end of the useful life of the building. Thus, the depreciable cost is $125,000. This is also the tax basis of the investment in the building.

Straight Line Depreciation

Partial-Year Depreciation, Beginning and End of Period

For income tax purposes especially, calendar-year or fiscal-year depreciation charges must be calculated. When the acquisition date (the start of the depreciation period) does not coincide with the start of the accounting year—which is usual—the amount of depreciation in the first and last accounting years must be taken as fractions of the full-year depreciation.

The keystrokes are:

1) Press CL FIN.

2) Calculate the amount of annual straight line depreciation (book value less salvage value all divided by life) and store in register 0.
   - Key in the starting book value and press f BOOK.
   - Key in the salvage value and press f SAL −.
   - Key in the life of the asset and press f LIFE.
   - Press ÷ STO 0 to determine the annual depreciation.

3) Calculate the portion of a year from the acquisition date to the start of the next accounting year, and store in register 1.

4) Multiply the step 2 result by the step 3 result to find the first year's depreciation.

5) Press RCL f BOOK x⇄y − f BOOK to find the balance at the beginning of the first full accounting year.

6) Press RCL f LIFE RCL 1 − f LIFE to determine the new life of the asset.

7) Key in the starting period for the schedule; press g N1. Key in the ending period of the schedule; press g N2. Set the printer switch to ALL.

8) Press SL to determine the amount of depreciation during the specified period(s). The display shows the remaining depreciable value at the end of the term.

9) For the last (partial) year, press RCL 0; calculate the portion of the year represented by the last partial year's depreciation; press × − to find the end-of-period balance.

Example 1:

If the example property were bought on September 1, what is the first 5-year schedule of depreciation?

Keystrokes:	Outputs:	
MAN ALL NORM		
CL FIN		
125000 f BOOK 25 f LIFE		
÷ STO 0 →	5000.00	Annual depreciation
4 ENTER↑ 12 ÷ STO 1 × →	1666.67	Partial-year (4 months) depreciation
RCL f BOOK x⇄y −		
f BOOK →	123333.33	Balance at end of 4 months; balance at beginning of calendar-year
RCL f LIFE RCL 1 −		
f LIFE →	24.67	Remaining term
1 g N1 4 g N2 →	4.00	Last year when full amount is depreciated
MAN ALL NORM		
SL →		

```
              SL
     1.00      N
  5000.00    DPN
118333.33    RDV

     2.00      N
  5000.00    DPN
113333.33    RDV

     3.00      N
  5000.00    DPN
108333.33    RDV

     4.00      N
  5000.00    DPN
103333.33    RDV

 20000.00   ΣDPN
103333.33    ***
```

Amount of depreciation during years 1-4 and remaining depreciable value

Keystrokes:	Outputs:	
RCL 0 8 ENTER↑ 12 ÷ →	0.67	
× →	3333.33	Partial-year (8 months) depreciation
− →	100000.00	Balance at end of 5 years

```
              ← 0
  5000.00    ***
     8.00   ENT↑
    12.00      ÷
     0.67    ***
               x
  3333.33    ***
               -
100000.00    ***
```

Declining Balance Depreciation

Partial-Year Depreciation, Beginning and End of Period

To find the amount of depreciation in the partial year at the beginning of the period, when the acquisition date is different from the beginning of the accounting year, first calculate the amount of first-year declining-balance depreciation and multiply by the percentage of a year from the acquisition date to the beginning of the first full accounting year. Then subtract that from the depreciable amount to obtain the balance at the beginning of the first full accounting year. Then apply the declining-balance routine over the income projection period, less the initial fractional-year period.

For the fractional year's depreciation at the end of the period, calculate the applicable percentage of a year and multiply it by the last full year's depreciation of the income projection period. Subtract that amount of depreciation from the previous year's balance to find the balance at the end of the income projection period.

Example 1:

What is the 150% declining balance depreciation schedule for the example property over 5 years, if the property was purchased on September 1 and depreciation is charged on a calendar-year basis?

Keystrokes:	Outputs:
ALL MAN [switch] NORM	
CL FIN	
125000 f BOOK	
25 ÷ 150 % 4 ENTER↑	
12 ÷ × ⟶	2500.00 Four months depreciation
RCL f BOOK x⇄y − f BOOK ⟶	122500.00 Balance after 4 months
150 g FACT	
25 f LIFE	
1 g N1 4 g N2	

ALL
MAN NORM

DB →

6104.79 ENTER↑ 8 ENTER↑
12 ÷ × → 4069.86 Last 8 months depreciation

− → 91571.89 Final balance

```
               DB
     1.00       N
  7350.00     DPN
115150.00     RDV

     2.00       N
  6909.00     DPN
108241.00     RDV

     3.00       N
  6494.46     DPN
101746.54     RDV

     4.00       N
  6104.79     DPN
 95641.75     RDV

 26858.25    ΣDPN
 95641.75     ***


  6104.79    ENT↑
     8.00    ENT↑
    12.00       ÷
     0.67     ***
                ×
  4069.86     ***
                -
 91571.89     ***
```

Calculation of Excess Depreciation

The steps and keystrokes are:

1) Calculate the total depreciation charged, as described in the Owner's Handbook.

2) Key in the depreciable amount, press ENTER↑; key in the useful life of the asset in years, press ÷; key in the number of years in the income projection period, press ×. This is the total straight line depreciation over the income projection period.

3) Press −. This is the "excess depreciation."

Example 1:

What is the excess depreciation charged on a 150% declining-balance basis for the example property over 10 years?

Keystrokes:	Outputs:
ALL MAN ▮ NORM	
CL FIN	
25 f LIFE 150 g FACT	
125000 f BOOK 1 g N1 10 g N2	
DB x⇄y ⟶	57673.11 Total depreciation charged
RCL f BOOK RCL f LIFE	
÷ RCL g N2 × − ⟶	7673.11 Excess depreciation

Crossover Point

To identify the "crossover" point where the straight line depreciation charge on the remaining balance exceeds the declining balance depreciation charge, first calculate the schedule of declining balance depreciation. Then start comparing the straight line depreciation for the remaining useful life on the remaining balance at the beginning of each year with the declining balance charge for that year. A good starting point is the year in which declining balance depreciation first falls below annual straight line depreciation on the original depreciable amount over the entire useful life.

The crossover point is the end of the year in which the declining balance depreciation last exceeds or equals the straight line depreciation amount on the remaining balance less salvage value at the beginning of the year.

Example 1:

What is the crossover point with 150% declining balance depreciation for the example property?

Keystrokes:	Outputs:
ALL MAN ▮ NORM	
CL FIN	CL F
25 f LIFE 150 g FACT	25.00 LIFE 150.00 FACT
125000 f BOOK 1 g N1	125000.00 BOOK 1.00 N1
12 g N2 DB ⟶	12.00 N2 DB

1.00	N
7500.00	DPN
117500.00	RDV
2.00	N
7050.00	DPN
110450.00	RDV
3.00	N
6627.00	DPN
103823.00	RDV
4.00	N
6229.38	DPN
97593.62	RDV
5.00	N
5855.62	DPN
91738.00	RDV
6.00	N
5504.28	DPN
86233.72	RDV
7.00	N
5174.02	DPN
81059.70	RDV
8.00	N
4863.58	DPN
76196.12	RDV
9.00	N
4571.77	DPN
71624.35	RDV
10.00	N
4297.46	DPN
67326.89	RDV
11.00	N
4039.61	DPN
63287.28	RDV

```
    12.00    N
  3797.24  DPN
 59490.04  RDV

 65509.96 ΣDPN
 59490.04  ***
```

Start testing at year 8, where 150% declining balance depreciation is $4863.58. This is the first year in which 150% declining balance depreciation is less than the original straight line depreciation of $5000.

1) Key in the remaining balance at the beginning of the year, press ENTER↑.
2) Key in the remaining years of useful life at the beginning of the year, press ÷.

This gives the annual straight line depreciation charge on the remaining balance over the remaining useful life.

Repeat steps 1 and 2 above until the result for a given year exceeds the declining balance depreciation charge for that year in the printed schedule of declining balance depreciation.

Year 8:

81059.70 ENTER↑ 18 ÷ ⟶ 4503.32

($4503.32 is less than $4863.58)

```
 81059.70 ENT↑
    18.00    ÷
  4503.32  ***
```

Year 9:

76196.12 ENTER↑ 17 ÷ ⟶ 4482.12

($4482.12 is less than $4571.77)

```
 76196.12 ENT↑
    17.00    ÷
  4482.12  ***
```

Year 10:

71624.35 ENTER↑ 16 ÷ ⟶ 4476.52

($4476.52 is more than $4297.46)

```
 71624.35 ENT↑
    16.00    ÷
  4476.52  ***
```

The "crossover" point is at the end of year 9. Depreciation in year 10 is $4297.46 which is less than $4476.52.

Sum-of-the-Years'-Digits Depreciation

Partial-Year Depreciation Beginning and End of Period

To find the amount of depreciation in the partial year at the beginning of the period, when the acquisition date is different from the beginning of the accounting year, first calculate the normal SOYD depreciation schedule. Then calculate the fraction of a year from the acquisition date to the beginning of the next accounting year (factor 1), and calculate the fraction of a year remaining from the beginning of the accounting year to the first full year of holding (factor 2). After these factors have been obtained, apply them to the normal SOYD schedule as shown below:

Normal year 1 scheduled depreciation [×] factor 1
= Year 1 value

Normal year 1 scheduled depreciation [×] factor 2 [+]

Normal year 2 scheduled depreciation [×] factor 1
= Year 2 value

Normal year 2 scheduled depreciation [×] factor 2 [+]

Normal year 3 scheduled depreciation [×] factor 1
= Year 3 value

Continue this procedure for each year. In the final year, which is one year beyond the useful life due to the partial years at the beginning and end of the schedule, the depreciation is simply:

Normal year N scheduled depreciation [×] factor 2
= Year N + 1 value

Example 1:

What is the SOYD depreciation schedule for the example property in the first 4 years if the property was purchased on September 1 and depreciation is charged on a calendar year basis?

Keystrokes:	Outputs:
ALL MAN [switch] NORM	
CL FIN	
25 f LIFE 125000 f BOOK	
1 g N1 4 g N2 SOYD →	

```
           CL F
     25.00 LIFE
 125000.00 BOOK
      1.00   N1
      4.00   N2
           SOYD
      1.00    N
   9615.38  DPN
 115384.62  RDV
```

```
    2.00    N
 9230.77  DPN
106153.85 RDV

    3.00    N
 8846.15  DPN
 97307.70 RDV

    4.00    N
 8461.54  DPN
 88846.16 RDV

 36153.84 ΣDPN
 88846.16  ***
```

4 ENTER↑ 12 ÷ STO 1	→	0.33 Factor 1
1 x⇄y − STO 2	→	0.67 Factor 2
9615.38 STO 0 RCL 1 ×	→	3205.13 Year 1 depreciation
RCL 0 RCL 2 ×		
9230.77 STO 0 RCL 1 × +	→	9487.18 Year 2 depreciation
RCL 0 RCL 2 ×		
8846.15 STO 0 RCL 1 × +	→	9102.56 Year 3 depreciation
RCL 0 RCL 2 ×		
8461.54 STO 0 RCL 1 × +	→	8717.95 Year 4 depreciation

Calculation of Excess Depreciation

The steps and keystrokes are:

1) Calculate the total depreciation charged.

2) Recall the starting book values; recall the useful life of the asset, press ÷; recall the number of years in the income projection period, press ×. This is the total straight line depreciation over the income projection period.

3) Press −. This is the ''excess depreciation.''

Example 1:

What is the excess depreciation charged on the example property, using SOYD depreciation over 10 years?

Keystrokes:	Outputs:
ALL MAN NORM (switch to ALL)	
CL FIN	
25 f LIFE 125000 f BOOK 1 g N1	
10 g N2 SOYD x⇄y ⟶	78846.15 Total depreciation charged
RCL f BOOK RCL f LIFE ÷	
RCL g N2 × − ⟶	28846.15 Excess depreciation

Crossover Point

There is no "crossover" point for SOYD depreciation schedules. At every point on the schedule, the SOYD depreciation charge exceeds the straight line depreciation charge on the remaining balance over the remaining useful life.

While it is legal to switch from SOYD to straight line depreciation at any time, it is not advantageous for the investor to do so.

FINANCIAL ANALYSIS

IMPACT OF FINANCING ALTERNATIVES

Terms of mortgage financing can and do vary. The equity investor (borrower) sometimes has a choice among two or more alternative financing packages. To select the alternative which is best for the borrower, the present value of the equity investment under each alternative financing package is calculated. The alternative which produces the highest present value of the equity investment is the one the equity investor should select.

Example:

An investor is considering the purchase of an income property for $200,000. The net operating income (NOI) is forecast at $22,000 annually. The investor plans to hold the property for 10 years, at which time the proceeds of resale are forecast to be 90% of the purchase price ($180,000). The investor is seeking a 13% rate of return on the equity investment.

The investor is considering four alternative financing plans which are available to him:

1) A 75% mortgage at 9.5% interest, with full amortization in level monthly payments over 20 years.
2) A 70% mortgage at 9.25% interest, with full amortization in level monthly payments over 20 years.

3) A 60% mortgage at 9% interest, with full amortization in level monthly payments over 20 years, plus a 15% second mortgage at 10.5% interest, with full amortization in level monthly payments over 10 years.

4) A 75% mortgage at 9.5% interest with full amortization in level monthly payments over 30 years.

Which is the best financing package for the equity investor?

Alternative 1: 75%, 20-year loan at 9.5%.

Keystrokes:	Outputs:	
BEGIN/NOTE ▬ END/BOND		
CL FIN		
200000 ENTER↑ 75 % PV 20		
f 12× 9.5 f 12÷ PMT →	-1398.20	Monthly payment necessary to amortize the loan
10 f 12× FV →	-108054.34	Remaining balance at end of 10th year
180000 + FV →	71945.66	Net cash proceeds of resale
RCL PMT 12 × →	-16778.36	Annual mortgage payment
22000 + PMT →	5221.64	Annual net cash flow
10 n →	10.00	Total term of investment
50000 CHS PV →	-50000.00	Total equity investment
i →	12.84	Percent annual yield

Alternative 2: 70%, 20-year loan at 9.25%

Keystrokes:	Outputs:	
BEGIN/NOTE ▬ END/BOND		
CL FIN		
20 f 12× 9.25 f 12÷ 200000		
ENTER↑ 70 % PV PMT →	-1282.21	Monthly payment necessary to amortize the loan
10 f 12× FV →	-100147.33	Remaining balance at end of 10th year
180000 + FV →	79852.67	Net cash proceeds of resale
RCL PMT 12 × →	-15386.56	Annual Mortgage payment
22000 + PMT →	6613.44	Annual net cash flow

10 n →	10.00	Total term of investment
60000 CHS PV →	-60000.00	Total equity investment
i →	12.83	Percent annual yield

Alternative 3: 60%, 20-year loan at 9%, plus 15%, 10-year second mortgage at 10.5%

Keystrokes:	**Outputs:**	
BEGIN/NOTE END/BOND		
CL FIN		
10 f 12× 10.5 f 12÷		
200000 ENTER↑ 15 % PV →	30000.00	Amount of second mortgage.
PMT STO 0 →	-404.80	Monthly payment of second
20 f 12× 9 f 12÷ 200000 ENTER↑ 60 % PV PMT		
STO + 0 →	-1079.67	Monthly payment of original mortgage
10 f 12× FV →	-85231.07	Remaining balance of original mortgage at end of 10th year
180000 + FV →	94768.93	Net cash proceeds of resale
22000 RCL 0 12 × + PMT →	4186.29	Net annual cash flow
10 n →	10.00	Total term of investment
50000 CHS PV →	-50000.00	Total equity investment
i →	13.19	Percent annual yield

Alternative 4: 75%, 30-year loan at 9.5%

Keystrokes:	**Outputs:**	
BEGIN/NOTE END/BOND		
CL FIN		
200000 ENTER↑ 75 % PV 30 f 12× 9.5 f 12÷ PMT →	-1261.28	Monthly payment to amortize the loan
10 f 12× FV →	-135311.57	Remaining balance at end of 10th year
180000 + FV →	44688.43	Net cash proceeds of resale

RCL PMT 12 ×		
22000 + PMT	6864.62	Annual net cash flow
10 n	10.00	Total term of investment
50000 CHS PV	-50000.00	Total equity investment
i	13.16	Percent annual yield

The following table summarizes the alternatives:

Alternative No.	Equity Investment	Annual Yield
1	$50000.00	12.84%
2	$60000.00	12.83%
3	$50000.00	13.19%
4	$50000.00	13.16%

The alternatives in order of attractiveness are:

1. 60%, 20-year loan at 9%, plus 15%, 10-year second mortgage at 10.5% (No. 3).
2. 75%, 30-year loan at 9.5% (No. 4).
3. 75%, 20-year loan at 9.5% (No. 1).
4. 70%, 20-year loan at 9.25% (No. 2).

CASH EQUIVALENT SALES PRICE

When a property is purchased with the assumption of an existing mortgage, or a purchase-money mortgage is taken back by the seller, and the mortgage interest rate is below the going market rate, the cash equivalent sales price is less than the nominal sales price. This adjustment process is useful in adjusting comparable sales data in Direct Sales Comparison Analysis.

The procedure involves calculating the monthly (periodic) payment at the below-market interest rate, and then capitalizing it at the market rate. This gives the present worth of the mortgage, which is added to the amount of equity payment to get the Cash Equivalent Sales Price.

Example 1:

A property was recently purchased for a nominal price of $60,000. The purchaser assumed the existing mortgage, which had a balance of $38,744. It had a remaining term of 20 years, with level monthly payments at 7% interest. The seller took back a $11,256 second purchase-money mortgage at 8% interest, with a maturity of 20 years (level monthly payments).

The going market interest rate is 8.75% for properties of this type. What is the cash equivalent price of this transaction?

Keystrokes:	Outputs:	
BEGIN/NOTE ▬ END/BOND		
CL FIN		
20 [f] [12×] 7 [f] [12÷] 38744 [PV]		
[PMT] [STO] [0] →	-300.38	Monthly payment on 1st mortgage
8 [f] [12÷] 11256 [CHS] [PV]		
[PMT] →	94.15	Monthly payment on 2nd mortgage
[RCL] [0] [−] [PMT] →	394.53	Total monthly payment
8.75 [f] [12÷] [PV] →	-44644.87	Present value of mortgage payments
60000 [ENTER↑] 38744 [−]		
11256 [−] →	10000.00	Equity investment
[−] →	-54644.87	Cash equivalent sales price

Example 2:

The trustees of a hospital are trying to negotiate the purchase of an adjoining property. They recently had it appraised at $385,000. They have been promised a mortgage by a local bank for $335,000 at 8.5% interest, fully amortized in level monthly payments over 25 years.

The owner of the property is insisting on a purchase price of $450,000. However, he is willing to take back a $400,000 purchase-money mortgage with a 25-year term at 6% interest. The trustees are holding out for $385,000.

What advice would you give the trustees as to the appropriate course of action?

Keystrokes:	Outputs:	
BEGIN/NOTE ▬ END/BOND		
CL FIN		
25 [f] [12×] 6 [f] [12÷]		
400000 [PV] [PMT] →	-2577.21	Monthly purchase-money mortgage payment
8.5 [f] [12÷] [PV] →	320059.48	Present value of mortgage (calculated)
50000 [+] →	370059.48	Cash equivalent sales price

Pay the nominal $450,000. The cash equivalent price is less than the appraised value.

REFINANCING

It can be mutually advantageous to both borrower and lender to refinance an existing mortgage which has an interest rate substantially below the current market rate, with a loan at a below-market rate. The borrower has the immediate use of tax-free cash, while the lender has substantially increased debt service on a relatively small cash outlay.

To find the benefits to both borrower and lender:

1) Calculate the monthly payment on the existing mortgage.
2) Calculate the monthly payment on the new mortgage.
3) Calculate the net monthly payment received by the lender (and paid by the borrower) by adding the figure found in Step 1 to the figure found in Step 2.
4) Calculate the net present value (NPV) to the lender of the net cash advanced.
5) Calculate the yield to the lender as an IRR.
6) Calculate the NPV to the borrower of the net cash advanced.

Example 1:

An investment property has an existing mortgage which originated 8 years ago with an original term of 25 years, fully amortized in level monthly payments at 6.5% interest. The current balance is $133,190.

Although the going current market interest rate is 10.5%, the lender has agreed to refinance the property with a $200,000, 17-year, level-monthly-payment loan at 9% interest.

What are the NPV and effective yield to the lender on the net amount of cash actually advanced?

What is the NPV to the borrower on this amount if he can earn a 14% equity yield rate on the net proceeds of the loan?

Keystrokes:	**Outputs:**	
BEGIN/NOTE ▬ END/BOND		
CL FIN		
17 f 12× 6.5 f 12÷		
133190 PV PMT STO 0 →	–1080.33	Monthly payment on existing mortgage
9 f 12÷ 200000 CHS PV		
PMT →	1917.61	Monthly payment on new mortgage

RCL 0 + PMT →	837.28	Net monthly payment
RCL PV 133190 + STO 0 →	-66810.00	Net amount of cash advanced
10.5 f 12÷ PV →	-79507.22	Present value of net monthly payment
RCL 0 − →	-12697.22	NPV to lender
RCL 0 PV i 12 × →	13.51	% nominal yield (IRR) to lender on new money
14 f 12÷ PV →	-65032.32	Present value of net monthly payment at 14%
RCL 0 − →	1777.68	NPV to borrower

WRAP-AROUND MORTGAGES

A wrap-around mortgage is essentially the same as a refinancing mortgage, except that the new mortgage is granted by a different lender, who assumes the payments on the existing mortgage, which remains in full force. The new (second) mortgage is thus "wrapped around" the existing mortgage. The "wrap-around" lender advances the net difference between the new (second) mortgage and the existing mortgage in cash to the borrower, and receives as net cash flow the difference between debt service on the new (second) mortgage and debt service on the existing mortgage.

When the terms of the original mortgage and the wrap-around are the same, the procedures in calculating NPV and IRR to the lender and NPV to the borrower are exactly the same as those presented in the preceding section on Refinancing.

Example 1:

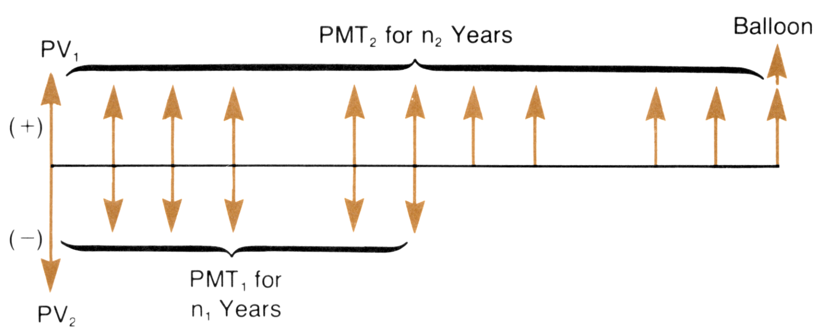
A mortgage loan on an income property has a remaining balance of $200,132.06. When the loan originated 8 years ago, it had a 20-year term with full amortization in level monthly payments at 6.75% interest.

A lender has agreed to "wrap" a $300,000 second mortgage at 9.5%, with full amortization in level monthly payments over 12 years. What is the effective yield (IRR) to the lender on net cash advanced?

Keystrokes:	Outputs:	
BEGIN/NOTE ■ END/BOND		
CL FIN		
20 ENTER↑ 8 − f 12× →	144.00	Total number of months remaining in original loan (into n)

6.75 f 12÷	0.56	Monthly interest rate (into i)
200136.06 PV	200136.06	Loan amount (into PV)
PMT g RND STO 0	-2031.59	Monthly payment on existing mortgage (calculated)
9.5 f 12÷	0.79	Monthly interest on wrap-around (into i)
300000 CHS PV	-300000.00	Amount of wrap-around (into PV)
PMT g RND	3499.12	Monthly payment on wrap-around (calculated)
RCL 0 + PMT	1467.53	Net monthly payment received (into PMT)
RCL PV 200132.06 + STO 0 PV	-99867.94	Net cash advanced (into PV)
i RCL f 12÷	14.51	Nominal yield (IRR) to lender (calculated)

Sometimes, the wrap-around mortgage will have a longer pay-back period than the original mortgage, or a balloon payment may exist. From the *lender's* point of view, the cash flows may be represented as follows:



where:

n_1 = number of years remaining in original mortgage
PMT_1 = yearly payment of original mortgage
PV_1 = remaining balance of original mortgage
n_2 = number of years in wrap-around mortgage
PMT_2 = yearly payment of wrap-around mortgage
PV_2 = total amount of wrap-around mortgage
BAL = balloon payment

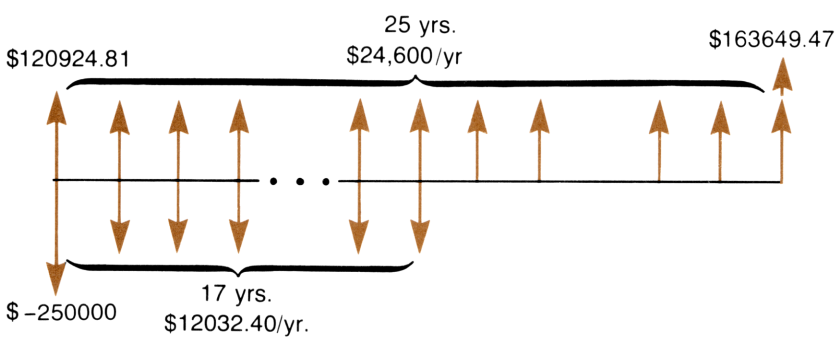
Note:

While the loan payments are assumed to occur monthly, the net cash flows are considered to be on an annual basis. An annual yield is calculated to best utilize the features of the HP-92. This calculated yield is *not* the APR or the interest rate which would amortize the loan.

If your wrap-around mortgage problem does not lend itself to this type of yearly evaluation (i.e., 3 months of one payment amount, and 9 months of a different payment), or you wish to determine the APR (nominal yield), refer to Example 4.

Example 2:

A customer has an existing mortgage with a balance of $120,924.81, a remaining term of 17 years, and a $1002.70 monthly payment. He wishes to obtain a $250,000, 9½% wrap-around with 25 years of monthly payments of $2050.00 and a balloon payment at the end of the 300th month of $163,649.47. If you, as a lender, accept the proposal, what is your ''annualized'' rate of return?



Keystrokes:	**Outputs:**
BEGIN/NOTE ▬ END/BOND	
f CLEAR	
12032.40 CHS ENTER↑ 24600 +	
STO 1 STO 2 STO 3 STO 4	
STO 5 STO 6 STO 7 STO 8	
STO 9 STO . 0 STO . 1	
STO . 2 STO . 3 STO . 4	
STO . 5 STO . 6 STO . 7 →	12567.60 Net yearly cash flow received for 17 years
f LAST X STO . 8	
STO . 9 STO Σ+ 0	
STO Σ+ 1 STO Σ+ 2	

STO Σ+ 3 STO Σ+ 4 →	24600.00	Net yearly cash flow received for 7 years
163649.47 + STO Σ+ 5 →	188249.47	Final net cash flow received
25 n →	25.00	Total term being considered
250000 CHS ENTER↑		
120924.81 + STO 0 →	-129075.19	Net amount advanced by lender
f IRR →	10.95	"Annualized" rate of return to lender

The nominal yield (IRR) to the lender is actually 10.8653% (calculated with an HP-67/97 Business Decisions Pac).

A situation may arise where the wrap-around lender advances additional funds to the borrower, but the term of the wrap-around is shorter than the remaining term of the first mortgage. During the period of the wrap-around loan, the borrower's payments will increase to reflect the amount of the new money advanced. When the wrap-around mortgage is paid off, the borrower's monthly payment will revert back to the payment amount of the original loan for the remaining life of that loan.

The wrap-around loan payment must amortize the wrap-around loan in the specified time and leave the first mortgage intact, with the proper remaining balance on the first mortgage as though the wrap-around loan had never existed.

Example 3:

A mortgage on a development project was originally made for $1,000,000 for 25 years at an interest rate of 7¾%. The loan is currently 10 years old and the borrower is seeking $250,000 in additional funds. The wrap-around mortgage contract interest rate will be 9¼%, with monthly payments for 10 years. As an investor, what would be your rate of return if you accepted the proposal?

Keystrokes:	**Outputs:**	
BEGIN/NOTE ▮ END/BOND		
CL FIN		
25 f 12× →	300.00	Term of original mortgage (into n)
7.75 f 12÷ →	0.65	Monthly interest of original mortgage (into i)
1000000 PV →	1000000.00	Original loan amount (into PV)

PMT →	-7553.29	Original monthly payment (calculated)
g RND PMT STO 0 →	-7553.29	Enter the rounded monthly payment actually made (into PMT)
10 f 12× FV STO 1 →	-802451.73	Remaining balance of original loan after 10 years (calculated)
20 f 12× FV →	-374722.20	Remaining balance of original loan after 20 years (calculated)
g RND CHS FV →	374722.20	Enter the rounded final value (into FV)
10 f 12× →	120.00	Term of wrap-around (into n)
9.25 f 12÷ →	0.77	Monthly interest rate on wrap-around (into i)
RCL 1 250000 STO 1 - PV →	-1052451.73	Face amount of wrap-around mortgage (into PV)
PMT →	11565.64	Monthly payment received (calculated)
RCL 0 + PMT →	4012.35	Net monthly payment received
RCL 1 CHS PV →	-250000.00	New money (into PV)
0 FV →	0.00	There is no additional balloon payment (into FV)
i RCL f 12÷ →	14.86	Percent annual yield (calculated)

You may also have the situation where the remaining term of the original mortgage is not a whole year. To find the annual yield to the lender, a trial and error (iterative) approach must be used. An initial guess is entered for the periodic IRR, and the present value of the cash flows is found. By subtracting the initial cash output, the net present value is found. If the NPV is equal to 0, the initial guess *is* the IRR. Otherwise, adjust the "guess" and repeat the procedure until the desired accuracy is acheived. The keystrokes are:

1) Set the Payment Mode switch to END and press CL FIN.
2) Key in a "best guess" periodic internal rate of return; press i.
3) To find the present value of the original mortgage:
    a) Key in the total number of periods remaining; press n.

b) Key in the periodic payment amount; press CHS PMT. (The lender makes the payments on the original mortgage.)
c) Press PV to find the present value, then press STO 0.

4) Find the present value of the wrap-around mortgage:
a) Key in the total number of periods in the wrap-around; press n.
b) Key in the periodic payment *received* by the lender; press PMT.
c) Key in the balloon amount (if it exists) at the end of the payback period; press FV.
d) Press PV to find the present value.

5) Press RCL 0 + CHS to obtain the present value of the cash flows.

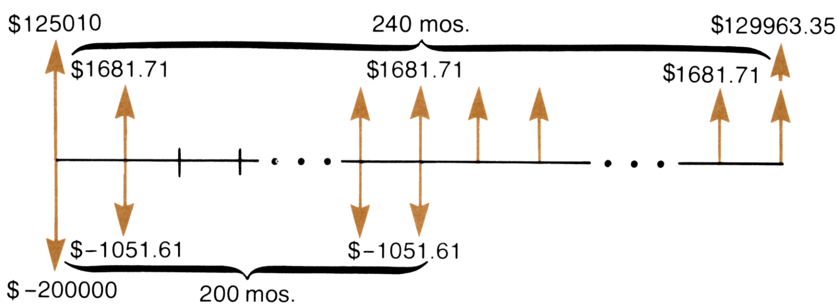
6) Key in the remaining balance on the original mortgage; press +.

7) Key in the total amount of the wrap-around mortgage; press − to obtain the NPV.

8) If the net present value is not equal to zero, adjust the guess and repeat steps 1-7 until the desired accuracy is obtained.

Example 4:

A customer has an existing mortgage with a balance of $125,010, a remaining term of 200 months, and a $1051.61 monthly payment. He wishes to obtain a $200,000, 9½% wrap-around with 240 monthly payemnts of $1681.71 and a balloon payment at the end of the 240th month of $129963.35. If you, as a lender, accept the proposal, what is your rate of return?



Keystrokes:	**Outputs:**
BEGIN/NOTE ■ END/BOND	
CL FIN	

Choose a 12% annual yield as the first IRR guess.

12 f 12÷ ⟶	1.00 % monthly IRR (into i)
200 n ⟶	200.00 Months (into n)
1051.61 CHS PMT ⟶	-1051.61 Payment (into PMT)

PV STO 0 →	90786.92	Present value of original mortgage at 12% annual IRR (calculated)
240 n →	240.00	Months (into n)
1681.71 PMT →	1681.71	Payment (into PMT)
129963.35 FV PV →	−164663.31	Present value of wrap-around at 12% annual IRR (calculated)
RCL 0 + CHS →	73876.39	Present value of the cash flows
125010 + 200000 − →	−1113.61	Net present value

Since the NPV is negative, the IRR is too high. Choose a lower rate of return, say 11.75%, and repeat the procedure. The actual IRR is 11.8391%.

If you, as a lender, know the yield on the entire transaction, and you wish to obtain the payment amount on the wrap-around mortgage to achieve this yield, use the following procedure. Once the monthly payment is known, the borrower's periodic interest rate may also be determined.

1) Set the Payment Mode switch to END and press CL FIN.
2) Key in the remaining term of the original mortgage and press n.
3) Key in the desired annual yield and press f 12÷.
4) Key in the monthly payment to be made by the lender on the original mortgage and press CHS PMT.
5) Press PV.
6) Key in the *net* amount of cash advanced and press + CHS PV.
7) Key in the total term of the wrap-around mortgage and press n.
8) If a *balloon payment exists,* key it in and press FV.
9) Press PMT to obtain the payment amount necessary to achieve the desired yield.
10) Press CHS PMT.
11) If a balloon payment exists, press RCL FV CHS FV.
12) Key in the amount of the wrap-around mortgage and press PV i to obtain the borrower's periodic interest rate.

Example 5:

Your firm has determined that the yield on a wrap-around mortgage should be 12% annually. In the previous example, what monthly payment must be received to achieve this yield on a $200,000 wrap-around? What interest rate is the borrower paying?

Keystrokes:	Outputs:
BEGIN/NOTE — END/BOND (switch)	
CL FIN	
200 n 12 f 12÷	
1051.61 CHS PMT PV	
74990 + CHS PV	
240 n 129963.35 FV	
PMT →	1693.97 Monthly payment received by lender
CHS PMT RCL FV CHS FV	
200000 PV i RCL f 12÷ →	9.58 Annual interest paid by borrower

MODIFIED IRR—VARYING REINVESTMENT RATE (FINANCIAL MANAGEMENT RATE OF RETURN)

The traditional IRR technique assumes that all positive cash flows are reinvested at the IRR to earn compound interest over the income projection period. It also assumes that all negative cash flows are to be discounted at the IRR. This means that cash can be invested today to earn compound interest at the IRR until it is needed to cover the forecasted negative cash flows.

Neither of these assumptions is necessarily realistic or valid. It is possible to compensate for either or both by using real-market rates to discount all negative flows (including Capital Outlay) to the present at a "safe" rate that will ensure liquidity when funds are needed; and to compound all positive flows at a realistic reinvestment rate to the end of the income projection period.

This procedure results in a single (negative) present value figure, and a single future value figure as well. IRR is then found by solving for i in a compounded amount procedure.

Example 1:

Negative Cash Flows, Reinvestment of Positive Flows at IRR.

A development project requires a total capital investment (development costs) of $600,000 staged as follows: $150,000 immediately, plus $150,000 at the end of years 1-3. Net sales proceeds over a total 10-year sellout period are projected as: Year 1—$0; Year 2—$50,000; Years 3-5—$125,000; Year 6—$140,000; Year 7—$150,000; Year 8—$175,000; Year 9—$100,000; Year 10—$50,000.

What is the indicated IRR for the developer, assuming he can earn 5.5% on the money required to cover future cash outlays (negative cash flows)?

The net cash flows projected are:

Year	Cash Flow	Year	Cash Flow
0	$-150,000	6	$140,000
1	-150,000	7	150,000
2	-100,000	8	175,000
3	- 25,000	9	100,000
4	125,000	10	50,000
5	125,000		

The steps in the procedure are:

1) Calculate the present value of the negative cash flows at the "safe" rate using the NPV routine.

2) Use the figure obtained in Step 1 as the initial investment in the IRR routine; store it in Register 0.

3) Entering 0 as the cash flow for years with a negative cash flow, find the IRR.

Keystrokes:	Outputs:
f CLEAR	
5.5 i 3 n 150000 CHS STO 0	
STO 1	
100000 CHS STO 2	
25000 CHS STO 3	
f NPV STO 0 →	-403315.68 Present value of negative cash flows at 5.5%
10 n	
0 STO 1 STO 2 STO 3	
125000 STO 4 STO 5	
140000 STO 6 150000 STO 7	
175000 STO 8 100000 STO 9	
50000 STO . 0	
f IRR →	12.38 % annual rate of return

Example 2:

Using the cash flow figures in Example 1, what is the IRR if the "safe" rate for negative cash flows is 5.5% and the reinvestment rate for positive cash flows is 10%?

Here the keystroke procedure is slightly different. The steps are:

1) Input the positive cash flows in the appropriate storage registers and calculate the future value of the cash flows at the reinvestment rate.
2) Calculate the present value of the negative cash flows at the "safe" rate.
3) Knowing n, PV, and FV, solve for i.

Keystrokes:	Outputs:
f CLEAR	
125000 STO 4 STO 5	
140000 STO 6	
150000 STO 7	
175000 STO 8	
100000 STO 9	
50000 STO • 0 →	50000.00 Cash flow in 10th year
10 i →	10.00 Reinvestment rate
10 n →	10.00 Total number of cash flows
f NPV →	462317.63 Present value of cash flows
CHS PV FV →	1199132.88 Future value of positive cash flows at 10%
f CL REG	
5.5 i →	5.50 Safe rate for negative cash flows
150000 CHS STO 0 STO 1	
100000 CHS STO 2	
25000 CHS STO 3	
f NPV PV →	-403315.68 Present value of negative cash flows at 5.5%
i →	11.51 % annual rate of return

RENT OR BUY

Frequently a tenant considers a real estate purchase as an alternative course of action. With the HP-92, it is possible to compare the financial or investment implications of purchase vs. rental. While the final decision may rest on personal or even emotional considerations, these can be weighed against the financial realities.

Note:

For illustrative purposes, only first-year financial requirements are shown.

Example 1:

A family is renting an apartment at $285 per month will all utilities paid. Tired of paying rent with no financial results, they decide to buy a comparable house.

They have found a house priced at $39,000, on which they can obtain a 90% mortgage loan at 9% interest with a 30-year maturity (level monthly payments).

Closing costs are estimated at $650. Property taxes are $1050; utilities charges (sewer and water) are $200 per year; insurance and maintenance is estimated at $850, and heating expenses at $1450.

The property is expected to follow the current trend of appreciation of 12% per year.

The family's tax bracket is 38%, applicable to tax-deductible interest and property tax payments. The savings from which the down payment and closing charges would be taken are earning 7% interest.

From a financial point-of-view (using first year figures), is it preferable to buy or to continue to rent?

Keystrokes:	Outputs:	
BEGIN/NOTE ▬ END/BOND		
CL FIN		
285 ENTER↑ 12 × STO 0 →	3420.00	Annual rent
39000 ENTER↑ 10 %		
650 + 7 % →	318.50	Interest on down payment and closing cost funds
RCL 0 x⇄y − STO 0 →	3101.50	Net first-year cost of renting
39000 ENTER↑ 90 % PV →	35100.00	Mortgage principal
30 f 12× 9 f 12÷ PMT →	−282.42	Monthly mortgage payment
CHS g RND 12 × STO 1 →	3389.04	Annual debt service
1 f P1 12 g P2		
AMORT x⇄y →	3149.28	First-year interest

STO 2 1050 + →	4199.28	Tax deductible items
38 % STO 3 →	1595.73	Tax saving
39000 ENTER↑ 12 % STO 4 →	4680.00	Appreciation
RCL 1 RCL 2 − STO 5 →	239.76	First-year equity build-up
RCL 1 1050 + 200 + 850 +		
1450 + RCL 5 − RCL 3 − RCL		
4 − →	423.55	First-year net cost of buying
RCL 0 →	3101.50	Net first-year cost of renting

Conclusion: Financially, it is preferable to purchase the house.

EQUITY INVESTMENT ANALYSIS

Equity Investment Analysis is a method of evaluating income producing real estate investment alternatives on a pretax basis. Two key factors in this type of analysis are the anticipated income stream that the property will provide and the property's projected resale value at the end of the investment horizon. Based on this and the current price of the property, an equity yield rate can be found giving an indication of the profitability of the investment.

One of the basic equations used in real estate equity analysis relates the income stream, sales price, projected appreciation or depreciation, and amount of mortgage as follows:

$$R = Y - MC \left\{ \begin{matrix} -\text{Apprec.} \\ \text{or} \\ +\text{Deprec.} \end{matrix} \right\} \frac{1}{S_{\overline{n|}}} = \frac{\text{NOI}}{\text{Price}}$$

Where

R = overall capitalization rate

Y = equity yield rate

M = mortgage to value ratio

C = mortgage coefficient (imbedded in calculation)

$1/S_{\overline{n|}}$ = annual sinking fund factor for depreciation or appreciation

NOI = net operating income

A brief explanation of terms frequently used in real estate analysis is given here in order to aid in understanding the problems and results more fully.*

Annual Net Cash Flow is the annual net operating income without depreciation minus the annual debt service (i.e., annual mortgage payments).

Reversion is the future sales price minus the mortgage balance at the end of the projection period.

Equity yield rate is that annual rate at which the present value of the net annual cash flows plus the present value of the equity reversion equals the equity investment value.

Equity investment value is the equity in the property at the beginning of the projection period.

Overall Capitalization Rate is the net operating income divided by the selling price.

Equity Yield Rate

Given the projection period in years, reversion amount, annual net cash flow, and equity investment value, the equity yield rate may be calculated as follows:

1) Set the Payment Mode switch to END and press **CL FIN**.
2) Key in the reversion; press **FV**.
3) Key in the number of years projection; press **n**.
4) Key in the net annual cash flow; press **PMT**.
5) Key in the equity investment value; press **CHS** **PV**.
6) Press **i** to obtain the equity yield rate.

Example 1:

An apartment complex is listed for $1,960,500 and has an annual net operating income of $166,315.37. The prospective buyer is considering a down payment of $572,500 and will finance the remaining $1,388,000 for 29 years at 8%. If the property appreciates a total of 20% over the next 10 years, what would the equity yield rate be?

Using calculations described in the Owner's Handbook, it is found that the monthly mortgage payments are $10,270.45 and therefore the annual net cash flow is $43,069.97

(NOI − debt service = net cash flow).

* For further information, refer to ELLWOOD TABLES, American Institute for Real Estate Appraisers, 1970.

The remaining mortgage balance at the end of 10 years will be $1,201,922.59.

To calculate the reversion at the end of the tenth year, find the future sales price and subtract the remaining balance.

Keystrokes:	**Outputs:**
BEGIN/NOTE ▬ END/BOND	
CL FIN	
1960500 ENTER↑ 20 %	
+ →	2352600.00 Future sales price
1201922.59 − FV →	1150677.41 Reversion
10 n 43069.97 PMT	
572500 CHS PV i →	13.00 Equity yield rate

Equity Investment Value and Present Value

Given the desired equity yield rate, projection period, annual net cash flow, and the reversion, this procedure solves for the equity investment value and present value of the investment (current sales price). Information is entered as follows:

1) Set the Payment Mode switch to END and press CL FIN.
2) Key in the reversion, press FV.
3) Key in the projection period in years; press n.
4) Key in the equity yield rate; press i.
5) Key in the annual net cash flow; press PMT.
6) Press PV for the equity investment value.
7) Key in the mortgage amount, press − to obtain the current sales price or present value.

Example 1:

An investor has some money he wants to invest in real estate. One of his alternatives is a warehouse, currently leased for 10 years, which generates $26,460 annually before debt service (NOI). Because the warehouse is located in a growth area, he estimates the property should sell for $420,000 at the end of 10 years. He can obtain an 8½%, 20 year mortgage for $240,000 which would have monthly payments of $2,082.78. If his desired yield is 11% over 10 years, what would his equity investment value be and how much could he pay for the property (what is the current sales price)?

Keystrokes:	**Outputs:**	
BEGIN/NOTE ▬ END/BOND		
CL FIN		
10 f 12× 8.5 f 12÷		
2082.78 CHS PMT 240000 PV		
FV →	-167984.38	Remaining loan balance after 10 years
420000 + →	252015.62	Reversion value
FV 10 n 11 i		
RCL PMT 12 ×		
26460 + PMT PV →	-97393.37	Equity investment value
240000 − →	-337393.37	Current sales price

Future Sales Price and Overall Depreciation/Appreciation Rate

This calculation solves for the sales price at the end of the projection period given the desired equity yield rate, annual net cash flow, equity investment value, projection period, and the mortgage balance at the end of the projection period. Information is entered as follows:

1) Set the Payment Mode switch to END and press CL FIN.
2) Key in the projection period in years; press n.
3) Key in the equity yield rate; press i.
4) Key in the annual net cash flow; press PMT.
5) Key in the equity investment value; press CHS PV.
6) Press FV to compute the reversion amount.
7) Key in the mortgage balance at the end of the projection period and press + to obtain the required future sales price.
8) Key in the purchase price, press x≷y f Δ% to obtain the overall appreciation (if the answer is positive) or depreciation (if the answer is negative).

Example 1:

A shopping center has an annual net cash flow of $14211.24. The desired equity yield rate is 14% over a 9 year period. If the current asking price is $616,000 what must the sales price at the end of year 9 be in order to achieve the desired 14% return? What overall appreciation does this represent?

(Assume 25% equity ($154,000), 25 year mortgage at 8%, monthly payment of $3,565.79, with a remaining balance of $385,522.31 at the end of year 9).

Keystrokes:		Outputs:
BEGIN/NOTE ▬ END/BOND		
CL FIN		
9 n 14 i 14211.24 PMT		
154000 CHS PV FV	→	272207.35 Reversion
385522.31 +	→	657729.66 Future sales price
616000 x⇄y f Δ%	→	6.77 Overall appreciation

CANADIAN MORTGAGES

In Canada, interest is compounded semi-annually with payments made monthly. This results in a different monthly mortgage factor than is used in the United States, and is programmed into the HP-92. This difference can be handled easily on the HP-92 by the addition of a few keystrokes. For any problem requiring an input for i, the Canadian mortgage factor is calculated first and then this value is entered for i in the calculation to give the answer for Canada.

The keystrokes to calculate the Canadian mortgage factor are:

1) Press CL FIN.
2) Key in 6, press n.
3) Key in 200; press PV.
4) Key in the annual interest rate as a percent; press + CHS FV.
5) Press i.

The Canadian mortgage factor is now stored in i for future use. The examples below show how this factor is used for i in Canadian mortgage problems.

Example 1—Periodic Payment Amount

What is the monthly payment required to fully amortize a 30-year, $30,000 Canadian mortgage if the interest rate is 9%?

Keystrokes:	Outputs:
BEGIN/NOTE — END/BOND (switch to END)	
CL FIN	
6 n 200 PV	
9 + CHS FV i →	0.74 Canadian mortgage factor
30 f 12× →	360.00 Total monthly periods in mortgage life (into n)
30000 PV 0 FV PMT →	–237.85 Monthly payment (calculated)

Example 2—Number of Periodic Payments to Fully Amortize a Mortgage

An investor can afford to pay $440 per month on a $56,000 Canadian mortgage. If the annual interest rate is 9¼%, how long will it take to completely amortize this mortgage?

Keystrokes:	Outputs:
BEGIN/NOTE — END/BOND (switch to END)	
CL FIN	
6 n 200 PV	
9.25 + CHS FV i →	0.76 Canadian monthly mortgage factor
440 CHS PMT →	–440.00 Monthly payment (into PMT)
56000 PV 0 FV n →	436.34 Total number of monthly payments
12 ÷ →	36.36 Total years

Example 3—Effective Interest Rate (Yield)

A Canadian mortgage has monthly payments of $612.77 with a maturity of 25 years. The principal amount is $75,500. What is the annual interest rate?

Keystrokes:	Outputs:
BEGIN/NOTE — END/BOND (switch to END)	
CL FIN	
25 f 12× 612.77 CHS PMT	
75500 PV i →	0.72 Canadian monthly mortgage factor
6 n 0 PMT 200 CHS PV	
FV + →	8.75 Annual interest rate

Example 4—Balance Remaining at End of Specified Period

A Canadian mortgage has monthly payments of $612.77 at 8.75% interest. The principal amount is $75,500. What will be the outstanding balance remaining at the end of 10 years?

Keystrokes:	**Outputs:**
BEGIN/NOTE ▬ END/BOND	
CL FIN	
6 n 200 PV	
8.75 ÷ CHS FV i ——→	0.72 Canadian monthly mortgage factor
612.77 CHS PMT 10 f 12×	
75500 PV FV ——→	–61877.18 Remaining balance outstanding at the end of 10 years

SAVINGS AND LEASING

INTEREST RATE CONVERSIONS

An annual effective rate demonstrates the effect of compounding for a full year of compounding periods at a particular periodic interest rate. The periodic interest rate to be used is determined by dividing the number of compounding periods in a year into the stated nominal interest rate. The effect is such that if the nominal rate is held constant, as the number of compounding periods per year is increased, the annual effective interest rate will increase. The ultimate or upper limit in this process is to have an infinite number of compounding periods in a year, commonly called continuous compounding.

Nominal Rate Converted to Effective Rate

Given a nominal interest rate and the number of compounding periods per year, this keystroke procedure computes the effective annual interest rate.

1) Press CL FIN.
2) Key in the nominal rate; press ENTER↑.
3) Key in the number of compounding periods per year; press n ÷ i.
4) As given key in 100; press CHS PV.
5) Press FV + to obtain the effective annual interest rate.

Example 1:

What is the effective annual rate of interest if the annual nominal rate of 5¼% is compounded quarterly?

Keystrokes:	**Outputs:**
CL FIN	
5.25 ENTER↑ 4 n ÷ i ⟶	1.31 % quarterly interest rate
100 CHS PV FV + ⟶	5.35 % effective interest rate

Effective Rate Converted to Nominal Rate

Given an effective interest rate and the number of compounding periods per year, this routine calculates the nominal interest rate.

1) Press CL FIN.
2) Key in the number of periods per year; press n.
3) Key in 100; press PV.
4) Key in the effective annual rate; press + CHS FV i.
5) Press × to obtain the nominal rate.

Example 1:

Find the nominal rate if the effective annual rate is 5.35% compounded quarterly.

Keystrokes:	Outputs:
CL FIN	
4 n 100 PV	
5.35 + CHS FV i	
× ——————→	5.25 % nominal interest rate

Nominal Rate Converted to Continuous Effective Rate

This procedure converts a nominal annual interest rate to the continuous effective rate.

1) Key in 1; press ENTER↑.
2) Key in the nominal rate; press %.
3) Press f e^x f Δ% to obtain the continuous effective rate.

Example 1:

What is the effective rate resulting from a 5¼% passbook rate with continuous compounding?

Keystrokes:	Outputs:
1 ENTER↑ 5.25 %	
f e^x f Δ% ——————→	5.39 % continuous rate

SAVINGS PLANS

This section gives keystroke procedures to evaluate frequently encountered savings problems. Also included is a generalized routine to evaluate a savings plan when deposits and withdrawals are made at irregular intervals.

Initial Deposit With Periodic Deposits

Given an initial deposit into a savings account, and a series of periodic deposits coincident with the compounding period, the future value (or accumulated amount) may be calculated as follows:

1) Set the Payment Mode switch to END and press CL FIN.
2) Key in the initial investment and press CHS PV.
3) Key in the number of additional periodic deposits and press n.
4) Key in the periodic interest rate and press i.

5) Key in the periodic deposit and press CHS PMT.

6) Press FV to determine the value of the account at the end of the time period.

Example 1:

You have just opened a savings account with a $200 deposit. If you deposit $50 a month, and the account earns 5¼% compounded monthly, how much will you have in 3 years?

Keystrokes:	Outputs:
BEGIN/NOTE ▭ END/BOND	
CL FIN	
200 CHS PV 3 f 12× 5.25 f 12÷	
50 CHS PMT FV ⟶	2178.94 Value of the account

Note:

If the periodic deposits do not coincide with the compounding periods, the account must be evaluated in another manner. First, find the future value of the initial deposit and store it. Then use the procedure for Compounding Periods Different from Payment Periods to calculate the future value of the periodic deposits. Recall the future value of the initial deposit and add to obtain the value of the account.

Number of Periods to Deplete a Savings Account or to Reach a Specified Balance

Given the current value of a savings account, the periodic interest rate, the amount of the periodic withdrawal, and a specified balance, this procedure determines the number of periods to reach that balance (the balance is zero if the account is depleted.)

1) Set the Payment Mode switch to END and press CL FIN.

2) Key in the value of the savings account and press CHS PV.

3) Key in the periodic interest rate and press i.

4) Key in the amount of the periodic withdrawal and press PMT.

5) Key in the amount remaining in the account and press FV. This step may be omitted if the account is depleted (FV = 0).

6) Press n to determine the number of periods to reach the desired balance.

Example 1:

Your savings account presently contains $18,000 and earns 5½% compounded monthly. You wish to withdraw $300 a month until the account is depleted. How long will this take? If you wish to reduce the account to $5,000, how many withdrawals can you make?

Keystrokes:	Outputs:
BEGIN/NOTE ▬ END/BOND	
CL FIN	
18000 CHS PV 5.5 f 12÷	
300 PMT n ⟶	70.32 Months
12 ÷ ⟶	5.86 Years to deplete account
5000 FV n ⟶	52.95 Months
12 ÷ ⟶	4.41 Years to reduce account to $5,000

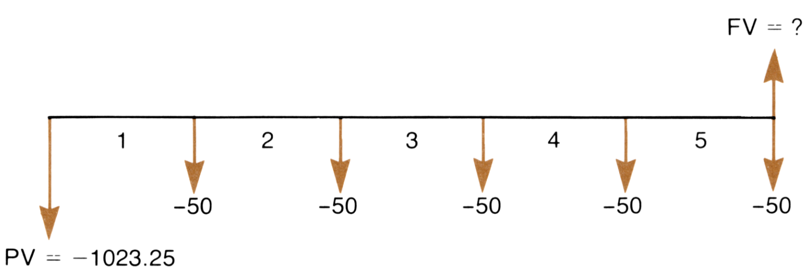
Periodic Deposits and Withdrawals

This section is presented as a guideline for evaluating a savings plan when deposits and withdrawals occur at irregular intervals. One problem is given, and a step-by-step method for setting-up and solving the problem is presented.

Problem:

You are presently depositing $50 at the end of each month into a local savings and loan, earning 5½% compounded monthly. Your current balance is $1023.25. How much will you have accumulated in 5 months?

The cash flow diagram looks like this:



The keystrokes are:

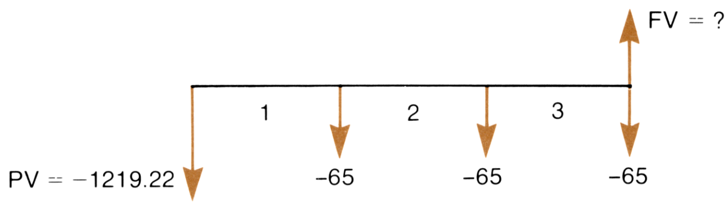
Keystrokes:	Outputs:
BEGIN/NOTE ■ END/BOND	
CL FIN	
50 CHS PMT 5.5 f 12÷ 1023.25	
CHS PV 5 n FV →	1299.22 Amount in account

Now suppose that at the beginning of the 5th month you withdraw $80. What is the new balance?

Keystrokes:	Outputs:
80 − →	1219.22 New balance

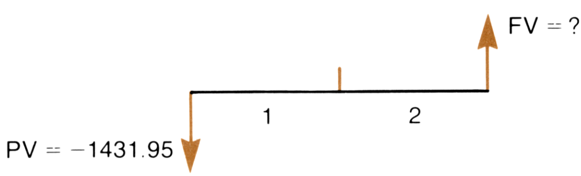
You increase your monthly deposit to $65. How much will you have in 3 months?

The cash flow diagram now looks like this:



Keystrokes:	Outputs:
CHS PV 65 CHS PMT 3	
n FV →	1431.95 Account balance

Suppose that for 2 months you decide not to make a periodic deposit. What is the balance in the account?



Keystrokes:	Outputs:
CHS PV 2 n 0 PMT FV →	1445.11 Account balance

This type of procedure may be continued for any length of time, and may be modified to meet the user's particular needs.

COMPOUNDING PERIODS DIFFERENT FROM PAYMENT PERIODS

In financial calculations involving a series of payments equally spaced in time with periodic compounding, both periods of time are normally equal and coincident. This is an assumption built into the HP-92 pre-programming.

In savings plans however, money may become available for deposit or investment at a frequency different from the compounding frequencies offered. The HP-92 can easily be used in these calculations. However, because of the assumptions mentioned, adjustments must be made to the data so that the two periods can be considered to occur at the same time. When the compounding periods occur more frequently than payment periods, additional keystrokes adjust the compounding period interest rate to an equivalent rate for the payment period. When payments occur more frequently than compounding, the payment amount is adjusted to reflect the fact that payments earn simple interest between compounding periods.

These procedures present solutions for future value, payment amount, and number of payments for both situations of compounding periods differing from payment periods. In addition it should be noted that only annuity due (payments at the *beginning* of payment periods) calculations are shown since this is most common in savings plan calculations.

Compounding Periods More Frequent

When the compounding periods occur more frequently than the payment periods, it is necessary to adjust the compounding period interest rate to an equivalent interest rate for the payment period. To solve a problem requiring an input for **i**, the equivalent payment period interest rate is calculated first and used for **i** in the standard procedure for the problem.

To calculate the equivalent payment period interest rate, information is entered as follows:

1) Set the Payment Mode switch to BEGIN and press **CL FIN**.
2) Key in the number of compounding periods per year and press **STO** **0**.
3) Key in the number of payments (deposits) per year and press **÷** **n**.
4) Key in the annual interest rate; press **RCL** **0** **÷** **i**.
5) Key in 100; press **CHS** **PV** **FV** **+**.
6) Press **CL FIN** **i**.

The interest rate which corresponds to the payment period is now in register "i" and you are ready to proceed.

Example 1: Solving for future value.

Starting today you make monthly deposits of $25 into an account paying 5% compounded daily (365-day basis). At the end of 7 years, how much will you receive from the account?

Keystrokes:	Outputs:	
BEGIN/NOTE ▮ END/BOND		
CL FIN		
365 STO 0 12 ÷ n ⟶	30.42	Compounding/month (into n)
5 RCL 0 ÷ i ⟶	0.01	Interest/month (into i)
100 CHS PV FV		
+ CL FIN i ⟶	0.42	Equivalent periodic interest rate
7 f 12× 25 CHS PMT		
FV ⟶	2519.61	Future value

Example 2: Solving for payment amount.

In 2 years you will need $2500. You have just opened a 5.5% savings account, compounded daily (365-day basis), where you intend to make semi-monthly deposits. What amount must you deposit each period to reach your goal?

Keystrokes:	Outputs:	
BEGIN/NOTE ▮ END/BOND		
CL FIN		
365 STO 0 24 ÷ n ⟶	15.21	Compounding/semi-month (into n)
5.5 RCL 0 ÷ i ⟶	0.02	Interest/semi-monthly (into i)
100 CHS PV FV		
+ CL FIN i ⟶	0.23	Equivalent periodic interest rate
2 ENTER↑ 24 × n		
2500 FV PMT ⟶	–49.22	Amount of semi-monthly deposit

Example 3: Solving for number of payment periods.

You can make weekly deposits of $10 into an account paying 5.25% compounded daily (365-day basis). How long will it take you to accumulate $1000?

Keystrokes:	Outputs:
BEGIN/NOTE ▯■ END/BOND	
CL FIN	
365 STO 0 7 n →	7.00 Compounding/week (into n)
5.25 RCL 0 ÷ i	0.01 Interest/week (into i)
100 CHS PV FV	
+ CL FIN i →	0.10 Equivalent periodic interest rate
10 CHS PMT 1000 FV n →	95.23 Weeks
52 ÷ →	1.83 Years

Payment Periods More Frequent

When the periodic payments occur more frequently than the compounding periods, the payment amount is adjusted by a payment factor. To evaluate a problem requiring an input for PMT, an equivalent payment amount that can be considered to occur with the same frequency as compounding is calculated and used for PMT in the standard procedure for the problem. To solve for the periodic payment, use the standard procedure to find the equivalent payment amount and then divide by the payment factor. The procedure assumes that payments are made at the beginning of the period.

To calculate the payment factor, information is entered as follows:

1) Key in the number of payment periods per year and press ENTER↑.
2) Key in the number of compounding periods per year; press STO 0 ÷ 1 +.
3) Key in the annual interest rate as a percent and press RCL 0 ÷ 2 ÷ % + 1 −.

Example 1: Solving for future value.

For 3 years you make monthly deposits of $75 into an account paying 6% compounded quarterly. How much will you have accumulated at the end of that time?

Keystrokes	**Outputs**	
BEGIN/NOTE ▭ END/BOND		
CL FIN		
12 ENTER↑ 4 STO 0 ÷		
1 + 6 RCL 0 ÷ 2 ÷ % +		
1 − ⟶	3.03	Payment factor
75 CHS × PMT ⟶	−227.25	Equivalent payment amount
3 RCL 0 × n		
6 RCL 0 ÷ i FV ⟶	2963.62	Balance in account

Example 2: Solving for periodic payment.

For 8 years you wish to make weekly deposits in a savings account paying 5.5% compounded quarterly. What amount must you deposit each week to accumulate $6000?

Keystrokes:	**Outputs:**	
BEGIN/NOTE ▭ END/BOND		
CL FIN		
8 ENTER↑ 4 STO 1 × n		
5.5 RCL 1 ÷ i		
6000 FV PMT ⟶	−150.53	Equivalent payment amount.
52 ENTER↑		
4 STO 0 ÷ 1 + 5.5 RCL 0 ÷		
2 ÷ % + 1 − ⟶	13.10	Payment factor
÷ ⟶	−11.49	Periodic payment

Example 3: Solving for number of payment periods.

You can make monthly deposits of $50 into an account paying 5.25% compounded quarterly. How long will it take to accumulate $1000?

Keystrokes:	**Outputs:**
BEGIN/NOTE ▬ END/BOND	
CL FIN	
12 ENTER↑ 4 STO 0 ÷	
1 + 5.25 RCL 0 ÷	
2 ÷ % + 1 − →	3.03 Payment factor
50 CHS × PMT →	-151.31 Equivalent payment amount
1000 FV	
5.25 RCL 0 ÷ i n →	6.38 Quarters
4 ÷ →	1.59 Years
12 × →	19.14 Months

ADVANCE PAYMENTS

Payments on loans are typically made at the end of the period (in arrears). However, there are situations where payments are made in advance (leasing is a good example). Sometimes these agreements call for extra payments to be made when the transaction is closed, before the payments would normally be due.

The first procedure finds the periodic payment amount necessary to achieve a desired yield when a number of payments are made in advance. And, given the periodic payment, the second procedure calculates the periodic yield.

To calculate the payment, information is entered as follows:

1) Set the Payment Mode switch to END and press CL FIN.

2) Key in the total number of payments in the lease and press ENTER↑.

3) Key in the total number of payments made in advance and press STO 0 − n.

4) Key in or calculate the periodic interest rate as a percent and press i.

5) Key in 1; press CHS PMT PV RCL 0 +.

6) Key in the initial loan amount and press x⇄y ÷ to obtain the periodic payment to be received by the lessor.

Example 1:

Equipment worth $750 is leased for 12 months. The equipment is assumed to have no salvage value at the end of the lease. The lessee has agreed to make 3 payments at the time of closing. What monthly payment is necessary to yield the lessor 10% annually?

Keystrokes:	Outputs:
BEGIN NOTE / END BOND (switch set to END)	
CL FIN	
12 ENTER↑ 3 STO 0 − n	
10 f 12÷ 1 CHS PMT PV RCL 0	
+ 750 x⇄y ÷ →	64.45 Monthly payment to be received

Example 2:

In the previous example, what monthly payment is necessary to yield the lessor 10% annually if one payment is due at the time of closing?

Keystrokes:	Outputs:
BEGIN NOTE / END BOND (switch set to END)	
CL FIN	
12 ENTER↑ 1 STO 0 − n	
10 f 12÷ 1 CHS PMT PV	
RCL 0 + 750 x⇄y ÷ →	65.39 Monthly payment to be received

Since this is an annuity due situation (payments at the beginning of the period) the calculation could also be done as follows:

Keystrokes:	Outputs:
BEGIN NOTE / END BOND (switch set to BEGIN)	
CL FIN	
12 n 10 f 12÷ 750 CHS PV	
PMT →	65.39 Monthly payment to be received

To calculate the periodic yield, information is entered as follows:

1) Set the Payment Mode switch to END and press CL FIN.
2) Key in the total number of payments in the lease and press ENTER↑.
3) Key in the total number of payments made in advance and press STO 0 − n.
4) Key in the periodic payment to be received and press PMT.
5) Key in the total amount of the loan and press CHS RCL 0 RCL PMT × + PV.
6) Press i to obtain the periodic yield.

Example 1:

A lease has been written to run for 60 months. The leased equipment has a value of $25,000 with a $600 monthly payment. The lessee has agreed to make 3 payments at the time of closing ($1800). What is the annual yield to the lessor?

Keystrokes:	**Outputs:**
BEGIN/NOTE — END/BOND (switch set to END)	
CL FIN	
60 ENTER↑ 3 STO 0 − n	
600 PMT 25000 CHS RCL 0	
RCL PMT × + PV i ⟶	1.44 Monthly yield
12 × ⟶	17.33 Annual yield as a %

Advance Payments With Residual

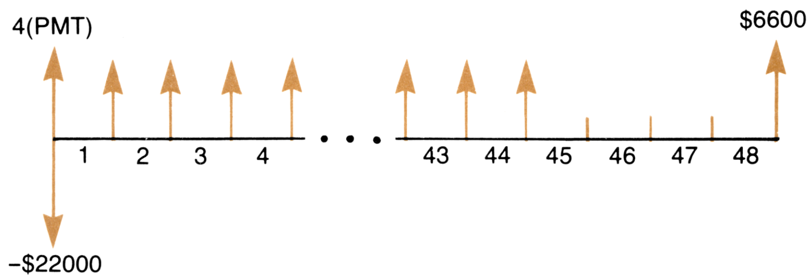
Situations may arise where a transaction has advance payments and a residual value (salvage value) at the end of the normal term.

To calculate the payment amount, information is entered as follows:

1) Set the Payment Mode switch to END and press CL FIN.
2) Key in the total number of payments and press n.
3) Key in or calculate the periodic interest rate and press i.
4) Key in the residual value and press FV PV.
5) Key in the loan amount and press + STO 0 0 FV RCL n.
6) Key in the total number of payments made in advance and press STO 1 − n 1 CHS PMT PV RCL 1 + RCL 0 x⇄y ÷ to obtain the payment amount received by the lessor.

Example 1:

A copier worth $22,000 is to be leased for 48 months. The lessee has agreed to make 4 payments in advance, with a purchase option at the end of 48 months enabling him to buy the copier for 30% of the purchase price. What monthly payment is necessary to yield the lessor 12% annually?



Keystrokes:	Outputs:
BEGIN NOTE ▬ END BOND	
CL FIN	
48 n 12 f 12÷ 22000 STO 2	
30 % FV PV	
RCL 2 + STO 0 0 FV	
RCL n 4 STO 1 − n 1 CHS	
PMT PV RCL 1 + RCL 0	
x⇄y ÷ ——→	453.84 Monthly payment received by lessor

To calculate the periodic yield, an iteration (trial and error) procedure must be used. Information is entered as follows:

1) Set the Payment Mode switch to END and press CL FIN.
2) Key in the total number of payments made in advance and press STO 0.
3) Key in the total number of payments and press STO 1.
4) Key in the periodic payment amount and press STO 2.
5) Key in the loan amount and press CHS STO 3.
6) Key in the residual value and press STO 4.
7) Choose a best-guess periodic interest rate and press i RCL 1 n RCL 4 FV PV STO 9 to obtain the present value of the residual value at the chosen interest rate.
8) Press 0 FV RCL n RCL 0 − n RCL 2 PMT PV STO + 9 to obtain the present value of the periodic cash flows at the chosen interest rate.

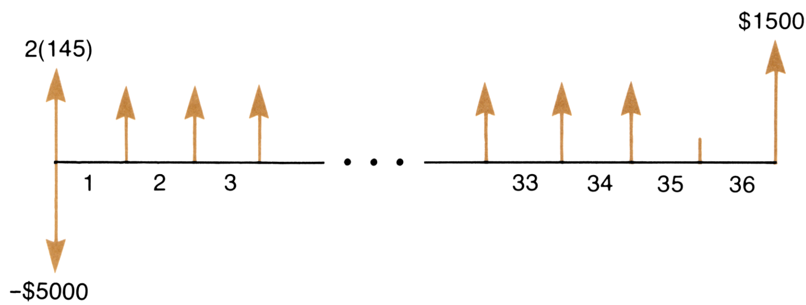
9) Press RCL 3 RCL 0 RCL 2 × + RCL 9 − to obtain the net present value (NPV) of all the cash flows at the chosen interest rate.

10) If the NPV is negative, the yield is lower than the value chosen in step 7. If the NPV is positive, the yield is higher than the value chosen in step 7.

11) Repeat steps 7-10 until the net present value is sufficiently close to zero.

Example 1:

Equipment worth $5000 is leased for 36 months, at $145 per month. The lessee has agreed to pay the first and last month's payments in advance. At the end of the lease, the equipment may be purchased for $1500. What is the annual yield to the lessor?



Keystrokes:	**Outputs:**
BEGIN NOTE ▬ END BOND	
CL FIN	
2 STO 0 →	2.00 Number of advance payments
36 STO 1 →	36.00 Total number of payments
145 STO 2 →	145.00 Monthly payment
5000 CHS STO 3 →	−5000.00 Amount given by lessor
1500 STO 4 →	1500.00 Residual value

Choose 20% as the first guess of the annual yield.

20 f 12÷ RCL 1 n RCL 4	
FV PV STO 9 →	−827.30 Present value of residual
0 FV RCL n RCL 0 − n	
RCL 2 PMT PV STO + 9 →	−3740.39 Present value of periodic payments
RCL 3 RCL 0 RCL 2 × +	
RCL 9 − →	−142.31 Net present value

Since the NPV is negative, the yield is less than 20%; choose 18% as next guess.

CL FIN		
18 f 12÷ RCL 1 n RCL 4		
FV PV STO 9 →	−877.63	Present value of residual
0 FV RCL n RCL 0 − n		
RCL 2 PMT PV STO + 9 →	−3839.85	Present value of periodic payments
RCL 3 RCL 0 RCL 2 × +		
RCL 9 − →	7.49	Net present value

The yield to the lessor is slightly higher than 18%. (It is actually 18.0978%.)

NOTES

NOTES

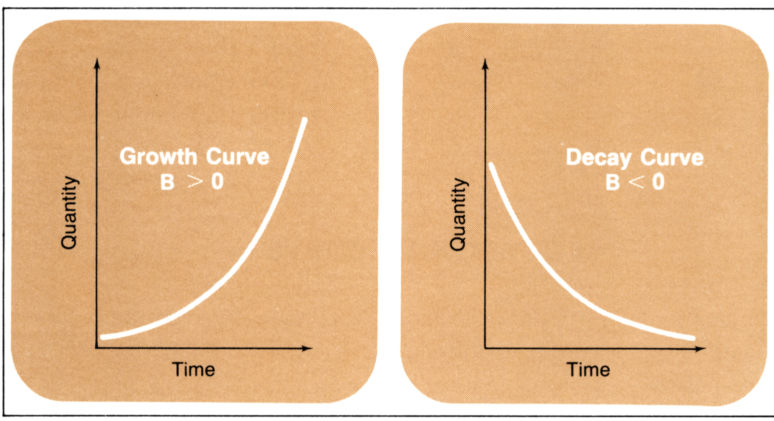
STATISTICS

CURVE FITTING

EXPONENTIAL CURVE FIT

Using the LN function of the HP-92, a least squares exponential curve fit may easily be calculated according to the equation $y = Ae^{Bx}$. The exponential curve fitting technique is often used to determine the growth rate of a variable such as a stock's value over time, when it is suspected that the performance is non-linear. The value for B is the decimal value of the continuous growth rate. For instance, assume after keying in several end-of-month price quotes for a particular stock, it is determined that the value for B is 0.10. This means that over the measured period the stock has experienced a 10% *continuous* growth rate. This decimal continuous growth rate may then be converted to an effective growth rate.

If $B > 0$, you will have a growth curve. If $B < 0$, you will have a decay curve. Examples of these are given below.



The procedure is as follows:

1) Press f CLΣ.
2) For each input pair of values, key in the y-value and press f LN; key in the corresponding x-value and press Σ+.
3) After all data pairs are input, press f L.R. f e^x to obtain A in the equation above.
4) Press x⇄y to obtain B in the equation above.
5) Press f e^x 1 − to obtain the effective growth rate (as a decimal).
6) To obtain the correlation coefficient, press g r.
7) To make a y-estimate, key in the x-value, press ŷ f e^x.

Example 1:

A stock's price history is listed below. What effective growth rate does this represent? If the stock continues this growth rate, what is the price projected to be at the end of 1976 (year 5)?

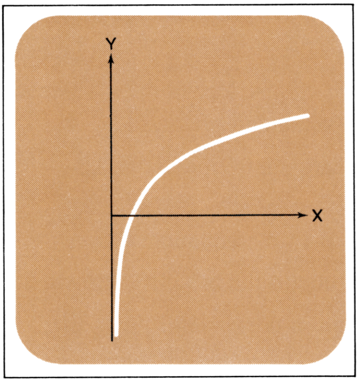
End of Year	Price
1972 (1)	52½
1973 (2)	55¼
1974 (3)	(missing data)
1975 (4)	61
1976 (5)	?

Keystrokes:	Outputs:	
f CLΣ		
52.5 f LN 1 Σ+ →	1.00	First data pair input
55.25 f LN 2 Σ+ →	2.00	Second data pair input
61 f LN 4 Σ+ →	3.00	Third data pair input
f L.R. f e^x →	49.96	y-intercept (A)
x⇄y →	0.05	B
f e^x 1 − →	0.05	Effective growth rate
g r →	1.00	Correlation coefficient
5 ŷ f e^x →	64.14	Projected price at the end of year 5 (1976)

LOGARITHMIC CURVE FIT

If your data does not fit a line or an exponential curve, try logarithmic curve fit. This is calculated according to the equation $y = A + B (\ln x)$, and all x values must be positive.

A typical logarithmic curve is shown below.



The procedure is as follows:

1) Press f CLΣ.
2) Key in the first y value, press ENTER↑. Key in the first x-value, press f LN Σ+. Repeat this step for each data pair.
3) After all data pairs are input, press f L.R. to obtain A in the equation above.
4) Press x⇄y to obtain B.
5) Press g r to obtain the correlation coefficient.
6) To solve for a projected y value, key in the x value, and press f LN ŷ.

Example 1:

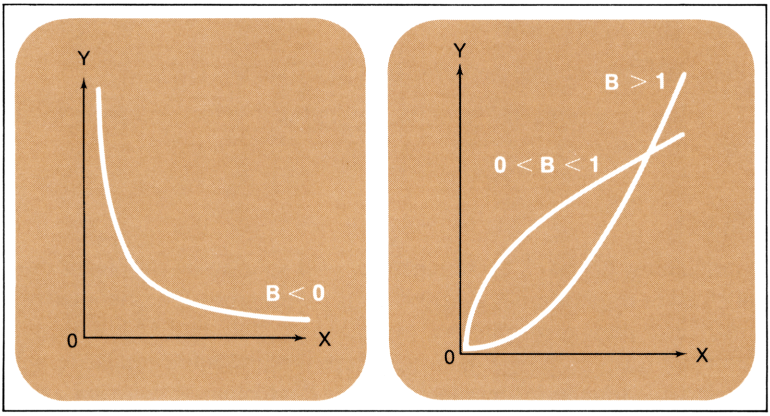
A manufacturer observes declining sales of a soon-to-be obsoleted product, of which there were originally 10,000 units in inventory. The cumulative sales figures over a number of months, given below, may be fit by a logarithmic curve of the form y = A + B ln x, where y represents cumulative sales in units and x the number of months since the beginning. How many units will be sold by the end of the eighth month?

Month	1	2	3	4	5	6
Cumulative Sales (units)	1431	3506	5177	6658	7810	8592

Keystrokes:	Outputs:	
f CLΣ		
1431 ENTER↑ 1 f LN Σ+		
3506 ENTER↑ 2 f LN Σ+		
5177 ENTER↑ 3 f LN Σ+		
6658 ENTER↑ 4 f LN Σ+		
7810 ENTER↑ 5 f LN Σ+		
8592 ENTER↑ 6 f LN Σ+ →	6.00	Six data pairs
f L.R. →	1066.15	Value of A
x⇄y →	4069.93	Value of B
g r →	0.99	Correlation coefficient
8 f LN ŷ →	9529.34	Total units sold by end of eighth month

POWER CURVE FIT

Another method of analysis is the power curve or geometric curve. The equation of the power curve is $y = Ax^B$, and the values for A and B are computed by calculations similar to linear regression. Some examples of power curves are shown below.



The following keystrokes fit a power curve according to the equation $\ln y = \ln A + B(\ln x)$:

1) Press f CLΣ.
2) Key in the first y-value, press f LN. Key in the first x-value, press f LN Σ+. Repeat this step for all data pairs.
3) Press f L.R. f e^x to obtain A in the above equation.
4) Press x⇄y to obtain B.
5) Press g r to obtain the correlation coefficient.
6) To find a projected y, key in the x-value and press ŷ f e^x to obtain y.

Example 1:

If Galileo had wished to investigate quantitatively the relationship between the time (t) for a falling object to hit the ground and the height (h) it has fallen, he might have released a rock from various levels of the Tower of Pisa (which was leaning even then) and timed its descent by counting his pulse. The following data are measurements Galileo might have made.

t (pulses)	2	2.5	3.5	4	4.5
h (feet)	30	50	90	130	150

Find the power curve formula that best expresses h as a function of t ($h = At^B$).

Keystrokes:	**Outputs:**
f CLΣ	
30 f LN 2 f LN Σ+	
50 f LN 2.5 f LN Σ+	
90 f LN 3.5 f LN Σ+	
130 f LN 4 f LN Σ+	
150 f LN 4.5 f LN Σ+ →	5.00 Five data pairs
f L.R. f e^x →	7.72 Value of A
x⇄y →	1.99 Value of B
g r →	1.00 Correlation coefficient

So, the formula that best expresses h as a function of t is

$$h = 7.72t^{1.99}$$

We know, as Galileo did not, that in fact $h \propto t^2$ (h is proportional to t^2).

STANDARD ERROR OF THE MEAN

The standard error of the mean is a measure of how reliable the mean of a sample (x) is as an estimator of the mean of the population from which the sample was drawn.

To calculate the standard error of the mean:

1) Press f CLΣ.
2) If you are summing **one** set of numbers, key in the first number and press Σ+. Continue until you have entered all of the values.
3) If you are summing **two** set of numbers, key in the y-value and press ENTER↑; key in the x-value and press Σ+. Continue until you have entered all of the values.
4) Press f $\bar{x}$ to obtain the mean of the x-values.
5) Press g s RCL Σ+ 0 f $\sqrt{x}$ ÷ to obtain the standard error of the mean of the x-values.
6) Alternatively press g s x⇄y RCL Σ+ 0 f $\sqrt{x}$ ÷ to obtain the standard error of the mean of the y-values.

Example 1:

A sample of 6 one-bedroom apartment rentals reveals that 1 rents for \$190 per month unfurnished, 1 rents for \$200 per month, 2 rent for \$205 per month, 1 rents for \$216 per month, and 1 rents for \$220 per month. What are the mean monthly rental and the standard deviation? What is the standard error of the mean?

Keystrokes:	Outputs:
f CLΣ	
190 Σ+ 200 Σ+	
205 Σ+ 205 Σ+	
216 Σ+ 220 Σ+	6.00 Total number of data inputs
f $\bar{x}$	206.00 Average monthly rent
g s	10.86 Standard deviation
RCL Σ+ 0 f $\sqrt{x}$ ÷	4.43 Standard error of the mean

MEAN, STANDARD DEVIATION, STANDARD ERROR FOR GROUPED DATA

Grouped data are presented in frequency distributions to save time and effort in writing down (or entering) each observation individually. Given a set of data points

$$x_1, x_2, \ldots, x_n$$

with respective frequencies

$$f_1, f_2, \ldots, f_n$$

this procedure computes the mean, standard deviation, and standard error of the mean.

1) Press f CLEAR.
2) Key in the first value and press ENTER↑ ENTER↑.
3) Key in the respective frequency and press STO + 0 × Σ+. The display shows the number of data points entered.
4) Repeat steps 2 and 3 for each data point.
5) To calculate the mean (average) press RCL 0 STO Σ+ 0 RCL Σ+ 5 STO Σ+ 2 f $\bar{x}$.
6) Press g s to find the standard deviation.
7) Press RCL 0 f $\sqrt{x}$ ÷ to find the standard error of the mean.

Example 1:

A survey of 266 one-bedroom apartment rentals reveals that 54 rent for $190 per month unfurnished, 32 rent for $195 per month, 88 rent for $200 per month, and 92 rent for $206 per month. What are the average monthly rental, the standard deviation, and the standard error of the mean?

Keystrokes:	**Outputs:**	
f CLEAR		
190 ENTER↑ ENTER↑ 54		
STO + 0 × Σ+ →	1.00	First data pair entered
195 ENTER↑ ENTER↑ 32		
STO + 0 × Σ+ →	2.00	Second data pair entered
200 ENTER↑ ENTER↑ 88		
STO + 0 × Σ+ →	3.00	Third data pair entered
206 ENTER↑ ENTER↑ 92		
STO + 0 × Σ+ →	4.00	Fourth data pair entered
RCL 0 STO Σ+ 0 RCL Σ+		
5 STO Σ+ 2 f $\bar{x}$ →	199.44	Average monthly rent
g s →	5.97	Standard deviation
RCL 0 f $\sqrt{x}$ ÷ →	0.37	Standard error of the mean

CHI-SQUARE STATISTIC

The chi-square statistic means the goodness of fit between two sets of frequencies. It is used to test whether a set of observed frequencies differ from a set of expected frequencies sufficiently to reject the hypothesis under which the expected frequencies were obtained.

In other words, you are testing whether discrepancies between the observed frequencies (O_i) and the expected frequencies (E_i) are significant, or whether they may reasonably be attributed to chance. The formula generally used is:

$$\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$$

If there is a close agreement between the observed and expected frequencies, χ^2 will be small. If the agreement is poor, χ^2 will be large.

The following keystrokes calculate the χ^2 statistic:

1) Press CLX ENTER↑.
2) Key in the first O_i value, press ENTER↑.
3) Key in the first E_i value, press STO 0 − ENTER↑ × RCL 0 ÷ +.
4) Repeat steps 2 and 3 for all data pairs. The χ^2 value is displayed.

Example 1:

A suspect die from a Las Vegas casino is brought to an independent testing firm to determine its bias, if any. The die is tossed 120 times and the following results obtained.

Number	1	2	3	4	5	6
Observed Frequency	25	17	15	23	24	16

The expected frequency = 120 throws/6 sides, or E = 20 for each number, 1 thru 6. (Since E is a constant is this example, there's no need to store it in R_0 each time.)

Keystrokes:	**Outputs:**
CLx ENTER↑ 25 ENTER↑	
20 STO 0 − ENTER↑ ×	
RCL 0 ÷ +	1.25
17 ENTER↑ 20 − ENTER↑	
× RCL 0 ÷ +	1.70
15 ENTER↑ 20 − ENTER↑	
× RCL 0 ÷ +	2.95
23 ENTER↑ 20 − ENTER↑	
× RCL 0 ÷ +	3.40
24 ENTER↑ 20 − ENTER↑	
× RCL 0 ÷ +	4.20
16 ENTER↑ 20 − ENTER↑	
× RCL 0 ÷ +	5.00 χ^2

The number of degrees of freedom is (n − 1). Since n = 6, the degrees of freedom = 5.

Consulting statistical tables, you look up χ^2 to a 0.05 significance level with 5 degrees of freedom, and see that $\chi^2_{0.05,5}$ = 11.07. Since χ^2 = 5 is within 11.07, we may conclude that to a 0.05 significance level (Probability = .95), the die is fair.

NOTES

SECURITIES

ALPHABETICAL INDEX

Note:

Remember that the issue date, settlement date, maturity date, coupon rate, price and yield are retained in separate storage registers, and do not need to be re-entered for similar problems.

Type 1: Semi-annual Coupon, 30/360 Day Basis

Examples:

Corporate Bonds, Commodity Credit Corporation, Export-Import Bank Certificates, Federal Home Loan Bank, Federal National Mortgage Association Debentures (FNMA), Federal Land Bank Bonds, General National Mortgage Association Bonds (GNMA), U.S. Postal Service Bonds, TVA Bonds, State and Local Government Issues, Certificate of Deposit, Inter-American Development Bank Bonds, International Bank for Reconstruction and Development, Merchant Marine Bonds, New Communities Act Debentures

To calculate price:

1) Set the Day Basis switch to 360, the Payment Mode switch to BOND, the Print Mode switch to ALL and press CL FIN.
2) Key in the settlement date, press f IS,ST.
3) Key in the maturity date and press g MT.
4) Key in the annual coupon rate as a percent and press g CPN.
5) Key in the annual yield as a percent and press YIELD.
6) Press PRICE to obtain the price and accrued interest of a semi-annual coupon, 30/360 day basis security.

Example 1:

Calculate the price of a Federal Home Loan Bank Note given the following information:

Settlement date June 20, 1977; maturity date August 30, 1978; coupon 5.65%; yield 6.84%.

Keystrokes:

ALL
MAN NORM

360 365 BEGIN/NOTE END/BOND

CL FIN

6.201977 f IS,ST 8.301978

g MT 5.65 g CPN 6.84

YIELD PRICE ⟶ 98.65 Price

Printer:

```
           CL F
6.201977     ST
8.301978     MT
    5.65    CPN
    6.84    YLD
BOND *360   PRC
    1.74     AI
   98.65    ***
```

To calculate yield:

1) Set the Day Basis switch to 360, the Payment Mode switch to BOND, the Print Mode switch to ALL and press CL FIN.
2) Key in the settlement date and press f IS,ST.
3) Key in the maturity date and press g MT.
4) Key in the annual coupon rate as a percent and press g CPN.
5) Key in the purchase price and press PRICE.
6) Press YIELD to obtain the yield of a semi-annual coupon, 30/360 day basis security.

Example 2:

Determine the yield of a Federal Land Bank Bond given the following data:

Settlement date May 19, 1972; maturity date June 30, 1983; price 98.50; coupon 5.85%.

Keystrokes: **Printer:**

MAN ALL NORM

360 365 BEGIN/NOTE END/BOND

CL FIN

5.191972 f IS,ST 6.301983

g MT 5.85 g CPN

98.5 PRICE YIELD → 6.04 Percent annual yield

```
                 CL F
5.191972           ST
6.301983           MT
    5.85          CPN
   98.50          PRC
BOND *360         YLD
    6.04          ***
```

Type 2: Semi-annual Coupon, Actual/Actual Day Basis

Examples:

U.S. Treasury Bonds and Notes, Farmers Home Administration Insured Notes (FHDA), Federal Housing Administration Debentures (FHA), Foreign Bonds

To calculate price:

1) Set the Day Basis switch to 365, the Payment Mode switch to BOND, the Print Mode switch to ALL, and press CL FIN.
2) Key in the settlement date, press f IS,ST.
3) Key in the maturity date and press g MT.

4) Key in the annual coupon rate as a percent and press g CPN.

5) Key in the annual yield as a percent and press YIELD.

6) Press PRICE to obtain the price and accrued interest of a semi-annual coupon, Actual/Actual day basis security.

Example 1:

Given the following security, find its price.

Settlement date September 17, 1976; maturity date December 15, 1976; interest 5.75%; annual yield 3.34%.

Keystrokes:

MAN ALL NORM

360 365 BEGIN/NOTE END/BOND

CL FIN

9.171976 f IS,ST 12.151976

g MT 5.75 g CPN 3.34

YIELD PRICE → 100.57 Purchase price

Printer:

```
          CL F
 9.171976   ST
12.151976   MT
     5.75  CPN
     3.34  YLD
BOND *365  PRC
     1.48   AI
   100.57  ***
```

To calculate yield:

1) Set the Day Basis switch to 365, the Payment Mode switch to BOND, the Print Mode switch to ALL, and press CL FIN.

2) Key in the settlement date, press f IS,ST.

3) Key in the maturity date and press g MT.

4) Key in the annual coupon rate as a percent and press g CPN.

5) Key in the purchase price and press PRICE.

6) Press YIELD to obtain the annual yield of a semi-annual coupon security.

Example 2:

Given the following security, find its yield.

U.S. Treasury Bond
Settlement date January 3, 1977; maturity date December 14, 1990; price 98.875 and coupon rate of 4¾%.

Keystrokes: **Printer:**

MAN ALL NORM

360 365 BEGIN/NOTE END/BOND

CL FIN

1.031977 f IS,ST 12.141990

g MT 4.75 g CPN 98.875

PRICE YIELD → 4.86 Percent annual yield

```
           CL F
1.031977     ST
12.141990    MT
4.75        CPN
98.875      PRC
BOND *365   YLD
4.86        ***
```

Type 3: Interest at Maturity, 30/360 Day Basis

Examples:

Banks for Cooperatives Debentures, Federal Intermediate Credit Bank Debentures, State and Local Government Issues, Warrants

To calculate price:

1) Set the Day Basis switch to 360, the Payment Mode switch to NOTE, the Print Mode switch to ALL, and press CL FIN.
2) Key in the issue date and press ENTER↑.
3) Key in the settlement date and press f IS,ST.
4) Key in the maturity date and press g MT.
5) Key in the annual coupon rate as a percent and press g CPN.
6) Key in the annual yield as a percent and press YIELD.
7) Press PRICE to obtain the price and accrued interest for both Actual/360 and 30/360 day basis.

Example 1:

Given the following security, find its price.

Banks for Cooperatives Debentures
Issued May 3, 1976; settlement date June 25, 1976; maturity date September 10, 1976; coupon 6.85%; yield 6.97%.

Keystrokes: **Printer:**

MAN ALL NORM

360 365 BEGIN/NOTE END/BOND

```
     CL F
5.031976 ENT↑
6.251976 ISST
9.101976   MT
    6.85  CPN
    6.97  YLD
NOTE *360 PRC
    1.01   AI
   99.96  ACT
    0.99   AI
   99.96  ***
```

CL FIN

5.031976 ENTER↑ 6.251976

f IS,ST 9.101976 g MT 6.85

g CPN 6.97 YIELD PRICE ⟶ 99.96 Purchase price

To calculate yield:

1) Set the Day Basis switch to 360, the Payment Mode switch to NOTE, the Print Mode switch to ALL, and press CL FIN.
2) Key in the issue date and press ENTER↑.
3) Key in the settlement date and press f IS,ST.
4) Key in the maturity date and press g MT.
5) Key in the annual coupon rate as a percent and press g CPN.
6) Key in the price and press PRICE.
7) Press YIELD to obtain the annual yield.

Example 2:

Given the following security, find its yield.

Banks for Cooperatives Debentures
Issued July 26, 1976; settlement date September 10, 1976; maturity date November 22, 1976; coupon 7.15%; price 99.45.

Keystrokes: **Printer:**

MAN ALL NORM

360 365

```
      CL F
 7.261976 ENT↑
 9.101976 ISST
11.221976   MT
     7.15  CPN
    99.45  PRC
NOTE *360  YLD
     9.83  ACT
     9.87  ***
```

CL FIN

7.261976 ENTER↑ 9.101976

f IS,ST 11.221976 g MT 7.15

g CPN 99.45 PRICE YIELD ⟶ 9.87 Percent annual yield

Type 4: Interest at Maturity, Actual/360 Day Basis

Examples:

Certificates of Deposit, Repurchase Agreements

To calculate price:

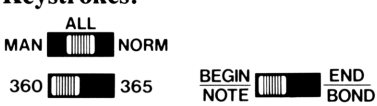
1) Set the Day Basis switch to 360, the Payment Mode switch to BOND, the Print Mode switch to ALL and press CL FIN.
2) Key in the issue date and press ENTER↑.
3) Key in the settlement date and press f IS,ST.
4) Key in the maturity date and press g MT.
5) Key in the annual coupon rate as a percent and press g CPN.
6) Key in the annual yield as a percent and press YIELD.
7) Press PRICE to obtain the price and accrued interest for both Actual/360 and 30/360 day basis.

Example 1:

Given the following security, find its price.

Certificate of Deposit
Settlement date August 23, 1976; maturity date December 20, 1976; issue date July 6, 1976; interest rate 5.8%; yield 5.65%.

Keystrokes: **Printer:**



CL FIN
7.061976 ENTER↑ 8.231976 f
IS,ST 12.201976 g MT 5.8
g CPN 5.65 YIELD PRICE ⟶ 100.03 Purchase price

```
          CL F
 7.061976 ENT↑
 8.231976 ISST
12.201976   MT
     5.80  CPN
     5.65  YLD
NOTE *360  PRC
     0.77   AI
   100.03  ACT
     0.76   AI
   100.03  ***
```

To calculate yield:

1) Set the Day Basis switch to 360, the Payment Mode switch to NOTE, the Print Mode switch to ALL, and press CL FIN.
2) Key in the issue date and press ENTER↑.

3) Key in the settlement date and press f IS,ST.
4) Key in the maturity date and press g MT.
5) Key in the annual coupon rate as a percent and press g CPN.
6) Key in the price and press PRICE.
7) Press YIELD R↓ to obtain the annual yield.

Example 2:

Having just performed the previous example, what would be the annual yield of the Certificate of Deposit if the security was purchased for $100.00?

Keystrokes:		Printer:
100 PRICE YIELD ⟶	5.76 Percent annual yield	100.00 PRC NOTE *360 YLD 5.76 ACT 5.76 ***

Note:

Since the other values are unchanged, it is not necessary to re-enter them in this calculation.

Type 5: Interest at Maturity, Actual/Actual Day Basis

Examples:

Certificates of Indebtedness

To calculate price:

1) Set the Day Basis switch to 365, the Payment Mode switch to NOTE, the Print Mode switch to ALL, and press CL FIN.
2) Key in the issue date and press ENTER↑.
3) Key in the settlement date and press f IS,ST.
4) Key in the maturity date and press g MT.
5) Key in the annual coupon rate as a percent and press g CPN.
6) Key in the annual yield and press YIELD.
7) Press PRICE to obtain the purchase price and accrued interest.

Example 1:

Find the price of a Certificate of Indebtedness given the following information.

5% interest; desired yield 6.35%; settlement date June 4, 1976; issue date April 15, 1976; maturity date November 15, 1976.

Keystrokes: **Printer:**

ALL
MAN NORM

360 365 BEGIN/NOTE END/BOND

CL FIN

4.151976 ENTER↑ 6.041976
f IS,ST 11.151976 g MT 5
g CPN 6.35 YIELD PRICE ⟶ 99.39 Purchase price

```
      CL F
4.151976 ENT↑
6.041976 ISST
11.151976   MT
     5.00  CPN
     6.35  YLD
NOTE *365 PRC
     0.68   AI
    99.39  ***
```

To calculate yield:

1) Set the Day Basis switch to 365, the Payment Mode switch to NOTE, the Print Mode switch to ALL, and press CL FIN.
2) Key in the issue date and press ENTER↑.
3) Key in the settlement date and press f IS,ST.
4) Key in the maturity date and press g MT.
5) Key in the annual coupon rate as a percent and press g CPN.
6) Key in the purchase price and press PRICE.
7) Press YIELD to obtain the annual yield as a percent.

Example 2:

Find the yield of a Certificate of Indebtedness given the following information:

Issue date April 1, 1976; settlement date June 1, 1976; maturity date October 15, 1976; price 99.65; interest 5%.

Keystrokes: **Printer:**

ALL
MAN NORM

360 365 BEGIN/NOTE END/BOND

CL FIN

4.011976 ENTER↑ 6.011976
f IS,ST 10.151976 g MT
5 g CPN 99.65 PRICE YIELD ⟶ 5.91 Percent annual yield

```
      CL F
4.011976 ENT↑
6.011976 ISST
10.151976   MT
     5.00  CPN
    99.65  PRC
NOTE *365 YLD
     5.91  ***
```

Type 6: Discount, Actual/360

Examples:

U.S. Treasury Bills, U.S. Treasury Tax-Anticipation Bills, FNMA Short Term Notes, TVA Notes, Bankers' Acceptances, Certificates of Deposit, Commercial Paper

To calculate yield given price:

1) Set the Day Basis switch to 360, the Payment Mode switch to NOTE, the Print Mode switch to ALL, and press CL FIN.
2) Key in the settlement date and press ENTER↑ f IS,ST.
3) Key in the maturity date and press g MT.
4) Key in the purchase price and press PRICE.
5) Press YIELD R↓ to obtain the annual yield as a percent for both Actual/360 and 30/360 day basis.

Example 1:

Calculate the yield of the following U.S. Treasury Bill:

Settlement date September 28, 1976; maturity date August 1, 1977; price 96.27041; no coupons.

Keystrokes:

ALL
MAN NORM

360 365 BEGIN/NOTE END/BOND

CL FIN
9.281976 ENTER↑ f IS,ST
8.011977 g MT
96.27041 PRICE YIELD R↓ → 4.54 Percent annual yield

Printer:

```
          CL F
9.281976 ENT↑
         ISST
8.011977   MT
96.27041  PRC
NOTE *360 YLD
    4.54  ACT
    4.60  ***
           R↓
    4.54  ***
```

To calculate the price and effective yield given the discount rate:

1) Set the Day Basis switch to 360, the Payment Mode switch to NOTE, the Print Mode switch to ALL, and press CL FIN.
2) Key in the settlement date and press ENTER↑ f IS,ST.
3) Key in the maturity date and press g MT.
4) Key in the discount rate (as a percent) and press CHS g CPN.

5) Press PRICE R↓ R↓ to find the price.

6) Press PRICE CL x g CPN YIELD R↓ to find the effective annual yield as a percent.

Example 2:

Calculate price and effective yield of the following U.S. Treasury Bill:

Settlement date October 8, 1976; maturity date March 21, 1977; discount rate 4.20%.

Keystrokes:

ALL
MAN NORM

360 365 BEGIN/NOTE END/BOND

CL FIN
10.081976 ENTER↑ f IS,ST
3.211977 g MT
4.20 CHS g CPN
PRICE R↓ R↓ → 98.09 Price
PRICE CL x g CPN
YIELD R↓ → 4.28 Percent effective annual yield

Printer:

```
              CL F
10.081976    ENT↑
             ISST
3.211977       MT
   -4.20      CPN
NOTE *360     PRC
    0.00       AI
   98.09      ACT
    0.00       AI
   98.10      ***
               R↓
    0.00      ***
               R↓
   98.09      ***
              PRC
             CL X
              CPN
NOTE *360     YLD
    4.28      ACT
    4.31      ***
               R↓
    4.28      ***
```

AFTER-TAX YIELD

This procedure calculates the after-tax yield (either to maturity or some other price). For after-tax computations, the interest or coupon payments are considered income, while the difference between the bond or note face value and its purchase price is considered capital gains.

The keystrokes are:

1) Set the Day Basis switch to 365, the Payment Mode switch to BOND, the Print Mode switch to ALL, and press CL FIN.
2) Key in the sale price; press STO 0.
3) Key in the purchase price; press STO 1 −.
4) Key in the capital gains tax rate (expressed as a percent); press % RCL 0 x≷y − to obtain the after-tax proceeds from the sale.
5) Press 100 ÷ STO 0.
6) Key in the purchase date; press f IS,ST.
7) Key in the assumed sell date; press g MT.
8) Key in the annual coupon rate (as a percent); press ENTER↑.
9) Key in the income tax rate (expressed as percent) and press % − RCL 0 ÷ g CPN.
10) Press RCL 1 RCL 0 ÷ PRICE YIELD to find the after-tax yield.

Example 1:

You can buy a 3% bond on November 1, 1976 for $70 and expect to sell it in 5 years for $90. What is your net (after-tax) yield over the 5-year period if interim coupon payments are considered as income, and your tax bracket is 50%?

(One-half of the long-term capital gain is taxable at 50%, so the tax on capital gains alone is 25%.)

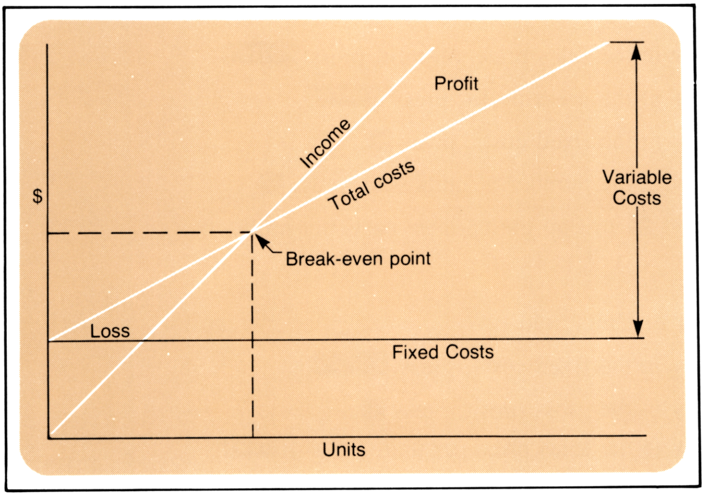
Keystrokes:	**Outputs:**
360 ▭ 365 BEGIN/NOTE ▭ END/BOND	
CL FIN	
90 STO 0 70 STO 1 −	
25 % RCL 0 x≷y − ⟶	85.00 After-tax proceeds from sale
100 ÷ STO 0	
11.011976 f IS,ST 11.011981	
g MT 3 ENTER↑ 50 % − RCL 0	
÷ g CPN RCL 1 RCL 0 ÷	
PRICE YIELD ⟶	5.89 % yield after taxes

NOTES

OTHER

BREAK-EVEN ANALYSIS

Break-even analysis is basically a technique for analyzing the relationships among fixed costs, variable costs, and income. Until the bread-even point is reached, at the intersection of the total income and total cost lines, the producer operates at a loss. After the break-even point, each unit produced and sold makes a profit. Break-even analysis may be represented as follows:



Given four of these variables: fixed costs, sales price per unit, variable costs per unit, number of units sold, and gross profit, the following procedures evaluate the remaining variable. To calculate the break-even values, simply let the gross profit equal zero.

Gross Profit

1) Key in the sales price per unit, press ENTER↑.
2) Key in the variable costs per unit, press −.
3) Key in the number of units sold, press ×.
4) To find the gross profit, key in the fixed costs and press −.

Fixed Costs

1) Key in the sales price per unit, press ENTER↑.
2) Key in the variable costs per unit, press −.
3) Key in the number of units sold, press ×.
4) To find the amount of fixed costs, key in the gross profit and press −.

Sales Price Per Unit

1) Key in the gross profit, press ENTER↑.
2) Key in the fixed costs, press +.
3) Key in the number of units sold, press ÷.
4) To find the sales price per unit, key in the variable costs per unit and press +.

Variable Costs Per Unit

1) Key in the sales price per unit, press ENTER↑.
2) Key in the gross profit, press ENTER↑.
3) Key in the fixed costs, press +.
4) Key in the number of units sold, press ÷.
5) Press − to find the variable costs per unit.

Number of Units Sold

1) Key in the fixed costs, press ENTER↑.
2) Key in the gross profit, press +.
3) Key in the sales price per unit, press ENTER↑.
4) Key in the variable costs per unit, press −.
5) Press ÷ to obtain the number of units sold.

Example 1:

The Cooper Company sells finance textbooks at $13 each. Given the costs and revenues below, how many textbooks must be sold to break-even?

Fixed costs	
Typesetting	$ 4,000
Graphics production	5,000
Printing and binding	3,000
Total fixed costs	$12,000
Variable costs per copy	
Distribution	$ 1.00
Commissions	3.75
Royalties	2.00
Total variable costs per copy	$ 6.75
Sales price per copy	$ 13.00

Keystrokes:	**Outputs:**
12000 ENTER↑ 0 +	
13 ENTER↑ 6.75 − ÷ ⟶	1920.00 Number of units.

The degree of operating leverage at a point is defined as the ratio of the percentage change in net operating income to the percentage change in units sold. The greatest degree of operating leverage is found near the break-even point, where a small change in sales may produce a very large increase in profits. This happens because the profits are close to zero near the break-even point. Likewise, firms with a small degree of operating leverage are operating farther from the break-even point, and they are relatively insensitive to changes in sales volume.

The necessary inputs to calculate the degree of operating leverage are fixed costs, sales price per unit, variable costs per unit, and number of units sold.

The keystrokes are:

1) Key in the sales price per unit, press ENTER↑.
2) Key in the variable costs per unit, press −.
3) Key in the number of units, press × ENTER↑ ENTER↑.
4) Key in the fixed costs, press −.
5) Press ÷ to determine the degree of operating leverage.

Example 2:

In the above example, what is the Cooper Company's degree of operating leverage at 2000 units? At 5000 units?

Keystrokes:	**Outputs:**
13 ENTER↑ 6.75 − 2000 ×	
ENTER↑ ENTER↑ 12000	
− ÷ ⟶	25.00 This is close to the break-even point.
13 ENTER↑ 6.75 − 5000 ×	
ENTER↑ ENTER↑ 12000	
− ÷ ⟶	1.62 The company is farther from the break-even point and less sensitive to changes in sales volume.

APPENDICES

APPENDIX A
FINANCIAL FORMULAS

Percentage

$$\% = \frac{\text{Base (y)} \cdot \text{Rate (x)}}{100}$$

$$\Delta\% = \left(\frac{\text{New Amount (x)} - \text{Base (y)}}{\text{Base (y)}}\right) \cdot 100$$

Compound Interest

n = number of compounding periods

i = periodic interest rate, expressed as %

PV = present value

FV = future value or balance

PMT = periodic payment

δ = BEGIN/END switch position factor (0 or 1) indicating treatment of PMT; 0 corresponds to END, 1 to BEGIN.

r = i/100, periodic interest rate expressed as decimal

$$0 = PV + (1 + r\delta)\,PMT\left[\frac{1 - (1 + r)^{-n}}{r}\right] + FV\,(1 + r)^{-n}$$

Net Present Value

NPV = net present value of a discounted cash flow

cf_j = cash flow at period j

$$NPV = cf_0 + \frac{cf_1}{(1 + r)^1} + \frac{cf_2}{(1 + r)^2} + \ldots + \frac{cf_n}{(1 + r)^n}$$

Amortization

P1 = first period of amortization schedule

P2 = last period of amortization schedule

INT_j = amount of PMT applied to interest in period j

PRN_j = amount of PMT applied to principal in period j

j = period number

$BAL_j = BAL_{j-1} - PRN_j; \qquad BAL_0 = |PV|$

$$\mathrm{INT}_1 = |\mathrm{PV}| \cdot \frac{i}{100}$$

$$\mathrm{PRN}_1 = |\mathrm{PMT}| - \mathrm{INT}_1$$

$$\mathrm{BAL}_1 = |\mathrm{PV}| - \mathrm{PRN}_1$$

$$\vdots$$

$$\mathrm{INT}_j = \mathrm{BAL}_{j-1} \cdot \frac{i}{100}$$

$$\mathrm{PRN}_j = |\mathrm{PMT}| - \mathrm{INT}_j$$

$$\Sigma\mathrm{INT} = \sum_{j=P1}^{P2} \mathrm{INT}_j = \mathrm{INT}_{P1} + \mathrm{INT}_{P1+1} + \ldots + \mathrm{INT}_{P2}$$

$$\Sigma\mathrm{PRN} = \sum_{j=P1}^{P2} \mathrm{PRN}_j = \mathrm{PRN}_{P1} + \mathrm{PRN}_{P1+1} + \ldots + \mathrm{PRN}_{P2}$$

Depreciation

LIFE = asset's depreciable life (years)

BOOK = starting book value

SAL = salvage value

N1 = first period of depreciation schedule

N2 = last period of depreciation schedule

FACT = declining balance factor expressed as %

j = period number

DPN_j = depreciation expense during period j

RDV_j = remaining depreciable value at end of period j = $\mathrm{RDV}_{j-1} - \mathrm{DPN}_j$ where RDV_0 = BOOK − SAL

RBV_j = remaining book value = $\mathrm{RBV}_{j-1} - \mathrm{DPN}_j$ where RBV_0 = BOOK

Straight-Line Depreciation

$$\mathrm{DPN}_j = \frac{\mathrm{BOOK} - \mathrm{SAL}}{\mathrm{LIFE}} \text{ for } j = 1, 2, \ldots$$

Sum-of-the-Years'-Digits Depreciation

$$SOYD = \frac{(W + 1)(W + 2F)}{2}$$

where W = integer part of LIFE

F = fractional part of LIFE

(i.e., for a LIFE of 12.25 years W = 12 and F = .25).

$$DPN_j = \frac{(LIFE - j + 1)}{SOYD}(BOOK - SAL)$$

Declining Balance Depreciation

$$DPN_j = RBV_{j-1}\left(\frac{FACT}{100 \cdot LIFE}\right)$$

Loans With A Constant Amount Paid Towards Principal

BAL_K = remaining balance after time period K

CPMT = constant payment to principal

$BAL_K = PV - (K \cdot CPMT)$

K^{th} payment to interest $= i(BAL_K) = (PMT_i)_K$

K^{th} total payment $= CPMT + (PMT_i)_K$

Add-On Interest Rate To APR

r = add-on rate as a decimal

n = number of monthly payments

APR = 1200i, where i is the solution in the following equation:

$$\frac{n}{1 + \frac{n}{12} r} = \frac{1 - (1 + i)^{-n}}{i}$$

Rule of 78's Rebate

PV = finance charge

I_k = interest charged at month k

$$I_k = \frac{2(n - k + 1)}{n(n + 1)} PV$$

$$Rebate = \frac{(n - k)\, I_k}{2}$$

Wrap-Around Mortgage

$$PV_2 - PV_1 = \frac{PMT_2\left[1-(1+r)^{-n_2}\right]}{r} - \frac{PMT_1\left[1-(1+r)^{-n_1}\right]}{r} + FV(1+r)^{-n_2}$$

Canadian Mortgages

$$\text{monthly factor} = \left[\left(1 + \frac{r}{2}\right)^{1/6} - 1\right] \times 100$$

Compounding Periods Different From Payment Periods

$$PMT = \frac{FV}{Z}\left[\frac{Q}{(1+Q)^n - 1}\right]$$

P = number of payment periods per year

C = number of compounding periods per year

when $P/C \leqslant 1$

$Q = (1+r)^{C/P} - 1$

$Z = (1 + Q)$

n = total number of payments

when $P/C > 1$

$Q = r$

n = (total number of payments) $\cdot$ C/P

$$Z = (P/C + 1) \cdot \left(\frac{Q}{2}\right) + (P/C)$$

Advance Payments

$$PMT = \frac{PV - FV(1+r)^{-n}}{\left[\dfrac{1-(1+r)^{-(n-A)}}{r} + A\right]}$$

where:

A = number of payments made in advance

Linear Regression

for

$$y = A + Bx$$

$$B = \frac{\Sigma x_i y_i - \dfrac{\Sigma x_i \Sigma y_i}{n}}{\Sigma x_i^2 - \dfrac{(\Sigma x_i)^2}{n}}$$

$$A = \overline{y} - B\overline{x}$$

where:

$$\overline{x} = \frac{\Sigma x_i}{n} \qquad \overline{y} = \frac{\Sigma y_i}{n}$$

$$R^2 = \frac{\left[\Sigma x_i y_i - \dfrac{\Sigma x_i \Sigma y_i}{n}\right]^2}{\left[\Sigma x_i^2 - \dfrac{(\Sigma x_i)^2}{n}\right]\left[\Sigma y_i^2 - \dfrac{(\Sigma y_i)^2}{n}\right]}$$

n = number of data pairs

Exponential Curve Fit

$$y = Ae^{Bx}$$

$$B = \frac{\Sigma x_i \ln y_i - \dfrac{1}{n}(\Sigma x_i)(\Sigma \ln y_i)}{\Sigma x_i^2 - \dfrac{1}{n}(\Sigma x_i)^2}$$

$$A = \exp\left[\frac{\Sigma \ln y_i}{n} - B\frac{\Sigma x_i}{n}\right]$$

$$\hat{y} = Ae^{Bx}$$

Logarithmic Curve Fit

$$y = A + B(\ln x)$$

$$= \frac{\Sigma y_i \ln x_i - \dfrac{1}{n}\Sigma \ln x_i \Sigma y_i}{\Sigma(\ln x_i)^2 - \dfrac{1}{n}(\Sigma \ln x_i)^2}$$

$$A = \frac{1}{n}(\Sigma y_i - B\Sigma \ln x_i)$$

$$\hat{y} = A + B(\ln x)$$

Power Curve Fit

$$y = Ax^B \qquad (A > 0)$$

$$\ln y = \ln A + B\ln x$$

$$B = \frac{\Sigma(\ln x_i)(\ln y_i) - \dfrac{(\Sigma\ln x_i)(\Sigma\ln y_i)}{n}}{\Sigma(\ln x_i)^2 - \dfrac{(\Sigma\ln x_i)^2}{n}}$$

$$A = \exp\left[\frac{\Sigma\ln y_i}{n} - B\frac{\Sigma\ln x_i}{n}\right]$$

$$\hat{y} = Ax^B$$

Standard Error of The Mean

$$s_{\bar{x}} = \frac{s_x}{\sqrt{n}} \qquad s_{\bar{y}} = \frac{s_y}{\sqrt{n}}$$

Mean, Standard Deviation, Standard Error For Grouped Data

$$\text{mean } \bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

$$\text{standard deviation } s_x = \sqrt{\frac{\Sigma f_i x_i^2 - (\Sigma f_i)\bar{x}^2}{\Sigma f_i - 1}}$$

$$\text{standard error } s_{\bar{x}} = \frac{s_x}{\sqrt{\Sigma f_i}}$$

Bonds and Notes

Reference:

Spence, Bruce M. and others, STANDARD SECURITIES CALCULATION METHODS, Securities Industry Association, 1973.

DIM/b = days between issue date and maturity date/day basis from calendar switch

DSM/b = days between settlement date and maturity date/day basis from calendar switch

DIS = days between issue date and settlement date

DIS/b = DIM/b − DSM/b

E = number of days in coupon period where settlement occurs

DSC = E − DIS = days from settlement date to next six month coupon date

N = number of semi-annual coupons payable between settlement date and maturity date or call date

$$\overline{\text{CPN}} = \frac{\text{CPN} \cdot \text{CALL}}{100}$$

DISC = discount rate (as a percent)

CALL = redemption value per $100 par value (call price)

Types 1 and 2

Price (given yield) with 6 months or less to maturity

$$\text{PRICE} = \frac{100\left(\text{CALL} + \dfrac{\overline{\text{CPN}}}{2}\right)}{100 + \left(\dfrac{\text{DSM}}{\text{E}} \cdot \dfrac{\text{YIELD}}{2}\right)} - \left[\frac{\text{DIS}}{\text{E}} \cdot \frac{\overline{\text{CPN}}}{2}\right]$$

Price (given yield) with more than 6 months to maturity.

$$\text{PRICE} = \frac{\text{CALL}}{\left(1 + \dfrac{\text{YIELD}}{200}\right)^{\text{N}-1+\frac{\text{DSC}}{\text{E}}}} + \left[\sum_{\text{K}=1}^{\text{N}} \frac{\dfrac{\overline{\text{CPN}}}{2}}{\left(1 + \dfrac{\text{YIELD}}{2}\right)^{\text{K}-1+\frac{\text{DSC}}{\text{E}}}}\right] - \left[\frac{\overline{\text{CPN}}}{2} \cdot \frac{\text{DIS}}{\text{E}}\right]$$

Types 3, 4, and 5

$$\text{PRICE} = \frac{100\left(\text{CALL} + \overline{\text{CPN}} \cdot \dfrac{\text{DIM}}{\text{b}}\right)}{\left(100 + \text{YIELD} \cdot \dfrac{\text{DSM}}{\text{b}}\right)} - \left(\overline{\text{CPN}} \cdot \frac{\text{DIS}}{\text{b}}\right)$$

Type 6

Price (given discount rate)

$$\text{PRICE} = (\text{CALL}) - \left(\text{DISC} \cdot \text{CALL} \cdot \frac{\text{DSM}}{\text{b}}\right)$$

Yield (given price)

$$\text{YiELD} = \left(\frac{\text{CALL} - \text{PRICE}}{\text{PRICE}}\right)\left(\frac{\text{b}}{\text{DSM}}\right)(100)$$

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