HEWLETT-PACKARD

HP-92 Real Estate and Investment Analysis

HEWLETT hp PACKARD

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Real Estate and Investment Analysis

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INTRODUCTION

This HP-92 REAL ESTATE AND INVESTMENT ANALYSIS handbook is a supplement to the HP-92 Investor Owner's Handbook and the HP-92 Investor Applications book. Together the books explain and illustrate how the HP-92 calculator can be used effectively and efficiently to solve a wide variety of recurring problems that confront the real estate practitioner or analyst. They cover as nearly as possible the full spectrum of computational problems related to real estate and investment transactions, derived from actual field experience. Of course these techniques also apply to other areas of financial analysis.

To help you quickly find the proper HP-92 keystroke procedure applicable to your particular real estate problem, this book features a quick reference Real Estate Solutions Guide. It is organized by general topic areas with each procedure in that area listed individually. Opposite the procedure is a table which directs you to the proper book and page where the solution is described. For example, if you want the procedure for finding the yield on a wrap-around mortgage, look under Investment Analysis and Feasibility Analysis: Financial (Mortgage) Analysis, and find the procedure 'Wrap-Around Mortgages'. In the table opposite the procedure you will find page 34 listed under the column HP-92 Investor Applications.

The keystroke procedures listed in the Real Estate Solutions Guide have been developed by many individuals in actual practice. They have been field-tested and they work. Many are adapted from HP-70, HP-80 and HP-81 routines. Our thanks go to all those who contributed suggestions and ideas for these procedures, as well as examples of their use. It is possible that with practice and experience you can add to the practical applications of the HP-92 in solving real estate problems, and perhaps develop short-cuts in some instances. The range of applications is not yet fully explored, although the examples provided here do appear to cover the most important recurring situations that confront professionals in real estate practice.

With the HP-92, you no longer need cumbersome printed tables of compound interest factors, or loan reduction schedules, or mortgage payments or mortgage constants, or depreciation schedules. All of these can be calculated accurately in less time than it normally takes to look them up in a printed book of tables. The same applies to components of mortgage-equity (Ellwood) analysis.

The HP-92 is also a highly flexible calculating instrument. In a number of instances, the operator can enter figures in any order. This means that you can use your own worksheet format instead of being forced into a specific pattern by the machine. Where this is possible with the HP-92, it is noted. In other cases, the steps and keystrokes must be followed exactly. This is also noted when it applies to the specific procedures.

Users of the HP-92 have a printed tape record of every calculation as long as the printer is on. This is a decided advantage in long or involved calculations so that you can check your data inputs as you go along.

Another outstanding distinguishing feature of the HP-92 is that it is pre-programmed to produce many calculations involving several related results automatically. This saves both time and effort (as well as minimizing chances of making a mistake) in producing such important printouts as Depreciation Schedules; Mortgage Amortization Schedules and summaries of Compound Interest calculations.

The terminology and symbols used in the examples in this handbook are those most widely accepted in professional real estate practice. A detailed listing of these symbols and terms is provided in the Appendix. In addition, the formulas used in the keystroke procedures are presented as they are covered throughout the handbook. However, it is not necessary to memorize these formulas, or even to use them, in order to apply the procedures that are presented. Just follow the steps as shown, and the proper answers will be produced every time.

In compiling and testing the routines and procedures for solving real estate problems that are contained in this manual, the authors received suggestions, assistance and criticisms from a number of sources. We wish to acknowledge publicly our appreciation for their help and advice. In particular, we are expecially indebted to our good friend and distinguished colleague, Dr. Stephen D. Messner, who is Director of the Center for Real Estate and Urban Economic Studies as well as Head of the Finance Department at the University of Connecticut. Dr. Messner gave unstintingly of his time and skill in commenting on materials prepared for inclusion in this manual. He is responsible for several of the routines and the ideas that underlie them, most especially in the areas of Rate-of-Return analysis and Investment-Feasibility analysis. The Modified IRR procedures are wholly his invention.

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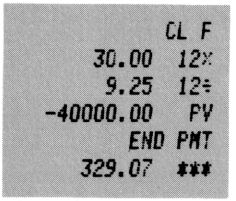
HOW TO READ THIS HANDBOOK

HP-92 Real Estate problem solutions in this book are presented in step-by-step keystroke form. The general procedure is shown first, followed by an example. To clarify the examples, intermediate results are shown and comments explaining the displayed answer are given where needed. In most cases the resulting tape printout is also shown.

The keys n, i, PV, FV and PMT which quickly, easily and accurately solve four and five variable compound interest problems are used extensively throughout this book. Proper data entry for PV FV and PMT requires that the user format each problem using the *cash flow sign convention* which is covered in the Owner's Handbook. Because the cash flow sign convention depends upon the point of view of the user it is not possible to always specify in general keystroke procedures the proper cash flow sign to be used. This is left to the user. It is therefore important for the user of this handbook to become familiar with Section 3 Financial Interest Calculations: Compound Interest Calculations and the Cash Flow Diagram in the Owner's Handbook before working the procedures presented in the following pages.

Problems and solutions appear throughout this book as follows:
Example—What is the monthly payment amount for a 30-year, fully amortized, level-monthly payment $40,000 mortgage at 9.25%?

CL FIN
30 [12×]
9.25 [12÷]
40000 CHS PV
PMT
The monthly payment is $329.07.



Note the keys [12×] and [12÷] do not appear on the HP-92 as individual keys but as gold labels beneath the n and i keys. In order to activate the computation denoted by gold (or blue) key labels you must first press the appropriate gold (or blue) key on the financial keyboard. The colored prefix keys are not indicated in the keystroke procedures in this book for conciseness. For further explanation of the operation of prefix keys see the Owner's Handbook page 15.

In all examples in this book the following initial conditions are applicable unless otherwise specified.

Payments occur at the END of the time period BEGIN BOND ▮▮▮ END NOTE The printer is in the ALL position MAN ALL ▮▮▮ NORM. The display is set to two decimal places g [2]. The position of the calendar switch 360 ▮▮▮ 365 is not relevant.

Now you're ready to start saving hours of calculation time.

REAL ESTATE SOLUTIONS GUIDE

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CHAPTER 1

COMPOUND INTEREST AND DISCOUNT FACTORS

Whenever a real estate analyst requires a compound interest or discount factor (any of the six functions of money at interest) for use in real estate problem-solving, it can be produced quickly and accurately on the HP-92. Printed tables of precalculated factors are rendered unnecessary because the HP-92 can produce anything—and more—that is available in any printed set of financial tables, no matter how voluminous or detailed they may be. Moreover, the analyst can usually calculate the factor in less time on the HP-92 than it takes to look up the factor in a printed set of tables.

Compounded interest and discount factor calculations on the HP-92 have the following characteristics (and advantages) for the real estate analyst:

1. Factors can be calculated for virtually any number of compounding periods and any rate of interest or discount.
2. Fractional time periods and interest (discount) rates can be employed, with accuracy to 10 decimal places.
3. The printer will provide 0-9 decimal places (see Owner's Handbook section 8).
4. Regardless of the number of decimal places set and displayed, the HP-92 retains and stores 10 decimal places for further calculations.
5. Unless the problem specifically calls for the factor alone, the HP-92 calculates answers with dollar amounts directly, thereby reducing calculating time as well as opportunities to make an operator error.

FUTURE WORTH OF ONE
(Compound Amount of One; Accumulation of One)

FV is the unknown. The known values are **n**, **i**, and **PV** which may be keyed in any order.

The formula is:

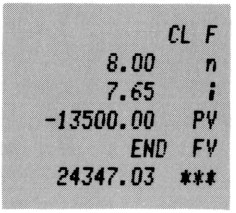
$$FV = PV\ (1 + i)^n$$

1. Key in (in any order) using the cash flow sign convention.
   a. Number of compounding periods, press **n**.
   b. Interest rate per period, press **i**.
   c. Present value (initial investment), press **PV**.
2. To calculate future value (future worth), press **FV**.

Example—An investor purchased a parcel of land 8 years ago for $13,500. Ignoring holding costs, how much must the property resell for in order for the investor to earn 7.65% per year?

CL FIN
8 n
7.65 i
13500 CHS PV
FV

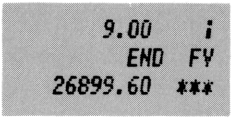
$24,347.03



To find the required resale price at 9% interest

9 i
FV

$26,899.60



FUTURE WORTH OF ONE PER PERIOD (Accumulation of One Per Period)

FV is the unknown. The known values are n, i and PMT, which may be keyed in any order.

The formula is:

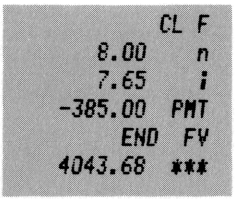
$$FV = PMT\frac{(1 + i)^n - 1}{i}.$$

1. Key in (in any order) using the cash flow sign convention.
   a. Number of compounding periods, press n.
   b. Interest rate per period, press i.
   c. Payment per period (at end of period), press PMT.
2. To calculate future value (future worth), press FV.

Example—An investor holding a parcel of land producing no income paid $385 per year in taxes (at the end of each year). At the end of 8 years, how much must be recovered on resale for the investor to earn 7.65% per year on his payments?

CL FIN
8 n
7.65 i
385 CHS PMT
FV

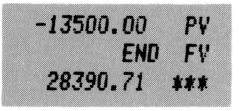
$4043.68



To combine the two previous examples and derive the resale price required to earn 7.65% on the original investment of $13,500 and the 8 annual tax payments of $385 it is necessary only to key in the original investment and solve for FV. n, i, and PMT remain stored in the financial memories from the previous solution and therefore do not have to be re-entered.

13500 CHS PV
FV

Total required resale price is $28,390.71.



SINKING FUND FACTOR
(Payment Amount For a Sinking Fund)

PMT is the unknown. The known values are n, i and FV, which may be keyed in any order.

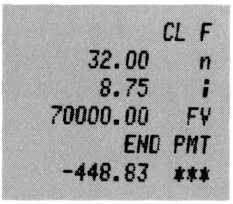
The formula is:

$$PMT = FV\frac{i}{(1 + i)^n - 1}.$$

1. Key in (in any order) using the cash flow sign convention.
   a. Number of compounding periods, press n.
   b. Interest rate per period, press i.
   c. Future value (future worth), press FV.
2. To calculate sinking fund payment, press PMT.

Example—An investor paid $70,000 for a building with an estimated remaining economic life of 32 years. What amount must be set aside annually at the end of each year to recover the full investment in the building over the remaining economic life, if the annual payments can accumulate at 8.75%, compounded annually at the end of each year?

CL FIN
32 n
8.75 i
70000 FV
PMT



The annual sinking fund payment is $448.83.

PRESENT WORTH OF ONE (Reversion Factor)

PV is the unknown. The known values are n, i and FV, which may be keyed in any order.

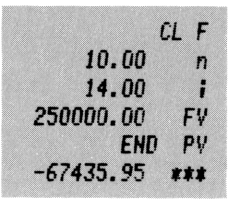
The formula is:

$$PV = FV\frac{1}{(1 + i)^n}$$

1. Key in (in any order) using the cash flow sign convention.
   a. Number of compounding periods, press n.
   b. Interest rate per period, press i.
   c. Future value (future worth), press FV.
2. To calculate present value (present worth), press PV.

Example 1—An income property is forecast to be worth $250,000 ten years hence. If 14% is regarded as an appropriate annual rate of return to compensate an investor for waiting and risk-taking, what should an investor pay for it today?

CL FIN
10 n
14 i
250000 FV
PV

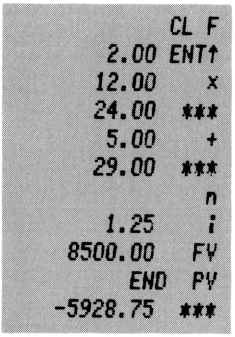


The investor should pay $67,435.95.

Example 2—A parcel of land recently sold for $8500. Market evidence indicates that competive land values have been increasing at 1.25% per month. What was it worth 2 years and 5 months ago, when the then-owner died?

CL FIN
2 ENTER↑
12 ×
5 +
n
1.25 i
8500 FV
PV

The land was worth $5928.75.



PRESENT WORTH OF ONE PER PERIOD (Level Annuity; Inwood Factor)

PV is the unknown. The known values are n, i and PMT, which may be keyed in any order.

The formula is:

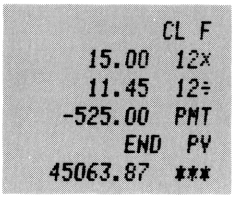
$$PV = PMT \left[\frac{1 - (1 + i)^{-n}}{i} \right]$$

1. Key in (in any order) using the cash flow sign convention.
   a. Number of compounding periods, press n.
   b. Interest rate per period, press i.
   c. Payment per period, press PMT.
2. To calculate present value (present worth), press PV.

Example—A 15-year lease calls for monthly rental payments of $525, payable at the end of each month. What is the present worth of the rental stream, discounted at 11.45%?

CL FIN
15 12×
11.45 12÷
525 CHS PMT
PV

The present worth of the rental stream is $45,063.87.



PRESENT WORTH OF LEVEL ANNUITY PLUS REVERSION

PV is the unknown. The known values are n, i, PMT and FV, which may be keyed in any order. This procedures combines Present Worth of One per Period and Present Worth of One routines.

The formula is:

$$PV = PMT\left[\frac{1 - (1 + i)^{-n}}{i}\right] + \frac{FV}{(1 + i)^n}$$

1. Key in (in any order) using the cash flow sign convention.
   a. Number of compounding periods, press n.
   b. Interest rate per period, press i.
   c. Payment per period, press PMT.
   d. Future value (future worth), press FV.
2. To calculate present value (present worth), press PV.

Example—A property is forecast to produce NOI of $21,750 annually for 10 years, with Annual Debt Service of $14,653. The property is expected to resell for $230,000 in 10 years. Sales commission and other disposition expenses paid by the seller in 10 years are estimated to be 7% of sales price. The mortgage balance in 10 years will be $122,175. If a purchaser can assume the existing mortgage, how much should he pay in cash for the equity investment position so as to reach 14% on that investment?

CL FIN
10 n
14 i
21750 ENTER↑
14653 −
PMT
230000 ENTER↑
7 %
−
122175 −
FV
PV

	CL F
10.00	n
14.00	i
21750.00	ENT↑
14653.00	-
7097.00	***
	PMT
230000.00	ENT↑
7.00	%
16100.00	***
	-
213900.00	***
122175.00	-
91725.00	***
	FV
END	PV
-61761.02	***

Present Worth of the equity investment position is $61,761.02. To earn the desired 14%, he should pay no more than this.

INSTALLMENT TO AMORTIZE ONE (Amortization Payment)

PMT is the unknown. The known values are n, i and PV, which may be keyed in any order.

The formula is:

$$PMT = \frac{PV}{\left[\frac{1 - (1 + i)^{-n}}{i}\right]}$$

1. Key in (in any order) using the cash flow sign convention.
   a. Number of compounding periods, press n.
   b. Interest rate per period, press i.
   c. Present value (present worth), press PV.
2. To calculate payment per period, press PMT.

Example 1—What monthly payment (principal plus interest) with fully amortize a mortgage of $45,500 in 22 years and 8 months, at 8.5% interest?

CL FIN
22 12×
8 +
n
8.5 12÷
45500 PV
PMT

	CL F
22.00	12×
8.00	+
272.00	***
	n
8.50	12÷
45500.00	PV
	END PMT
-377.67	***

The monthly payment is $377.67.

Example 2—What is the mortgage constant (f) on a 20-year, level-monthly payment, fully amortized mortgage with a 9.25% interest rate?

CL FIN
g 6
20 12×
9.25 12÷
1 CHS PV
PMT
12 ×

	CL F
20.000000	12×
9.250000	12÷
-1.000000	PV
	END PMT
0.009159	***
12.000000	×
0.109904	***

The annual constant is .109904 or 10.9904%.

SOLVING FOR INTEREST RATE, DISCOUNT RATE, RATE OF RETURN (i)

The HP-92 automatically calculates the interest rate (or rate of discount, or rate of return) per period, as an Internal Rate of Return.

The unknown is **i**. One of the known values must be **n**. **PV**, **FV**, and **PMT** are the other possible known values.

Example 1—Future Worth of One, Present Worth of One—A house was purchased for $41,990 four years ago. It just resold for $53,500. What annual rate of interest did the owner earn (ignoring holding and disposition costs)?

CL FIN
41990 **CHS** **PV**
53500 **FV**
4 **n**
i

6.24% per year.

```
                CL F
-41990.00        PV
53500.00         FV
    4.00          n
     END          i
    6.24         ***
```

Example 2—Future Worth of One Per Period, Sinking Fund Payment—An investment in a building was $170,000. The investor has been setting aside $1600 per year to provide for full recovery of the $170,000 in 25 years. What is the indicated level-annuity Capital Recovery Rate (or implicit reinvestment rate)?

CL FIN
170000 **FV**
1600 **CHS** **PMT**
25 **n**
i

10.51% per year.

```
                CL F
170000.00        FV
-1600.00        PMT
   25.00          n
     END          i
   10.51         ***
```

Example 3—Present Worth of One Per Period; Installment To Amortize One. A $75,000 fully amortized loan has level monthly payments of $637.13. The maturity is 22 years 5 months; What is the annual interest rate?

CL FIN
75000 **PV**
637.13 **CHS** **PMT**
22 **12×**
5 **+**
n
i
12 **×**

8.75% annual interest rate.

```
                CL F
75000.00         PV
-637.13         PMT
   22.00        12×
    5.00          +
  269.00         ***
                 n
     END          i
    0.73         ***
   12.00          ×
    8.75         ***
```

CHAPTER 2

RATES OF RETURN AND RATES OF CAPITALIZATION

There are three types of rates of capitalization used in real estate appraisal and investment analysis.

The first group includes Rates of Return on the investment. These are the Discount Rate or Basic Rate (r) on the total property investment; the Equity Yield Rate (y) on the equity investment; and the Mortgage Interest Rate (i) on the mortgage investment or principal. These are all calculated as Internal Rates of Return over a specified time span.

The second category includes Capital Recovery Rates, providing for recovery of forecast capital loss over a specified time span. All are calculated as a Sinking Fund Factor at specified rate of discount (IRR), which may be the Rate of Return, a lower rate, or zero.

The third category are Capitalization Rates, which are the sum of a given Rate of Return and its associated Capital Recovery Rate. They are rates used to capitalize level-annuity income streams to Present Worth or Value figures. They include the Capitalization Rate, applied to NOI for property value estimates; the Equity Dividend Rate (e), applied to CTO for equity investment valuation; and the Mortgage Constant (f), applied to Annual Debt Service for mortgage value estimation. All are calculated using the Installment to Amortize One routine.

All rates of return and rates of capitalization for real estate problems are calculated and expressed as annual rates.

DISCOUNT RATE— BAND OF INVESTMENT

When non-amortized mortgages are involved, the Discount Rate or Basic Rate (r) is found by the following formula:

$$r = m\,i + (1 - m)\,y$$

In this formula, m is the loan-to-value ratio of the mortgage, i is the annual mortgage interest rate, (l-m) is the equity investment (down payment) ratio, y is the equity yield rate.

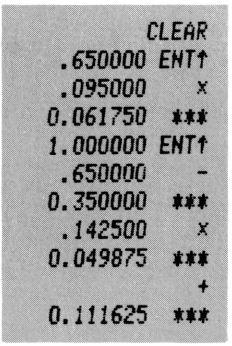
Example—An investor can obtain a 65% mortgage on a property he is planning to purchase, with interest at 9.5%. The investor is looking for at least a 14.25% return on his equity. What Discount or Basic Rate is required to meet these standards?

Note:

Enter all figures as decimals.

CLEAR
g 6
.65 ENTER↑
.095 ×
1 ENTER↑
.65 −
.1425 ×
+

r = .111625 or
11.1625%



CAPITAL RECOVERY RATE

Capital Recovery Rates apply particularly to investments in improvements (buildings) in total property valuation and analysis. They may be calculated on a straight-line basis (which is the same as the Sinking Fund Factor at a zero rate), a "sinking fund" basis (which is the Sinking Fund Factor at the "safe" rate), and a level-annuity basis (which is the Sinking Fund Factor at the Discount or Base Rate.)

The procedures all use the Sinking Fund Factor routine illustrated in Chapter 1.

Example—An investment property includes a building which has an estimated Remaining Economic Life of 28 years. What is the annual Capital Recovery Rate:

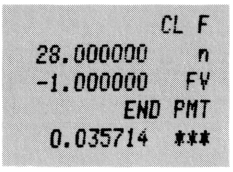
on a straight-line basis?

on a sinking fund basis at a "safe" rate of 5%?

on a level annuity basis if the discount rate (r) is 10.45%?

Straight-Line Capital Recovery:

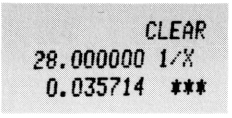
CL FIN
g 6
28 n
1 CHS FV
PMT



CRR

or CRR $= \frac{1}{n}$

CLEAR
28 1/x



CRR

"Safe"-Rate Capital Recovery:

CL FIN
28 n
5 i
1 CHS FV
PMT

```
          CL F
28.000000   n
 5.000000   i
-1.000000  FV
       END PMT
 0.017123 ***
```

CRR

Level-Annuity Capital Recovery With Sinking Fund at "Safe"-Rate:

CL FIN
28 n
10.45 i
1 CHS FV
PMT

```
          CL F
28.000000   n
10.450000   i
-1.000000  FV
       END PMT
 0.006890 ***
```

CRR

CAPITALIZATION RATE

Capitalization Rates are applied to the amount of property investment (typically buildings) to be fully recovered via periodic income payments over the income projection period. They are calculated according to the method of providing for capital recovery: straight-line sinking-fund at the "safe" rate, or level annuity.

As noted in an earlier section, Capitalization Rates are equal to the sum of a Discount Rate plus the appropriate Capital Recovery Rate.

Example—An income-producing property contains buildings with an estimated remaining economic life of 45 years. The indicated Discount Rate (r) applicable to this investment is 9.85%. What is the annual Capitalization Rate:

with straight-line capital recovery?

with sinking-fund capital recovery at a 5.25% "safe" rate?

with level-annuity capital recovery?

Note:

For arithmetic functions, enter the Discount Rate as a decimal figure.

Straight-Line Capital Recovery:

CL FIN
g 6
45 n
1 CHS FV
PMT
.0985 +

```
          CL F
45.000000   n
-1.000000  FV
       END PMT
 0.022222 ***
  .098500   +
 0.120722 ***
```

CR

Sinking-Fund Capital Recovery At "Safe" Rate:

CL FIN
45 n
5.25 i
1 CHS FV
PMT
.0985 +

```
          CL F
45.000000   n
 5.250000   i
-1.000000  FV
       END PMT
 0.005833 ***
  .098500   +
 0.104333 ***
```

CR

Level-Annuity Capital Recovery:

CL FIN
45 n
9.85 i
1 CHS PV
PMT

```
          CL F
45.000000   n
 9.850000   i
-1.000000  PV
       END PMT
 0.099958 ***
```

CR

or

CL FIN
45 n
9.85 i
1 CHS FV
PMT
.0985 +

```
          CL F
45.000000   n
 9.850000   i
-1.000000  FV
       END PMT
 0.001458 ***
  .098500   +
 0.099958 ***
```

CR

RATE OF RETURN ON LEVEL ANNUITY PLUS REVERSION

Example—An investment property has just been purchased for $60,000. NOI is forecast to be $8,500 per year for 15 years. Resale proceeds are forecast at $75,000 in 15 years. If the investor holds the property for the full 15 years, and expectations about NOI and Resale Proceeds are fully realized, what rate of return will he earn on the investment?

CL FIN
15 n
8500 PMT
60000 CHS PV
75000 FV
i

```
          CL F
 15.00      n
8500.00   PMT
-60000.00  PV
75000.00   FV
       END   i
 14.70 ***
```

Investor's annual yield.

CHAPTER 3

MORTGAGE-EQUITY (ELLWOOD) ANALYSIS

Analyzing and appraising real estate investment properties in terms of their mortgage and equity investment components constitutes Mortgage-Equity Analysis. It was formalized and popularized by the late L.W. Ellwood. This is why it is frequently referred to as "Ellwood Analysis."

This framework of analysis is used to estimate the Present Worth (Market Value or Investment Value) of the total property investment and of the equity investment position. Property value is estimated by capitalizing Net Operating Income at the Overall Rate:

$$V = \frac{NOI}{R}$$

Present Worth of the equity investment position is estimated by capitalizing Cash Throw-off to equity at the Equity Dividend Rate:

$$Ve = \frac{CTO}{Re}$$

The Mortgage-Equity framework is also used to estimate the dollar amount of resale proceeds (PR), or the percentage of increase (app.) or decrease (dep.) in resale proceeds over initial investment (Capital Outlay), required to achieve a given Basic Rate (r) or Equity Yield Rate (y).

Finally, the analysis can be used to calculate the Basic Rate (r) on the total property investment, or the Equity Yield Rate (y) on the equity investment.

If all figures were available in dollar amounts, it would be unnecessary to have a separate Mortgage-Equity framework. However, often the dollar value of Present Worth, Resale Proceeds (reversion) and Mortgage Principal are unknown. Only NOI is given as a dollar figure, with mortgage loan terms and capital gain (app.) or loss (dep.) on resale given as percentages. Thus it is necessary to calculate the Basic Rate (r) and the Overall Rate (R) to apply to NOI to estimate value.

Note:

Specifications for the income stream are that NOI be a level annuity. The total income stream is thus a level annuity plus a reversion receivable at the end of the payment period. Also, all cash flows (NOI, ADS and CTO) are before-tax cash flows, and all rates of return (r, i and y), as well as all capitalization rates (R, R and R_e) are before-tax annual rates.

CALCULATION OF BASIC RATE AND OVERALL RATE

The basic formulas are (see the appendix for definitions of all symbols):

Basic Rate:

$$r = mf + (1 - m)y - mp\ 1/s_n$$

Overall Rate:

$$R = r + \text{dep. } 1/s_n$$

$$R = r - \text{app. } 1/s_n$$

$$R = mf + (1 - m)y - mp\ 1/s_n \begin{matrix} +\text{dep. } 1/s_n \\ -\text{app. } 1/s_n \end{matrix}$$

The given values required are:

i = mortgage interest rate

m = loan-to-value ratio of mortgage

n_t = total number of mortgage payments to full amortization

y = equity yield rate

n_p = income projection period (investment holding period)

dep./app. = capital loss or gain or resale as a percentage of present worth or value of property

With these values, it is then possible to calculate:

f = mortgage constant

$1/s_n$ = sinking fund factor at the equity yield rate over the income projection period

p = percentage of mortgage principal paid off over the income projection period

After these values are calculated, r and R can be calculated.

Example—To illustrate all the required calculations to derive the Basic Rate (r) and the Overall Rate (R), the following conditions are assumed: An investor plans to purchase an income property, hold it for 10 years, and then resell it. It is estimated that the proceeds of resale will result in a 15% capital loss. A 25-year mortgage loan with level monthly payments at 8.75% interest can be obtained, with a loan-to-value ratio of 70%. The investor is seeking a 14% yield on his equity investment.

Thus,

i = 8.75% or .0875

m = 70% or .70

n_t = 25 years or 300 months

y = 14% or .14

n = 10 years

dep. = 15% or .15

Calculation of Mortgage Constant (f):

CL FIN	CL F	
g 6	25.000000 12x	
25 12×	8.750000 12÷	
8.75 12÷	.700000 PV	P
.7 PV	END PMT	
PMT	-0.005755 ***	
12 ×	12.000000 x	
STO 0	-0.069060 ***	f
	→ 0	

Calculate Mortgage Balance (b) at end of Income Projection Period (n):

10 12×	10.000000 12x	
FV	END FV	
1 +	-0.575818 ***	b
	1.000000 +	
	0.424182 ***	p

Calculation of Basic Rate (r):

CL FIN	CL F	
FV	FV	
10 n	10.000000 n	
14 i	14.000000 i	
.3 CHS PV	-.300000 PV	
PMT	END PMT	
RCL 0	0.035578 ***	
−	← 0	
1 ÷ *	-0.069060 ***	
	-	
	0.104638 ***	
	1.000000 ÷	V
	0.104638 ***	r

Calculation of Overall Rate (R)—Ellwood Format:

1 ENTER↑	1.000000 ENT↑	
15 %	15.000000 %	
RCL FV	0.150000 ***	
x≷y −	← FV	
FV	0.424182 ***	
	X≷Y	
PMT	0.150000 ***	
RCL 0	−	
−	0.274182 ***	NCPR
1 ÷ *	FV	
	END PMT	
	0.043335 ***	
	← 0	
	-0.069060 ***	
	−	
	0.112395 ***	
	1.000000 ÷	V
	0.112395 ***	R

* When dealing with factors this step appears redundant, however when using dollar values this step must be included to calculate both r and R.

CALCULATION OF VALUE (PRESENT WORTH) WITH R, GIVEN ONLY NOI

When R is calculated, as above, Value is estimated by the formula:

$$V = \frac{NOI}{R}$$

Example 1—In the preceding examples R = .112395. If NOI is forecast at $33,500, what is the estimated value or present worth of the property?

33500 ENTER↑	33500.00 ENT↑	
.112395 ÷	.112395 ÷	
	298055.96 ***	V or PW

V or PW = $298,055.96 (probably rounded to $300,000).

Example 2—A property is forecast to produce NOI of $24,550 annually. The most probable mortgage loan terms are an 82% loan with level monthly payments at 9.25% interest over a maturity of 22 years 8 months. The investor expects to hold the property for 12 years and then sell it at 20% above its present value. If the investor is looking for a 15.35% rate of return on equity investment, what is the value (present worth) of the property? What is the indicated present worth of the equity investment position?

The given values are:

n_t = 272 months
n = 12 (years)
m = .82
app. = .20 or 20%
y = .1535
NOI = $24,550
i = .0925 or 9.25%

To calculate and store f:

CL FIN
g 6
22 ENTER↑
12 ×
8 +
n
9.25 12÷
.82 PV
PMT
12 ×
STO 0

```
                CL F
  22.000000 ENT↑
  12.000000    x
 264.000000  ***
   8.000000    +
 272.000000  ***
               n
   9.250000  12÷
    .820000   PV      P
         END PMT
  -0.007214  ***
  12.000000    x
  -0.086573  ***     f
             → 0
```

To calculate b and NCPR:

12 12×
FV
1 ENTER↑
20 %
+
+

```
  12.000000  12×
         END  FV
  -0.585670  ***     b
   1.000000 ENT↑
  20.000000    %
   0.200000  ***
               +
   1.200000  ***     CPR
               +
   0.614330  ***     NCPR
```

To calculate R:

CL FIN
FV
12 n
15.35 i
.18 CHS PV
PMT
RCL 0
−

```
                CL F
                  FV
  12.000000    n
  15.350000    i      y
   -.180000   PV      CO
         END PMT
   0.012974  ***
             → 0
  -0.086573  ***
               -
   0.099547  ***     R
```

To calculate V:

24550 [x≷y]
[÷]

24550.00000	X≷Y	NOI
0.099547	***	
	÷	
246618.1479	***	V

To calculate V_e, P:

[ENTER↑]
[ENTER↑]
[RCL] [PV]
[×]
[+]

	ENT↑	
	ENT↑	
	÷ PV	
-0.180000	***	
	×	
-44391.26662	***	V_e
	+	
202226.8813	***	P

CALCULATION OF EQUITY DIVIDEND RATE (e)

The Equity Dividend Rate (e) is applied directly to Cash Throw-Off to Equity to find the present worth of the equity investment position:

$$V_e = \frac{CTO}{R_e}$$

The Equity Dividend Rate is calculated when CTO and the amount of the equity investment are known in dollar amounts by the formula:

$$e = \frac{CTO}{V_e}$$

Example—The equity investment in an income property is $44,391 NOI is forecast at $24,550, while Annual Debt Service is $21,350. What is the indicated Equity Dividend Rate (e)?

[g] [6]
24550 [ENTER↑]
21350 [−]
44391 [÷]

24550.00000	ENT↑	
21350.00000	-	
3200.000000	***	
44391.00000	÷	
0.072087	***	R_e

When dollar amounts are not available, the Equity Dividend Rate can be calculated with all the data used to calculate R, as illustrated in the preceding examples.

Example—An income property has an 82% mortgage with level monthly payments at 9.25% interest fully amortized in 22 years 8 months. The equity investor is seeking a 15.35% Equity Yield Rate over the income projection of 12 years. What is the indicated Equity Dividend Rate, if the proceeds of resale are forecast to be 20% above present value of the property?

Calculate and store f:

CL FIN	CL F	
g 6	22.000000 ENT↑	
22 ENTER↑	12.000000 ×	
12 ×	264.000000 ***	
8 +	8.000000 +	
n	272.000000 ***	
	n	
9.25 12÷	9.250000 12÷	
.82 PV	.820000 PV	P
PMT	END PMT	
12 ×	-0.007214 ***	
	12.000000 ×	
STO 0	-0.086573 ***	f
	→ 0	

Calculate b, CPR and NCPR:

12 12×	12.000000 12×	
FV	END FV	
	-0.585670 ***	b
1 ENTER↑	1.000000 ENT↑	
20 %	20.000000 %	
+	0.200000 ***	
	+	
	1.200000 ***	CPR
+	+	
	0.614330 ***	NCPR

Calculate CTO, R_e:

CL FIN	CL F	
FV	FV	
12 n	12.000000 n	
15.35 i	15.350000 i	
.18 CHS PV	-.180000 PV	
PMT	END PMT	
	0.012974 ***	CTO
	÷ PV	
RCL PV	-0.180000 ***	
CHS	CHS	
÷	0.180000 ***	
	÷	
	0.072078 ***	R_e

CALCULATION OF VALUE (PRESENT WORTH) WITH DOLLAR AMOUNTS GIVEN

This procedure involves calculating the present worth of the future income stream and reversion, to derive the present worth of the investment. It can be used to estimate property value using NOI, the discount or basic rate (r), and the Proceeds of Resale. It can also be used to estimate the present worth of the equity investment position using CTO, y, and the Net Cash Proceeds of Resale.

Example 1—An income property is forecast to produce NOI of $20,575 per year. It has just been financed with a $160,000 mortgage, to be fully amortized in level monthly payments at 8.75% interest over 25 years. The anticipated proceeds of resale of the property in 10 years is $191,250. The equity investor expects an Equity Yield Rate of 14.75%.

What is the present worth of the equity investment position? What is the present worth (value) of the property?

Calculate Annual Debt Service:

CL FIN	CL F	
25 12×	25.00 12×	
8.75 12÷	8.75 12÷	
160000 PV	160000.00 PV	
PMT	END PMT	
12 ×	-1315.43 ***	
	12.00 ×	
STO 0	-15785.16 ***	ADS
	→ 0	

Calculate Mortgage Balance in 10 Years:

10 12×	10.00 12×	
FV	END FV	
	-131615.55 ***	b

Calculate Net Cash Proceeds of Resale:

191250 +	191250.00 +	CPR
	59634.45 ***	NCPR
CL FIN	CL F	
FV	FV	

Calculate Equity Investment Position, Present Worth (Value:)

10 n	10.00 n	
14.75 i	14.75 i	y
20575 RCL 0	20575.00 ↓ 0	NOI
	-15785.16 ***	ADS
+	+	
	4789.84 ***	CTO
PMT	PMT	
	END PV	
PV	-39334.99 ***	V_e
	160000.00 -	P
160000 −	-199334.99 ***	V or PW

CALCULATION OF CAPITAL APPRECIATION OR DEPRECIATION ON RESALE, PLUS RESALE PRICE REQUIRED TO ACHIEVE A GIVEN EQUITY YIELD RATE:

The percent of capital appreciation or depreciation on resale required to achieve a given equity yield rate can be calculated using either rates or dollar amounts. In addition, when dollar amounts are available, it is possible to calculate the dollar amount of resale proceeds required. The calculations can be applied to either NOI or CTO cash flows.

a. Calculation of dep. or app. Using Rates

The formula for dep. or app. is:

$$\% \text{ app. or dep.} = \left(\frac{\text{CPR}}{\text{V}} - 1 \right) \times 100$$

Where positive results represent app., negative results dep.

Example—An investment is producing NOI at an Overall Rate (R) of 10.25%. It has just been financed with a 70% mortgage at 9% interest, fully amortized in level monthly payments over 20 years. What must resale proceeds be at the end of 10 years for the investor to earn a 13% rate of return on the equity investment?

Calculate f, CTO and store:

CL FIN	CL F	
g 6	20.000000 12x	
20 12×	9.000000 12÷	
9 12÷	.700000 PV	P
.7 PV	END PMT	
PMT	-0.006298 ***	
12 ×	12.000000 x	
	-0.075577 ***	f
.1025 +	.102500 +	NOI
	0.026923 ***	CTO
STO 0	→ 0	

Calculate and store b:

10 12×	10.000000 12x	
FV	END FV	
	-0.497181 ***	b
STO 1	→ 1	

Calculate NCPR and CPR:

CL FIN	CL F	
10 n	10.000000 n	
RCL 0	← 0	
	0.026923 ***	CTO
PMT	PMT	
13 i	13.000000 i	y
.30 CHS PV	-.300000 PV	1 − m
FV	END FV	
RCL 1	0.522455 ***	NCPR
	← 1	
−	-0.497181 ***	b
	-	
	1.019636 ***	CPR

Calculate app. or dep.:

1 x≷y	1.000000 X≷Y	V
△%	1.019636 ***	
	△%	
	1.963618 ***	% app.

What if the desired Equity Yield Rate is 16%?

Calculate NCPR and CPR:

16 [i]	16.000000 i	y
[FV]	END FV	
	0.749392 ***	NCPR
[RCL] [1]	÷ 1	
	-0.497181 ***	b
[−]	−	
	1.246573 ***	CPR

Calculate app. or dep.:

1 [x≷y]	1.000000 X≷Y	V
[Δ%]	1.246573 ***	
	Δ%	
	24.657342 ***	% app.

To earn an Equity Yield Rate of 13%, Proceeds of Resale must be 1.96% higher than the original purchase price or value.

To earn an Equity Yield Rate of 16%, Proceeds of Resale must be 24.66% higher than the original purchase price or value.

b. Calculations of dep. or app. Using Dollar Figures

These procedures center on the calculation of what the dollar amount of the reversion (PR or NCPR) must be to achieve a given or desired rate of return (r or y).

In one procedure, the net amount of Future Worth is derived as the amount of the reversion. In another procedure, the net amount of Present Worth of the investment position not covered by periodic income is derived, and the amount of reversion required to cover that net amount of Present Worth is then calculated.

Example 1—Future Sales Price, Amount of Equity Reversion and app./dep. Required to Achieve a Given Equity Yield Rate.

A investment property is for sale for $100,000. It is expected to produce NOI of $11,000 per year. It can be financed with a $70,000 mortgage at 9% interest, fully amortized in level monthly payments over 20 years. What must the property sell for in 10 years for the investor to earn a 13% rate of return (y) on the equity investment? What must the equity reversion be? What percentage of dep. or app. is involved?

Calculate and store ADS:

CL FIN	CL F	
20 12×	20.00 12x	
9 12÷	9.00 12÷	
70000 PV	70000.00 PV	
PMT	END PMT	
12 ×	-629.81 ***	
	12.00 x	
STO 0	-7557.70 ***	ADS
	→ 0	

Calculate and store b:

10 12×	10.00 12x	
FV	END FV	
	-49718.12 ***	b
STO 1	→ 1	

Calculate NCPR, CPR:

CL FIN	CL F	
10 n	10.00 n	
13 i	13.00 i	
11000 RCL 0	11000.00 ← 0	NOI
	-7557.70 ***	ADS
+	+	
	3442.30 ***	CTO
PMT	PMT	
30000 CHS PV	-30000.00 PV	CO
FV	END FV	
	38430.68 ***	NCPR
	← 1	
RCL 1	-49718.12 ***	b
	-	
−	88148.81 ***	CPR

Calculate app. or dep.:

100000 x≷y	100000.00 X≷Y	V
Δ%	88148.81 ***	
	Δ%	
	-11.85 ***	% dep.

Continuing with this example, if the investor had instead desired a 15% rate of return on his equity investment, what must the cash proceeds from the sale of the property be in 10 years? Appreciation or depreciation?

Calculate NCPR, CPR:

15 i	15.00 i	y
FV	END FV	
	51475.20 ***	NCPR
RCL 1	÷ 1	
	-49718.12 ***	b
−	−	
	101193.33 ***	CPR

Calculate app. or dep.:

100000 x≷y	100000.00 X≷Y	V
	101193.33 ***	CPR
△%	Δ%	
	1.19 ***	% app.

Example 2—Future Sales Price (Resale Proceeds) and Percentage app. or dep. Required to Achieve a Given Discount Rate (r).

An investment property was recently acquired for $65,800. NOI is forecast to be $6,350 per year. What must it resell for (net) in 12 years to produce a rate of return (r) of 10.45% on the total property investment? What percentage app. or dep. over the original purchase price does this represent?

CL FIN	CL F	
12 n	12.00 n	
10.45 i	10.45 i	
6350 PMT	6350.00 PMT	
65800 CHS PV	-65800.00 PV	
FV	END FV	
	77359.15 ***	
CHS	CHS	
	-77359.15 ***	PR
△%	Δ%	
	17.57 ***	% app.

Rather than purchasing the building outright, if the investor had instead financed the property with 20% down and the balance at 8.75% for 25 years, what must the property app./dep. at to earn the desired 10.45%?

Calculate and store ADS:

CL FIN	CL F	
25 12×	25.00 12×	
8.75 12÷	8.75 12÷	
65800 ENTER↑	65800.00 ENT↑	
20 %	20.00 %	1 − m
	13160.00 ***	CO
−	-	
	52640.00 ***	P
PV	PV	
PMT	END PMT	
12 ×	-432.78 ***	
	12.00 ×	
	-5193.32 ***	ADS
STO 0	→ 0	

Calculate and store b:

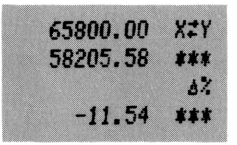
12 12×	12.00 12×	
FV	END FV	
	-40244.02 ***	b
STO 1	→ 1	

Calculate NCPR, CPR:

CL FIN	CL F	
12 n	12.00 n	
10.45 i	10.45 i	
6350 RCL 0	6350.00 ← 0	NOI
	-5193.32 ***	ADS
+	+	
	1156.68 ***	CTO
PMT	PMT	
65800 ENTER↑	65800.00 ENT↑	
20 %	20.00 %	
CHS	13160.00 ***	
	CHS	
	-13160.00 ***	CO
PV	PV	
FV	END FV	
	17961.56 ***	NCPR
RCL 1	← 1	
	-40244.02 ***	
−	-	
	58205.58 ***	CPR

Calculate app. or dep.:

65800 [x⇄y]
[Δ%]



V

% dep.

CALCULATION OF EQUITY YEILD RATE (y) FROM DOLLAR FIGURES

The income stream conventionally forecast in Mortgage-Equity or Ellwood Analysis is a level annuity plus a reversion. It is either a level NOI flow plus Proceeds of Resale, or a level CTO flow plus NCPR.

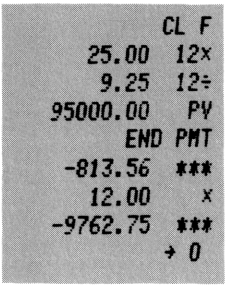
The equity yield rate (y) can be calculated on the equity investment using CTO and NCPR; the discount rate or basic rate (r) can be calculated on the property investment using NOI and PR.

Example—An investor has just purchased an income property for $123,750. A mortgage of $95,000 was obtained financed at 9¼% for 25 years. NOI is forecast at $13,200. The investor plans to hold the property for 12 years and then resell it. Anticipated resale proceeds are $135,000. What is the indicated equity yield rate?

Calculate and store Annual Dept. Service (ADS):

[CL FIN]
25 [12×]
9.25 [12÷]
95000 [PV]
[PMT]
12 [×]

[STO] [0]

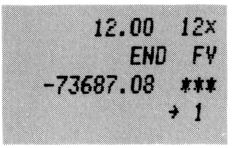


ADS

Calculate and store Mortgage Balance (b):

12 [12×]
[FV]

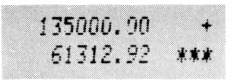
[STO] [1]



b

Calculate Net Cash Proceeds from Resale (NCPR):

135000 [+]



CPR

NCPR

Calculate Equity Yield Rate (y):

CL FIN	CL F	
FV	FV	
12 n	12.00 n	
13200 RCL 0	13200.00 ↓ 0	NOI
	-9762.75 ***	ADS
+	+	
	3437.25 ***	CTO
PMT	PMT	
123750 ENTER↑	123750.00 ENT↑	
95000 −	95000.00 −	P
CHS	28750.00 ***	
	CHS	
	-28750.00 ***	CO
PV	PV	
i	END i	
	15.70 ***	y

CHAPTER 4
INCOME PROJECTION AND ESTIMATION

In most real estate investment an valuation problems (among others), it is necessary to calculate the future income and expense flows that are utilized in appraisal, financing and investment analysis.

The HP-92 has an advantage in calculating Before-Tax Cash Flows in that sequential or chain calculation capabilities can be utilized to work from Potential Gross Income to Cash Throw-Off to Equity in one continuous operation. With After-Tax Cash Flows, however, the ability of the HP-92 to store values and to calculate schedules of depreciation and annual interest payments considerably shortens calculating time, as well as reducing the possibilities of manual entry error.

BEFORE-TAX CASH FLOWS

The several before-tax cash flows applicable to real estate analysis and problems are:

PGI: Potential Gross Income

EGI: Effective Gross Income

NOI: Net Operating Income (also called Net Income Before Recapture)

CTO: Cash Throw-Off to Equity (also called Gross Spendable Cash)

All are annual flows in real estate analysis.

These terms and symbols are further explained in the Appendix.

The derivation of these cash flows follows a set sequence:

1. Potential Gross Income is calculated by multiplying the rental per unit times the number of units, and that product times the number of rental payment periods per year. This gives what the property would generate in rental income if it were fully occupied.
2. Deduct Allowance for Vacancy and Rantal Loss. The result is Rent Collections, which is also Effective Gross Income if there is no "Other Income".
3. Add "Other Income", such as receipts from concessions (laundry equipment, etc.), which is produced from sources other than the rental of space. This produces Effective Gross income.
4. Deduct Operating Expenses. These are expenditures the landlord-investor must make, by contract or custom, to preserve the property and keep it capable of producing the forecast gross income. The result is Net Operating Income.
5. Deduct Annual Debt Service on the mortgage. This produces Cash Throw-Off to Equity.

Thus:

$$PGI - Vac + Other = EGI$$

$$EGI - OE = NOI$$

$$NOI - ADS = CTO$$

Example—A 60-unit apartment building has rentals of $250 per unit per month. Three units are currently vacant, which is a typical vacancy ratio for competitive properties. Concession income from coin-operated laundry equipment averages $6 per occupied unit per month.

Management fees are 3.5% of rent collections. Other operating expenses are: Property Taxes $27,350; Insurance $3,255; Repairs and Maintenance $14,285 plus a free apartment for the building superintendent; Utilities (sewer and water) $7,850; Heat and Air Conditioning $11,450; Replacements $3,975; Other (Miscellaneous) $3,125.

The property has just been financed with a $700,000 mortgage, fully amortized in level monthly payments at 9.5% interest over 20 years.

a. What is Effective Gross Income?

b. What is Net Operating Income?

c. What is Cash Throw-Off to Equity?

d. What is the Operating Expense Ratio? $\left(OER = \frac{OE}{EGI} \right)$

e. What is the Debt Service Coverage Ratio? $\left(DS\ COV = \frac{NOI}{ADS} \right)$

(a) Effective Gross Income:

60 ENTER↑	60.00 ENT↑	
250 ×	250.00 ×	
12 ×	15000.00 ***	
	12.00 ×	
ENTER↑	180000.00 ***	PGI
ENTER↑	ENT↑	
	ENT↑	
3	3.00 ENT↑	
ENTER↑	60.00 ÷	
60 ÷	0.05 ***	Vacancy ratio
	×	
×	9000.00 ***	
−	−	
STO 2	171000.00 ***	
57 ENTER↑	→ 2	
6 ×	57.00 ENT↑	
12 ×	6.00 ×	
	342.00 ***	
	12.00 ×	
	4104.00 ***	Laundry concessions

[+]		
	175104.00 ***	EGI
[STO] [3]	→ 3	

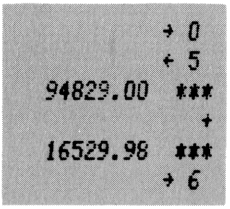
(b) Net Operating Income:

[RCL] [2]	÷ 2	
3.5 [%]	171000.00 ***	
	3.50 %	
27350 [+]	5985.00 ***	Management fees
	27350.00 +	Property taxes
3255 [+]	33335.00 ***	
	3255.00 +	Insurance
14285 [+]	36590.00 ***	
250 [ENTER↑]	14285.00 +	Repairs and maintenance
12 [×]	50875.00 ***	
	250.00 ENT↑	
	12.00 ×	
[+]	3000.00 ***	Building superintendent
7850 [+]	+	
11450 [+]	53875.00 ***	
3975 [+]	7850.00 +	Utilities
3125 [+]	61725.00 ***	
	11450.00 +	Heat, air conditioning
[STO] [4]	73175.00 ***	
[RCL] [3]	3975.00 +	Replacements
	77150.00 ***	
[x≷y]	3125.00 +	Miscellaneous
[−]	80275.00 ***	OE
	→ 4	
[STO] [5]	÷ 3	
	175104.00 ***	EGI
	X≷Y	
	80275.00 ***	
	−	
	94829.00 ***	NOI
	→ 5	

(c) Cash Throw-Off to Equity (CTO):

[CL FIN]	CL F	
20 [12×]	20.00 12×	
9.5 [12÷]	9.50 12÷	
700000 [PV]	700000.00 PV	
[PMT]	END PMT	
12 [×]	-6524.92 ***	
	12.00 ×	
	-78299.02 ***	ADS

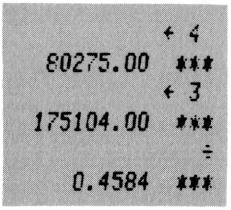
STO 0
RCL 5
+
STO 6



CTO

(d) Operating Expense Ratio:

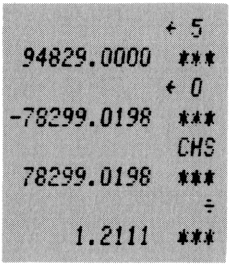
RCL 4
RCL 3
g 4
÷



OER

(e) Debt Service Coverage Ratio:

RCL 5
RCL 0
CHS
÷



DS COV

BEFORE-TAX REVERSIONS (RESALE PROCEEDS)

The reversion receivable at the end of the income projection period is usually based on forecast or anticipated resale of the property at that time. The several before-tax reversion amounts applicable to real estate analysis and problems are:

SP: Resale Price

PR: Proceeds of Resale

b: Outstanding Mortgage Balance

NCPR: Net Cash Proceeds of Resale to Equity

These terms and symbols are further explained in the Appendix.

The derivation of these reversions is as follows:

1. Forecast or estimated Resale Price. Deduct sales and disposition expenses (brokerage commission, legal fees, etc.). The result is Proceeds of Resale.

2. Calculate Outstanding Balance of the Mortgage at the end of the Income Projection Period and subtract it from Proceeds of Resale. The result is Net Cash Proceeds of Resale.

Thus: SP − Disp. Exp. = PR

PR − b = NCPR

Example—The apartment property in the preceding example is expected to be resold in 10 years. The forecast resale price is $800,000. The broker's commission is expected to be 6% and other selling or disposition expenses are 2.5%. The mortgage is the same as that indicated in the preceding example.

a. What will the Mortgage Balance be in 10 years?

b. What are the Forecast Cash Proceeds of Resale, Net Cash Proceeds of Resale?

Calculate Annual Debt Service (ADS), Mortgage Balance (b):

CL FIN	CL F	
20 12×	20.00 12×	
9.5 12÷	9.50 12÷	
700000 PV	700000.00 PV	
PMT	END PMT	
12 ×	-6524.92 ***	
	12.00 ×	
STO 0	-78299.02 ***	ADS
10 12×	→ 0	
FV	10.00 12×	
	END FV	
	-504253.59 ***	b

Forecast Cash Proceeds of Resale (CPR), Net Cash Proceeds (NCPR):

800000 ENTER↑	800000.00 ENT↑	
6 ENTER↑	6.00 ENT↑	
2.5 +	2.50 +	
%	8.50 ***	
−	%	
	68000.00 ***	
+	−	
	732000.00 ***	CPR
	+	
	227746.41 ***	NCPR

ANNUAL INTEREST AND PRINCIPAL PAYMENTS FROM DEBT SERVICE

In calculating taxable income for deriving After-Tax Cash Flow it is necessary to develop annual payments of mortgage interest and principal, when mortgage payment periods are less than one year (monthly, quarterly, semi-annual).

The routine is a modification of the Accumulated Interest Paid and Remaining Balance procedure. Found on page 43 of the HP-92 Owner's Handbook.

1. Press CL FIN.
2. Set the Print Mode switch ALL / MAN / NORM to NORM.
3.* Input the following in any order:
   - Key in the periodic interest rate, and press i.
   - Key in the payment amount, and press CHS PMT.
   - Key in the initial principal, and press PV.
4. Key in 1, press P1.
5. Key in 12, press P2.
6. Press AMORT to obtain the accrued interest, principal and remaining balance for the year's mortgage payments.
7. Press PRINT x PV to print the remaining balance and update the present value to the amount outstanding.
8. Repeat steps 6 and 7 for each succeeding year of interest.

* If the payment amount is calculated, it becomes necessary to round the payment to two decimal place accuracy by pressing 2 RND PMT. By rounding the payment, we eliminate the problems created by the fractional cents when we attempt to reconcile our figures with the bank's schedule.

Example—A $97,000 mortgage loan has monthly payments of $830.69 with interest at 9.25%. Construct the schedule of annual interest and principal payments over the first four years of the loan term.

ALL
MAN NORM
CL FIN
9.25 12÷
830.69 CHS PMT
97000 PV

```
          CL F
  9.25   12÷
-830.69  PMT
97000.00  PV
```

Year 1:

1 P1
12 P2
AMORT
PRINT x
PV

```
   1.00   P1
  12.00   P2
          AMRT
1039.10  ΣPRN
8929.18  ΣINT
95960.90  ***
          PV
```

Year 2:

AMORT
PRINT x
PV

```
          AMRT
1139.39 ΣPRN
8828.89 ΣINT
94821.51  ***
          PV
```

Year 3:

AMORT
PRINT x
PV

```
          AMRT
1249.38 ΣPRN
8718.90 ΣINT
93572.13  ***
          PV
```

Year 4:

AMORT
PRINT x

```
          AMRT
1369.98 ΣPRN
8598.30 ΣINT
92202.15  ***
```

AFTER-TAX CASH FLOW

After-tax cash flow is found for each year by deducting Income Tax Liability for that year from CTO. (ATCF = CTO − Tax Liability.)

To derive Income Tax Liability for each year, it is necessary first to calculate Taxable Income. Then ATCF can be found:

1. Calculate and store annual Net Operating Income (NOI) in storage register corresponding with year of the projection period (i.e., yr. 1 = R1, yr. 15 = R.5, 19 year maximum projection period).
2. Calculate and deduct the yearly interest payment as illustrated in the previous section and store in appropriate storage register using storage register arithmetic.*
3. Calculate and deduct yearly depreciation in appropriate storage register using register arithmetic. Each storage register now contains the Taxable Income for each year at the projection period.
4. Multiply Taxable Income by the r, where r equals the appropriate tax rate to obtain tax liability.
5. Calculate and add the annual CTO to each years tax liability to obtain ATCF.

* Because register arithmetic can only be performed on registers 0–9, when the projection period exceeds 9 years it becomes necessary to recall directly from the register, perform the calculation and restore.

Thus:

$$\text{Taxable Income} = \text{NOI} - \text{Int.} - \text{Dep.}$$

$$\text{Tax Liability} = \text{Taxable Income} \times r$$

$$\text{ATCF} = \text{CTO} - \text{Tax Liability}$$

or

$$\text{Tax Liability} \times (r/1 - r)$$

Example—The property used in the example in the preceding section on Before-Tax Cash Flows was purchased for $900,000, of which $150,000 was allocated to land. Therefore the "depreciable amount" of investment in the buildings is $750,000. The buildings have an estimated remaining useful life of 25 years, and are to be depreciated on a 125% declining-balance basis.

The mortgage loan terms are those stipulated in the earlier example: Principal of $700,000; Interest rate of 9.5%; Full amortization in level monthly payments over 20 years. The applicable income tax rate is 48%.

What is the schedule of ATCF for 10 years?

Note:
From the preceeding example:
NOI = $94,829.00
CTO = $16,529.98

(1) Store NOI:

CL REG
94829 STO 1
STO 2
STO 3
STO 4
STO 5
STO 6
STO 7
STO 8
STO 9
STO • 0

```
                CL R
94829.00 → 1
         → 2
         → 3
         → 4
         → 5
         → 6
         → 7
         → 8
         → 9
         →.0
```

NOI

(2) Calculate and deduct yearly accumulated interest:

ALL
MAN NORM
CL FIN
20 12×
9.5 12÷
700000 PV
PMT
1 P1
12 P2
AMORT

```
                CL F
20.00           12×
9.50            12÷
700000.00       PV
          END   PMT
1.00            P1
12.00           P2
                AMRT
12326.58        ΣPRN
65972.46        ΣINT
```

Yr. 1 accumulated interest

PV
R↓
STO − 1
AMORT

```
          PV
          R↓
        →-1
        AMRT
13549.98 ΣPRN
64749.06 ΣINT
```

Yr. 2 accumulated interest

PV
R↓
STO − 2
AMORT

```
          PV
          R↓
        →-2
        AMRT
14894.78 ΣPRN
63404.26 ΣINT
```

Yr. 3 accumulated interest

PV
R↓
STO − 3
AMORT

```
          PV
          R↓
        →-3
        AMRT
16373.05 ΣPRN
61925.99 ΣINT
```

Yr. 4 accumulated interest

PV
R↓
STO − 4
AMORT

```
          PV
          R↓
        →-4
        AMRT
17998.04 ΣPRN
60301.00 ΣINT
```

Yr. 5 accumulated interest

PV
R↓
STO − 5
AMORT

```
          PV
          R↓
        →-5
        AMRT
19784.29 ΣPRN
58514.75 ΣINT
```

Yr. 6 accumulated interest

PV
R↓
STO − 6
AMORT

```
          PV
          R↓
        →-6
        AMRT
21747.85 ΣPRN
56551.19 ΣINT
```

Yr. 7 accumulated interest

PV
R↓
STO − 7
AMORT

```
          PV
          R↓
        →-7
        AMRT
23906.27 ΣPRN
54392.77 ΣINT
```

Yr. 8 accumulated interest

PV R↓ STO − 8 AMORT	PV R↓ →-8 AMRT 26278.90 ΣPRN 52020.14 ΣINT	Yr. 9 accumulated interest
PV R↓ STO − 9 AMORT	PV R↓ →-9 AMRT 28887.02 ΣPRN 49412.02 ΣINT	Yr. 10 accumulated interest
R↓ RCL • 0 x≷y − STO • 0	R↓ →.0 X≷Y - →.0	

(3) Calculate and deduct yearly depreciation charges:

CL FIN 25 LIFE 750000 BOOK 125 FACT 1 N1 N2 DB	CL F 25.00 LIFE 750000.00 BOOK 125.00 FACT 1.00 N1 N2 DB 37500.00 ΣDPN	Assets depreciable life Assets book value Declining balance factor 1st yr. depreciation
BOOK R↓ STO − 1 DB	BOOK R↓ →-1 DB 75625.00 ΣDPN	2nd yr. depreciation
BOOK R↓ STO − 2 DB	BOOK R↓ →-2 DB 33643.75 ΣDPN	3rd yr. depreciation
BOOK R↓ STO − 3 DB	BOOK R↓ →-3 DB 32151.56 ΣDPN	4th yr. depreciation

BOOK R↓ STO − 4 DB	BOOK R↓ →-4 DB 30543.98 ΣDPN	5th yr. depreciation
BOOK R↓ STO − 5 DB	BOOK R↓ →-5 DB 29016.79 ΣDPN	6th yr. depreciation
BOOK R↓ STO − 6 DB	BOOK R↓ →-6 DB 27565.95 ΣDPN	7th yr. depreciation
BOOK R↓ STO − 7 DB	BOOK R↓ →-7 DB 26187.65 ΣDPN	8th yr. depreciation
BOOK R↓ STO − 8 DB	BOOK R↓ →-8 DB 24878.27 ΣDPN	9th yr. depreciation
BOOK R↓ STO − 9 DB	BOOK R↓ →-9 DB 23634.35 ΣDPN	10th yr. depreciation
R↓ RCL • 0 x⇄y − STO • 0	R↓ →.0 X⇄Y - →.0	

Print out Taxable Income:

REG	LIST 0.00 - 0 -8643.46 - 1 -5545.06 - 2 -2419.01 - 3 751.45 - 4 3984.02 - 5

```
 7297.46 - 6
10711.86 - 7
14248.58 - 8
17930.59 - 9
21782.63 -.0
```

(4) Multiply Taxable Income by tax rate (r) and print out tax liability r:

.48 CHS STO × 1
STO × 2
STO × 3
STO × 4
STO × 5
STO × 6
STO × 7
STO × 8
STO × 9
RCL • 0
×
STO • 0

```
-.48 ÷x1
     ÷x2
     ÷x3
     ÷x4
     ÷x5
     ÷x6
     ÷x7
     ÷x8
     ÷x9
     ÷.0
       x
     ÷.0
```

REG

```
          LIST
  0.00 - 0
4148.86 - 1
2661.63 - 2
1161.12 - 3
-360.70 - 4
-1912.33 - 5
-3502.78 - 6
-5141.69 - 7
-6839.32 - 8
-8606.68 - 9
-10455.66 -.0
```

Note:

Assume that in years 1-3 other income is available to offset the positive tax inflow or that we may use the loss cary back, carry forward feature of Federal Income Tax statutes.

Add CTO to each year's tax liability to obtain ATCF:

16529.98 STO + 1
STO + 2
STO + 3
STO + 4

```
16529.98 ÷+1
         ÷+2
         ÷+3
         ÷+4
```

STO + 5
STO + 6
STO + 7
STO + 8
STO + 9
RCL • 0
+
STO • 0

```
          ++5
          ++6
          ++7
          ++8
          ++9
          +.0
            +
          +.0
```

Print out After Tax Cash Flows (ATCF):

REG

```
          LIST
     0.00 - 0
 20678.84 - 1
 19191.61 - 2
 17691.10 - 3
 16169.28 - 4
 14617.65 - 5
 13027.20 - 6
 11388.29 - 7
  9690.66 - 8
  7923.30 - 9
  6074.32 -.0
```

Note:

These values should remain stored in the storage registers for they will be used in the next section.

AFTER-TAX NET CASH PROCEEDS OF RESALE
ATNCPR = NCPR − Tax Liability

To calculate Tax Liability, it is necessary to find the Gain on Resale. This is divided between Excess Depreciation, which is taxed (fully or partially, depending on whether any Excess Depreciation is "forgiven") as ordinary income; and the remainder, which is Capital Gain taxed at the capital gains tax rate.

The steps are:

1. Calculate total depreciation charged. Subtract this from the original purchase price (Capital Outlay) to obtain Tax Basis.
2. Subtract Tax Basis from Proceeds of Resale. The result is Gain on Resale.
3. Subtract total straight-line depreciation over the income projection period from total depreciation charged. This produces Excess Depreciation.
4. Subtract Excess Depreciation from Gain on Resale to obtain Capital Gain.
5. Multiply Excess Depreciation by ordinary income tax rate. This produces ordinary income tax liability on resale.

6. Multiply Capital Gain by capital gains tax rate. This produces capital gains tax liability on resale.
7. Add the figures derived in steps 5 and 6 to obtain total Tax Liability on resale.
8. Subtract total Tax Liability from Net Cash Proceeds of Resale to obtain After-Tax Net Cash Proceeds of Resale.

Thus:

$$CO - \text{Total Dep.} = \text{Tax Basis}$$

$$PR - \text{Tax Basis} = \text{Gain on Resale}$$

$$\text{Total Dep.} - \text{S-L Dep.} = \text{Excess Dep.}$$

$$\text{Gain on Resale} - \text{Excess Dep.} = \text{Capital Gain}$$

$$(\text{Excess Dep.} \times \text{Ord. Tax Rate}) + (\text{Cap. Gain} \times \text{CG Tax Rate}) = \text{Tax Liability}$$

$$NCPR - \text{Tax Liability} = ATNCPR$$

Example—The apartment complex which has been used as an example throughout this chapter is forecast in year 10 to sell for $1,750,000. Disposition expenses will amount to 8%. The applicable ordinary income tax rate is 48% and the capital gain tax rate is 30%.

Recalling from previous examples:

P = $700,000 @ 9.5% for 20 years,

CO = $200,000

Estimated remaining useful life at 25 years, depreciated on a 125% declining balance basis.

What is the indicated ATNCPR? After Tax IRR?

Calculate and store NCPR:

ALL MAN ▮▮▮▮ NORM		
CL FIN	CL F	
20 12×	20.00 12×	
9.5 12÷	9.50 12÷	
700000 PV	700000.00 PV	
PMT	END PMT	
10 12×	-6524.92 ***	
FV	10.00 12×	
	END FV	
	-504253.59 ***	b
1750000 ENTER↑	1750000.00 ENT↑	
8 %	8.00 %	
−	140000.00 ***	
	-	
	1610000.00 ***	CPR

STO Σ+ 0
+
STO Σ+ 1

```
          →Σ0
            +
1105746.41  ***
          →Σ1
```

NCPR

Calculate and store accumulated depreciation over projection period:

ALL
MAN NORM

CL FIN
25 LIFE
750000 BOOK
125 FACT
1 N1
10 N2
DB
R↓
STO Σ+ 2

```
          CL F
    25.00 LIFE
750000.00 BOOK
   125.00 FACT
     1.00   N1
    10.00   N2
            DB
300947.30 ΣDPN
          R↓
        →Σ2
```

Assets useful life
Assets depreciation base
Declining balance factor
Accumulated depreciation

Calculate and store tax basis:

ALL
MAN NORM

900000 x⇄y
−
STO Σ+ 3

```
900000.00  X⇄Y
300947.30  ***
             -
599052.70  ***
         →Σ3
```

Tax basis

Calculate and store Gain or Resale:

RCL Σ+ 0
x⇄y
−

```
          ←Σ0
1610000.00  ***
          X⇄Y
599052.70   ***
             -
1010947.30  ***
```

Gain on resale

Calculate Excess Depreciation, Capital Gain:

ALL
MAN NORM

SL
R↓
RCL Σ+ 2
x⇄y
−
STO Σ+ 4

```
            SL
300000.00 ΣDPN
          R↓
        ←Σ2
          X⇄Y
           -
        →Σ4
```

PRINT x	947.30 ***	
−	-	Excess depreciation
PRINT x	1010000.00 ***	Capital gain

Calculate ordinary, capital gain taxes, total tax liability on resale:

ALL MAN NORM	LSTX	
LAST x	947.30 ***	
.48 ×	.48 x	
	454.70 ***	Tax on excess depreciation
x≷y	X≷Y	
	1010000.00 ***	
.3 ×	.30 x	
	303000.00 ***	Capital gain tax
+	+	
	303454.70 ***	Tax liability

Calculate After-Tax Net Cash Proceeds from Resale:

RCL Σ+ 1	→Σ1	
	1105746.41 ***	NCPR
x≷y	X≷Y	
	303454.70 ***	Tax liability
−	-	
	802291.70 ***	ATNCPR

Calculate 10th year Cash Flow:

RCL • 0	→.0	
	6074.32 ***	10th yr. ACTF
+	+	
STO • 0	808366.02 ***	10th yr. cash flow
	→.0	

Store Cash Outlay (CO) in R_0:

200000 CHS STO 0	-200000.00 → 0	CO

Calculate After-Tax IRR:

10 n	10.00 n	
IRR	IRR	
	19.72 ***	After-Tax IRR

CHAPTER 5
INVESTMENT AND FEASIBILITY ANALYSIS

The HP-92 can be used effectively in real estate decision making, using both routines and procedures that have been described and illustrated in previous chapters of this manual, as well as other procedures illustrated below.

Decision making involves making a choice from amoung two or more alternative courses of action. The routines and procedures available on the HP-92 make it possible for the analyst to consider almost any combination of outcomes, and compare them with one another to select the "best" alternative, or to compare them with some standard of acceptability to make an accept-reject decision.

Feasibility Analysis is a process of measuring and testing whether a proposed investment is expected to meet an investor's minimum standard(s) of acceptability. If the investment or project proposal meets the investor's standard(s), then it is "feasible".

Investment Analysis consists essentially of comparing alternative investment or project proposals, and making them according to the results of their feasibility tests. The highest-ranking alternative is the "best" in terms of the investor's standard(s) of acceptability.

In addition, there are decisions about the selection of the "best" or optimum financing alternative, decisions concerning rent-buy and sell-lease alternatives, and measures of financial safety or coverage that enter into real estate problem solving. All these are considered and illustrated here in Chapter 5, along with measures and tests of feasibility and sensitivity analysis.

FEASIBILITY TESTS

A feasibility test measures whether a project or investment is likely to meet an investor's standard of acceptability. These standards of acceptability include:

1. The investment should be worth to the investor at least as much as it will cost the investor to acquire it. This criterion is tested by calculating the Present Worth of the Forecast Future Cash Flows from the investment at a rate of discount reflecting the rate of return minimally acceptable to the investor, and comparing that Present Worth to the Capital Outlay required. This procedure uses Present Worth, Net Present Value and the Profitability Index.
2. The investment should produce a rate of return to the investor at least as high as the rate of return desired or required. This criterion is tested by calculating the Internal Rate of Return or Modified Internal Rate of Return on the investment, and comparing it with the investor's desired or required rate of return.

3. The investment should provide for full recovery of the investor's Capital Outlay within the time period desired or required by the investor. This criterion is tested by calculating the Payback Period and comparing it with the investor's desired or required payback period.

a. Present Worth

The PW of any investment is calculated with the routines and procedures illustrated in Chapters 1 and 3. This involves discounting the Forecast Future Cash Flows at a specified rate. For feasibility analysis, that specified rate is the minimally acceptable rate of return to the investor. It is y for estimating equity investment value, r for estimating total property value, and i for estimating the present worth of a mortgage.

1.) Level Annuity, No Reversion

The routine is the one illustrated in Chapter 1.
Enter: Number of Payment Periods [n]. Rate of Return per Period [i]. Cash Flow per Period [PMT] [PV].

2.) Level Annuity with Reversion or Balloon Payment

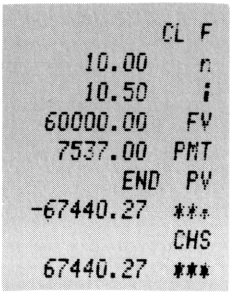
The routines are as illustrated in Chapters 1 and 3. The Present Worth of the level cash flows is added to the Present Worth of the reversion, both at the investor's minimally acceptable rate of return. The sum is the Present Worth of the investment.

Example—An income property purchased for $62,500 is forecast to produce NOI of $7,537 per year. The investor expects to hold it for 10 years, and then sell it. The Forecast Proceeds of Resale are $60,000.

The property has just been financed with a $50,000 mortgage at 9% interest, with level monthly payments over a 25-year term.

What is the Present Worth of the property assuming no mortgage and a Basic Rate (r) of 10.5%?

[CL FIN]
10 [n]
10.5 [i]
60000 [FV]
7537 [PMT]
[PV]
[CHS]



Present Worth (PW)

What is the Present Worth of the equity investment position with an Equity Yield Rate of 14%?

Mortgage payment:

CL FIN	CL F	
25 12×	25.00 12×	
9 12÷	9.00 12÷	
50000 PV	50000.00 PV	
PMT	END PMT	
	-419.60 ***	Monthly payment amount

Net Cash Proceeds of Resale to Equity (NCPR):

10 12×	10.00 12×	
FV	END FV	
	-41369.62 ***	b
60000 +	60000.00 +	CPR
	18630.38 ***	NCPR
FV	FV	

Cash Throw-Off to Equity (CTO):

7537 RCL PMT	7537.00← PMT	NOI
12 ×	-419.60 ***	
	12.00 ×	
+	-5035.18 ***	Annual Debt Service
	+	
PMT	2501.82 ***	CTO
	PMT	

Present Worth of Equity (PW):

10 n	10.00 n	
14 i	14.00 i	y
PV	END PV	
CHS	-18075.22 ***	
	CHS	
	18075.22 ***	PW

3.) Variable Annuity (Increasing/Decreasing) Annuity

The majority of real estate financing arrangements deal with equal periodic payments. It is possible however to consider a payment stream where the payment amounts change, such as income, repair costs or price changes reflecting inflation or deflation. The present worth of such a payment stream may be easily calculated by summing each individual moved in time. A quicker and easier way to sum the payments is with the following keystrokes:

1) Set the Payment switch to the END position BEGIN BOND END NOTE and press CL FIN.

2) Key in the total number of payment periods, press n.

3) Key in the payment percentage increase per period expressed as one plus the decimal interest rate, press ENTER↑. If there is a percentage decrease, express it as one minus the decimal interest rate.

4) Key in the discount (interest) rate per period expressed as one plus the decimal interest rate, press Δ% i.

5) Press CLx, key in the starting payment amount, press x≷y ÷ PMT.*

6) Press PV to obtain the present worth of the payment stream.

* Positive for cash received, negative for cash paid out.

Note:

This procedure assumes that payments occur at the end of the period.

Example 1—Repair costs on the new duplex that you just purchased are expected to start at $200 one year from now and increase at the rate of 12% per year. What sum must be put in the back today to cover the next three years of maintenance if the bank pays 5.5% interest?

CL FIN	CL F	
3 n	3.00 n	Years
1.12 ENTER↑	1.12 ENT↑	Payment percentage increase
1.055 Δ%	1.055 Δ%	Interest rate per period
	-5.80 ***	Adjusted i
i	i	
CLx	CL X	
200 x≷y	200.00 X≷Y	Starting payment amount
÷	1.12 ***	
PMT	÷	
PV	178.57 ***	
	PMT	
	END PV	
	-604.48 ***	Present worth of cash outflows

When the payment amounts change by a constant amount instead of by a constant ratio, as in the previous problem, the following keystrokes will give the present value:

1) Set the Payment switch to BEGIN / BOND — END / NOTE and press CL FIN.
2) Key in the periodic discount (interest) rate as a percent; press i.
3) Key in the starting payment; press ENTER↑.
4) Key in the amount that the payment increases each period; press ENTER↑.
5) Key in the periodic discount (interest) rate as a decimal; press ÷ STO 0 + PMT.
6) Key in the total number of payment periods; press ENTER↑ n RCL 0 × CHS FV.
7) Press PV to obtain the present worth of the payments.

Example 2—If the repair costs in the previous example increased each year by $35, what would be the sum required?

CL FIN	CL F	
5.5 i	5.50 i	Periodic interest rate
200 ENTER↑	200.00 ENT↑	Starting payment
35 ENTER↑	35.00 ENT↑	Periodic payment increase
.055 ÷	.055 ÷	
STO 0	636.36 ***	
+	→ 0	
	+	
	836.36 ***	Adjusted payment
PMT	PMT	
3 ENTER↑	3.00 ENT↑	Years
n	n	
RCL 0	← 0	
×	636.36 ***	
CHS	×	
	1909.09 ***	
FV	CHS	
PV	-1909.09 ***	Adjusted future value
	FV	
	END PV	
	-630.65 ***	Present worth of cash out flows

b. Net Present Value

Net Present Value is the difference between Present Worth and Capital Outlay required: NPV = PW − CO.

The test of feasibility is NPV ≥ 0.

If Present Worth at the investor's required or desired rate of return is equal to or greater than the Capital Outlay required to acquire the investment position, then the investment is "feasible".

Example 1—An investment property has just been purchased for $62,500 including a 50,000 mortgage financed at 9% for 25 years. NOI is forecast at $7,537 the first year and increasing at 6% a year thereafter. The property is expected to be resold in 10 years with CPR being $60,000. The investor desired a equity yield rate (y) of 10.5%.

What is the indicated NPV of this investment if the property is sold in year 10?

Note:

This procedure utilizes the keystroke procedure presented in the previous section, Increasing/Decreasing Annuity.

Annual Debt Service (ADS):

CL FIN	CL F	
25 12×	25.00 12×	
9 12÷	9.00 12÷	
50000 PV	50000.00 PV	
PMT	END PMT	
12 ×	-419.60 ***	
	12.00 ×	
STO 0	-5035.18 ***	ADS
	→ 0	

Mortgage Balance yr. 10, Net Cash Proceeds at Resale (NCPR):

10 12×	10.00 12×	
FV	END FV	
	-41369.62 ***	b
60000 +	60000.00 +	CPR
	18630.38 ***	NCPR
CL FIN	CL F	
FV	FV	
10 n	10.00 n	
10.5 i	10.50 i	
PV	END PV	
	-6864.34 ***	Present Worth NCPR
STO 1	→ 1	

Present Worth Annual Debt Service:

RCL 0	→ 0	
PMT	-5035.18 ***	
0 FV	PMT	
PV	0.00 FV	
	END PV	
STO 0	30285.45 ***	Present Worth ADS
	→ 0	

Present Worth Cash Throw-Off to Equity with increasing annuity:

CL FIN	CL F	
10 n	10.00 n	
1.06 ENTER↑	1.06 ENT↑	(1 + growth factor/100)
1.105 Δ%	1.105 Δ%	(1 + y/100)
	4.25 ***	
i	i	
CL x	CL X	
7537 x≷y	7537.00 X≷Y	NOI
÷	1.06 ***	
PMT	÷	
PV	7110.38 ***	
	PMT	
RCL 0	END PV	
	-56973.72 ***	Present Worth NOI
+	+ 0	
	30285.45 ***	Present Worth ADS
	+	
	-26688.27 ***	Present Worth CTO

Net Present Value (PW-CO):

RCL 1	+ 1	
	-6864.34 ***	Present Worth NCPR
+	+	
	-33552.62 ***	PW
CHS	CHS	
	33552.62 ***	
62500 ENTER↑	62500.00 ENT↑	
50000 −	50000.00 −	
	12500.00 ***	Down payment
−	−	
	21052.62 ***	NPV

Example 2—A rental property has 7 years remaining on the lease to the single tenant. The property is for sale for $200,000. A mortgage in the amount of $137,500 can be obtained.

A potential investor seeking an after-tax rate of return on his equity investment of 12% has forecast the after-tax cash flows and reversion, based on lease terms, as follows:

Year	ATCF
1	$11,846
2	$ 9,673
3	$ 8,217
4	$ 6,743
5	$ 5,018
6	$ 3,716
7	$ 2,284
7 (Reversion)	$51,883

Should the investor purchase the property?

CLEAR	CLEAR	
200000 ENTER↑	200000.00 ENT↑	
137500 −	137500.00 −	
CHS	62500.00 ***	
	CHS	
STO 0	-62500.00 ***	CO
	→ 0	
11846 STO 1	11846.00 → 1	
9673 STO 2	9673.00 → 2	
8217 STO 3	8217.00 → 3	
6743 STO 4	6743.00 → 4	
5018 STO 5	5018.00 → 5	
3716 STO 6	3716.00 → 6	
2284 STO 7	2284.00 → 7	
51883 STO + 7	51883.00 →+7	CPR
7 n	7.00 n	
12 i	12.00 i	
NPV	NPV	
	-4845.57 ***	NPV

Because the NPV is negative, the property does not reach the investor's desired return.

To obtain the desired 12% after-tax rate of return, what is the minimum amount the reversion may be?

PV	PV	
FV	END FV	
RCL 7	10712.02 ***	
+	← 7	
2284 −	54167.00 ***	
	+	
	64879.02 ***	
	2284.00 −	
	62595.02 ***	CPR

Therefore, any reversion in year 7 greater than $65,595.02 will generate the investor's desired 12% return, or a NPV > 0.

c. Profitability Index

The Profitability Index is the Ratio of Present Worth to Capital Outlay:

$$PI = \frac{PW}{CO}$$

The test of feasibility is: PI ≥ 1.

Example—An investment property has just been purchased for $62,500, including a $50,000 mortgage. NOI is forecast at $7537 annually, while CTO is $2502. The property is expected to be resold in 10 years for $60,000, at which time NCPR would be $18,630. What is the Profitability Index for the property investment at a Basic Rate of 10.5%?

CL FIN	CL F	
10 n	10.00 n	
10.5 i	10.50 i	
60000 FV	60000.00 FV	
7537 PMT	7537.00 PMT	
PV	END PV	
CHS	-67440.27 ***	PW
	CHS	
	67440.27 ***	
62500	62500.00 ÷	CO
÷	1.08 ***	PI

d. Internal Rate of Return

As noted previously, an Internal Rate of Return is that rate of discount at which the Present Worth of Forecast Future Cash Flows from an investment exactly equals the required Capital Outlay.

The Test of feasibility is: IRR $\geq$ Desired Rate of Return

The calculated IRR from the property investment and forecast cash flow data must be equal to or greater than the rate of return required or desired by the investor.

Example—An investment property is forecast to produce the following after-tax cash flow over a 10-year income projection period.

Year	ATCF
1	$ 2,861
2	$ 2,753
3	$ 2,631
4	$ 2,508
5	$ 2,387
6	$ 2,162
7	$ 1,894
8	$ 1,583
9	$ 1,212
10	$ 808
10 (Reversion, ATNCPR)	$14,765

The investor can acquire the equity investment position for $12,500. What is the after-tax rate of return, calculated as an IRR?

CLEAR	CLEAR	
12500 CHS	-12500.00	
STO 0	→ 0	
2861 STO 1	2861.00 → 1	
2753 STO 2	2753.00 → 2	
2631 STO 3	2631.00 → 3	
2508 STO 4	2508.00 → 4	
2387 STO 5	2387.00 → 5	
2162 STO 6	2162.00 → 6	
1894 STO 7	1894.00 → 7	
1583 STO 8	1583.00 → 8	
1212 STO 9	1212.00 → 9	
808 ENTER↑	808.00 ENT↑	
14765 +	14765.00 +	
STO • 0	15573.00 ***	(Yr. 10 cash flow = ATCF + ATNCPR)
10 n	→.0	
IRR	10.00 n	
	IRR	
	19.71 ***	IRR

Continuing with this example, if the investor had instead desired a 20% equity yield rate (y), what After Tax Net Proceeds from Resale (ATNCPR) must he realize in year 10 to achieve the desired yield?

20 i	20.00 i	y
NPV	NPV	
	-141.88 ***	NPV
PV	PV	
FV	END FV	
RCL • 0	878.49 ***	
+	→.0	
808 −	15573.00 ***	
	+	
	16451.49 ***	
	808.00 −	
	15643.49 ***	ATNCPR

e. Payback Period

The Payback Period is the number of years required to return or "payback" the amount of Capital Outlay, disregarding any rate of discount. It is n when:

$$\sum_{t=1}^{n} (CF_t) \geq CO$$

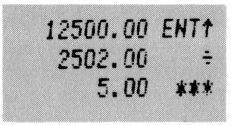
The test of feasibility is: $n \leq$ Target

An investment is feasible when the Capital Outlay is forecast to be repaid within the period required or desired by the investor.

If cash flows are level, $n \geq \frac{CO}{CTO}$

Example—An investor has just purchased an income property for $62,500, of which $12,500 was equity. CTO is forecast at $2502 annually. What is the Payback Period?

12500 ENTER↑
2502 ÷



Payback Period

GROSS INCOME MULTIPLIER

The Gross Income Multiplier (or Gross Rent Multiplier) is the ratio of Sales Price to Gross Income. The Gross Income figure used can be either Potential Gross Income or Effective Gross Income, depending on which is appropriate or applicable in the particular case.

$$GIM = \frac{SP}{PGI} \qquad \text{or} \qquad GIM = \frac{SP}{EGI}$$

Example—An apartment property recently sold for $885,700. It contains 63 units, renting for $247.50 per month each. Five units were vacant at the time of sale, which is a normal vacancy ratio in this market. What is the indicated Gross Income Multiplier, using both PGI and EGI?

Calculate GIM using Potentially Gross Income (PGI):

885700 ENTER↑	885700.00 ENT↑	SP
63 ENTER↑	63.00 ENT↑	
247.5 ×	247.50 ×	
12 ×	15592.50 ***	
	12.00 ×	
÷	187110.00 ***	PGI
	÷	
	4.73 ***	GIM

Calculate GIM using Effective Gross Income (EGI):

885700 ENTER↑	885700.00 ENT↑	SP
63 ENTER↑	63.00 ENT↑	
5 −	5.00 −	
247.5 ×	58.00 ***	
12 ×	247.50 ×	
	14355.00 ***	
÷	12.00 ×	
	172260.00 ***	EGI
	÷	
	5.14 ***	GIM

FINANCIAL COVERAGE (SAFETY) RATIOS

In evaluating investment proposals, the safety or ability of forecast income to cover required payments (cash outflows) is often as important a consideration as profitability.

The required payments are Operating Expenses and Debt Service. They are compared with the Cash Flows available to cover them.

a. Operating Expense Ratio.

This is the ratio of Operating Expenses to Effective Gross Income:

$$\text{OER} = \frac{\text{OE}}{\text{EGI}}$$

Example—An apartment property has forecast Effective Gross Income of $42,866 and annual Operating Expenses of $17,694. What is the indicated Operating Expense Ratio?

g 4	17694.0000 ENT↑	OE
17694 ENTER↑	42866.0000 ÷	EGI
42866 ÷	0.4129 ***	OER

b. Debt Service Coverage Ratio

This is the ratio of Net Operating Income to Annual Debt Service:

$$\text{Debt Service Coverage Ratio} = \frac{\text{NOI}}{\text{ADS}}$$

For safety or coverage purposes, the higher this ratio, the better.

Example—An apartment with Forecast Effective Gross Income of $42,866 and Operating Expenses of $17,694 per year also has a mortgage with level monthly payments of $1,395.75. What is the indicated Debt Service Coverage Ratio?

42866 ENTER↑	42866.0000 ENT↑	
17694 −	17694.0000 −	
	25172.0000 ***	NOI
1395.75 ENTER↑	1395.7500 ENT↑	
12 ×	12.0000 ×	
	16749.0000 ***	ADS
÷	÷	
	1.5029 ***	Debt Service Coverage Ratio

c. Breakeven Cash Throw-Off Ratio

This is the ratio of Effective Gross Income to the sum of Operating Expenses and Annual Debt Service:

$$\text{Breakeven CTO Ratio} = \frac{\text{EGI}}{\text{OE} + \text{ADS}}$$

The ratio measures the extent to which EGI covers required cash outlays. For safety or coverage purposes, the higher this ratio, the better.

Example—For the apartment property illustrated in the preceding examples in this section on Financial Coverage Ratios, what is the Breakeven Cash Throw-Off Ratio?

42866 ENTER↑	42866.0000 ENT↑	EGI
17694 ENTER↑	17694.0000 ENT↑	OE
1395.75 ENTER↑	1395.7500 ENT↑	
12 ×	12.0000 ×	
	16749.0000 ***	ADS
+	+	
	34443.0000 ***	
÷	÷	
	1.2445 ***	Breakeven CTO Ratio

FINANCIAL MORTGAGE ANALYSIS

a. Mortgagee Participation Loans (Equity Kickers)

Mortgage lenders, especially life insurance companies, sometimes require a share of property income as part of the price of granting a mortgage loan, in addition to contractual debt service. This sharing in property income is called a mortgagee participation or "equity kicker". The participation may be a percentage of Gross Income, of NOI, of CTO, or even forecast NCPR.

The analysis of mortgage participation loans takes the same general format used in the sections on Refinancing and Wrap—Around Mortgages in the HP—92 Applications Book. The NPV and/or effective yield (IRR) to borrower and lender are calculated and compared to ascertain which alternative is preferable to each participant.

Example—An investor has agreed to purchase an income property for $270,000. A mortgage loan of $210,000 has been arranged with an institutional lender, with full amortization over 25 years in level monthly payments at 9% interest.

The property has a lease with 10 years at $50,000 per year. Stabilized annual operating expenses are forecast at $22,000 per year. The investor plans to sell the property at the end of the lease term. The forecast proceeds of resale are $250,000.

The lender has just offered the investor two alternative financing plans:

1. An 8.6% interest rate, plus a 4% lender participation in gross income (all other loan terms the same);
2. An 8.35% interest rate, plus a $^1/_3$ lender participation in CTO and NCPR (all other loan terms the same).

Assuming the investor goes through with resale plans, and all income and resale forecasts are realized, which alternative is preferable to the lender? Which is preferable to the borrower (investor)?

1. Original loan, no participation
Calculate and store ADS:

CL FIN	CL F	
25 12×	25.00 12×	
9 12÷	9.00 12÷	
210000 PV	210000.00 PV	
PMT	END PMT	
12 ×	-1762.31 ***	
	12.00 ×	
STO 0	-21147.75 ***	ADS
	→ 0	

Calculate b, NCPR:

10 12×	10.00 12×	
FV	END FV	
	-173752.38 ***	b
250000 +	250000.00 +	CPR
	76247.62 ***	NCPR
CL FIN	CL F	
FV	FV	

Calculate y:

10 n	10.00 n	
50000 ENTER↑	50000.00 ENT↑	
22000 −	22000.00 −	
	28000.00 ***	NOI
RCL 0 +	→ 0	
	-21147.75 ***	ADS
	+	
PMT	6852.25 ***	CTO
270000 ENTER↑	PMT	
210000 −	270000.00 ENT↑	
CHS	210000.00 −	
	60000.00 ***	
PV	CHS	
i	-60000.00 ***	CO
	PV	
	END i	
	12.90 ***	y

Note:
NPV to lender = 0, effective yield = 9%.

2. 8.6% loan with 4% participation in gross income, 10 years:
Calculate and store ADS:

CL FIN	CL F	
25 12×	25.00 12×	
8.6 12÷	8.60 12÷	
210000 PV	210000.00 PV	
PMT	END PMT	
12 ×	-1705.15 ***	
	12.00 ×	
STO 0	-20461.82 ***	ADS
	→ 0	

Calculate b, NCPR:

10 12×	10.00 12×	
FV	END FV	
	-172131.50 ***	b
STO 1	→ 1	

250000 +	250000.00 +	CPR
	77868.50 ***	NCPR
CL FIN	CL F	
FV	FV	

Calculate investors' y:

10 n	10.00 n	
50000 ENTER↑	50000.00 ENT↑	
4 %	4.00 %	
STO 2	2000.00 ***	
−	→ 2	
22000 −	−	
	48000.00 ***	
RCL 0	22000.00 −	
	26000.00 ***	NOI
+	← 0	
	-20461.82 ***	ADS
PMT	+	
	5538.18 ***	CTO
270000 ENTER↑	PMT	
210000 −	270000.00 ENT↑	
CHS	210000.00 −	
	60000.00 ***	
PV	CHS	
i	-60000.00 ***	CO
	PV	
	END i	
	11.01 ***	y

Calculate lenders' effective yield:

RCL 2	← 2	
RCL 0 −	2000.00 ***	
PMT	← 0	
210000	-20461.82 ***	
CHS PV	−	
RCL 1	22461.82 ***	
	PMT	
CHS	-210000.00 PV	
FV	← 1	
i	-172131.50 ***	b
	CHS	
	172131.50 ***	
	FV	
	END i	
	9.54 ***	y

3. 8.35% loan with 1/3 participation in CTO and NCPR, 10 years:

Calculate and store ADS:

CL FIN	CL F	
25 12×	25.00 12×	
8.35 12÷	8.35 12÷	
210000 PV	210000.00 PV	
PMT	END PMT	
12 ×	-1669.80 ***	
	12.00 ×	
STO 0	-20037.63 ***	ADS
	→ 0	

Calculate b, NCPR:

10 12×	10.00 12×	
FV	END FV	
	-171092.12 ***	b
STO 1	→ 1	
250000 +	250000.00 +	CPR
ENTER↑	78907.88 ***	
ENTER↑	ENT↑	
3 1/x	ENT↑	
×	3.00 1/X	
STO 2	0.33 ***	
−	×	
	26302.63 ***	
	→ 2	
	−	
	52605.25 ***	NCPR

Calculate investors' y:

CL FIN	CL F	
FV	FV	
10 n	10.00 n	
50000 ENTER↑	50000.00 ENT↑	
22000 −	22000.00 −	
	28000.00 ***	NOI
RCL 0	→ 0	
	-20037.63 ***	ADS
+	+	
	7962.37 ***	
ENTER↑	ENT↑	
ENTER↑	ENT↑	
3 1/x	3.00 1/X	
×	0.33 ***	
STO 3	×	
−	2654.12 ***	
	→ 3	
	−	
	5308.25 ***	CTO

PMT	PMT	
270000 ENTER↑	270000.00 ENT↑	
210000 −	210000.00 −	
	60000.00 ***	
	CHS	
CHS	-60000.00 ***	CO
PV	PV	
i	END i	
	8.00 ***	y

Calculate lenders' effective yield:

RCL 3	← 3	
RCL 0	2654.12 ***	
−	← 0	
PMT	-20037.63 ***	
210000 CHS	−	
PV	22691.75 ***	
RCL 2	PMT	
RCL 1	-210000.00 PV	
−	← 2	
FV	26302.63 ***	
i	← 1	
	-171092.12 ***	
	−	
	197394.75 ***	
	FV	
	END i	
	10.44 ***	y

Summary	IRR to Lender	IRR to Borrower
1. 9% loan	9.00%	12.90%
2. 8.6% loan with 4% Gross Participation	9.54%	11.01%
3. 8.35% loan with $^1/_3$ CTO and NCPR Participation	10.44%	8.00%

Clearly, the mortgagee would prefer the $^1/_3$ participation in CTO and NCPR, while the mortgagor would perfer the original loan.

b. Maturity Associated with Specific Mortgage Yield

By varying the nominal maturity, usually by extending it, it is possible to achieve a given effective yield for the lender or maintain a maximum mortgage constant for the borrower, while keeping the periodic payment amount constant.

Example—An $80,000 mortgage was recently negotiated, with full amortization over 20 years in level monthly payments at 9% interest. Subsequently, interest rates have risen, and the lender now wants a 9.25% yield to maturity. The borrower wants to keep the same monthly payments.

1. What maturity term would give a 9.25% yield to the lender with the same monthly payments?
2. What maturity term would be required if the borrower insisted on paying no higher than 10.5% mortgage constant (f)?

1. Solve for term with 9.25% effective yield:

CL FIN
20 12×
9 12÷
80000 PV
PMT
9.25 12÷
n

	CL F
20.00	12×
9.00	12÷
80000.00	PV
END	PMT
-719.78	***
9.25	12÷
END	n
253.05	***

253.05 months or 21.09 years (21 years 2 months)

2. Solve for term with 9.25% effective yield and 10.5% constant:

RCL PV
10.5 %
12 ÷
CHS
PMT
n

÷	PV
80000.00	***
10.50	%
8400.00	***
12.00	÷
700.00	***
	CHS
-700.00	***
	PMT
END	n
277.16	***

f

277.16 months or 23.10 years (23 years 2 months)

Note:

In this case, monthly payment can be no higher than $700.

c. Refinancing Decisions

Selection among alternative financing packages has been covered and illustrated in the Financial Analysis section of the HP-92 Investor Application book.

Impact of Financing Alternatives
Refinancing
Wrap-Around Mortgages

In the examples provided in those sections, it is demonstrated that the NPV and IRR to the mortgagee (lender) can be calculated, and then compared to find the alternative most preferable to the lender.

It is also possible to calculate IRR and/or NPV to the mortgagor (borrower) on the net amount of cash received on refinancing or wrap-around mortgage proposals. A comparison of these measures indicates which alternative is most preferable to the borrower.

One other method of evaluating refinancing or wrap-around mortgage proposals for the borrower is to calculate the periodic income forecast to be receivable by the borrower on the net cash proceeds of the loan at the rate the borrower expects to earn on the reinvested proceeds. If this income exceeds the additional dept service required to pay off the net cash proceeds of refinancing, it is an attractive or "feasible" transaction for the borrower.

Example—An existing mortgage has a current balance of $133,190. It has a remaining term of 17 years, with monthly payments of $1080.33.

It is proposed to refinance (or "wrap") with a new mortgage of $180,000 at 9% interest, payable in level monthly payments over 17 years.

The equity investor can reinvest the net proceeds at 13.5% on an annual basis. Should he accept the refinancing (or wrap-around) proposal?

Calculate and store net additional ADS:

CL FIN		CL F
17 12×	17.00	12×
9 12÷	9.00	12÷
180000 PV	180000.00	PV
PMT	END	PMT
1080.33 +	-1725.85	***
12 ×	1080.33	+
	-645.52	***
	12.00	×
STO 0	-7746.20	***
		→ 0

Net additional ADS

Calculate coverage of net ADS:

		← PV
RCL PV	180000.00	***
133190 −	133190.00	−
PV	46810.00	***
17 n		PV
13.5 i	17.00	n
PMT	13.50	i
RCL 0	END	PMT
x≷y	-7149.91	***
−		← 0
	-7746.20	***
		X≷Y
	-7149.91	***
		−
	-596.29	***

Coverage of net ADS

No. The income produced from reinvesting the cast proceeds at 13.5% does not cover the net annual debt service on the refinancing (or wrap-around).

d. Selection Among Disparate Alternatives: Size and Time Disparity

Not all investments require the same capital outlay, nor do all produce income streams of the same length. In order to make investment opportunities requiring different capital outlays comparable for selection of the best for the investor, it is necessary to identify what can or must be earned on the portion of available investment funds not committed to the smaller investment.

Example—An investor is considering two alternative investments. The first requires a Capital Outlay of $88,000. It has forecast CTO of $10,000 per year plus forecast NCPR of $100,000 in 10 years. The second requires an equity investment of $102,500, with forecast CTO of $11,500 per year and NCPR of $115,000 in 10 years. The minimum acceptable equity yield rate (y) is 11%.

NPV, PI and y of $88,000 investment:

CL FIN	CL F	
10 n	10.00 n	
11 i	11.00 i	
10000 PMT	10000.00 PMT	
100000 FV	100000.00 FV	
PV	END PV	
CHS	-94110.77 ***	
88000 −	CHS	
	94110.77 ***	
	88000.00 −	
LAST X	6110.77 ***	NPV
RCL PV	LSTX	
CHS	88000.00 ***	
x≷y	÷ PV	
÷	-94110.77 ***	
	CHS	
LAST X	94110.77 ***	
CHS	X≷Y	
PV	88000.00 ***	
i	÷	
	1.07 ***	PI
	LSTX	
	88000.00 ***	
	CHS	
	-88000.00 ***	
	PV	
	END i	
	12.14 ***	y

NPV, PI, and y of $102,500:

CL FIN	CL F	
10 n	10.00 n	
11 i	11.00 i	
11500 PMT	11500.00 PMT	
115000 FV	115000.00 FV	
PV	END PV	
CHS	-108227.38 ***	
102500	CHS	
−	108227.38 ***	
	102500.00 −	
LAST X	5727.38 ***	NPV
RCL PV	LSTX	
CHS	102500.00 ***	
x≷y	↑ PV	
÷	-108227.38 ***	
	CHS	
	108227.38 ***	
LAST X	X≷Y	
CHS	102500.00 ***	
PV	÷	
i	1.06 ***	PI
	LSTX	
	102500.00 ***	
	CHS	
	-102500.00 ***	
	PV	
	END i	
	11.92 ***	y

On a direct comparison basis, the $88,000 investment is clearly preferable:

	$88,000	**$102,500**
NPV	+$6,110.77	+$5,727.38
PI	1.07	1.06
IRR	12.14%	11.92%

However, there remains the question of what rate must be earned on the unused portion of available investment funds.

Calculate required y on unused funds:

10 n	10.00 n
11500 ENTER↑	11500.00 ENT↑
10000 −	10000.00 −
PMT	1500.00 ***
102500 ENTER↑	PMT
	102500.00 ENT↑

88000 −
CHS
PV
115000 ENTER↑
100000 −
FV
i

88000.00	
14500.00	***
	CHS
-14500.00	***
	PV
115000.00	ENT↑
100000.00	-
15000.00	***
	FV
END	i
10.56	***

Required y

If the investor can earn 10.56% on the unused $14,500, the two investments are exactly equal. If he can earn more than 10.56% on the unused funds, the $88,000 investment is preferable; if less, the $102,500 investment is preferable.

OTHER REAL ESTATE DECISIONS

There are other types of real estate problems and decisions which do not fall neatly into the foregoing categories. However the same general process of comparative analysis applies in these cases as well.

Leased Fee (Lessor's Interest) valuation and analysis involves estimating the Present Worth of the cash flows of the lease, and the Present Worth of the reversion (forecast value of the property when the lease expires) during the term of the lease. Effective yield to the lessor is calculated as an IRR.

However, leasehold valuation and calculation of yield on a leasehold purchase also require attention, especially since it is usually not possible to obtain rates of discount readily from the market. The leasehold valuation procedure is to calculate it as a residual from Market Value minus Leased Fee Value. The effective yield on a leasehold is calculated as an IRR for a fully amortized annuity.

Example—A property is leased at $1,000 per month with a remaining term of 12 years. The property is forecast to be worth $125,000 when the lease expires. The rate of discount for the leased fee is 10%.

The market rental for this type of property is $1100 per month. The rate of discount for market value purposes is 10.5% What is the present worth of the leasehold?

Market Value Estimate:

CL FIN
12 n
10.5 i
1100 ENTER↑
12 ×

	CL F
12.00	n
10.50	i
1100.00	ENT↑
12.00	x

PMT
125000 FV
PV
CHS
STO 0

```
 13200.00  ***
           PMT
125000.00   FV
       END  PV
-125498.75  ***
           CHS
 125498.75  ***
          → 0
```

PW Leased Fee:

10 i
1000 ENTER↑
12 ×
PMT
PV

```
    10.00    i
  1000.00 ENT↑
    12.00    x
 12000.00  ***
           PMT
       END  PV
-121593.15  ***
```

PW Leased Fee

PW Leasehold:

RCL 0
+

```
          ← 0
 125498.75  ***
             +
  3905.59   ***
```

PW Leasehold

REAL ESTATE SYMBOLS AND TERMINOLOGY USED IN THE HP-92 REAL ESTATE AND INVESTMENT ANALYSIS BOOK.

1. INCOME SYMBOLS

PGI: Potential Gross Income (Number of rental units times rental per unit, at 100% occupancy, annually)

v: Allowance for vacancy and income loss (annual)

EGI: Effective Gross Income: Rent Collections plus "Other Income" (PGI − v + "Other" = EGI, annual)

OE: Operating Expenses (annual)

NOI: Net Operating Income (annual: EGI − OE = NOI) Also: NIBR = NOI

ADS: Annual Debt Service (Monthly mortgage payment × 12)

CTO: Cash Throw-Off to Equity (annual: NOI − ADS = CTO); Gross Spendable Income

ATCF: After-Tax Cash Flow (annual: NOI − Income Tax Liability = ATCF); Net Spendable Income

2. VALUE (Present Worth, Reversion) SYMBOLS

V: Value (Present Worth)

PW: Present Worth (Value, Present Value)

SP: Sales Price

V_m: Value, Principal, Present Worth of Mortgage

V_e: Value, Present Worth of Equity

P: Principal of Mortgage

CPR: Cash Proceeds of Resale; Reversion (forecast; before tax) CPR = SP − Selling of Disposition Expenses

b: Balance of Mortgage Outstanding

NCPR: Net Cash Proceeds of Resale (to equity; before tax); Equity Reversion NCPR = CPR − b

ATNCPR: After-Tax Net Cash Proceeds of Resale (to equity); After-Tax Equity Reversion

FW: Future Worth (Reversion; Resale Proceeds)

3. COMPOUND INTEREST AND DISCOUNT FACTOR SYMBOLS

FW 1: Future Worth of One; Compound Amount of One

FW 1/A: Future Worth of One per Period; Accumulation of One per Period

SFF: Sinking Fund Factor; $1/s_n$

$1/s_n$:	Sinking Fund Factor; SFF
PW 1:	Present Worth of One; Reversion Factor
PW 1/A:	Present Worth of One per Period; Level Annuity Factor; Inwood Factor; a_n
a_n:	Present Worth of One per Period; Level Annuity Factor; Inwood Factor; PW 1/A
Amort.:	Installment to Amortize One; $1/a_n$
$1/a_n$:	Installment to Amortize One

4. RATE, CAPITALIZATION RATE, RATE OF RETURN SYMBOLS

R:	Overall Rate (on property investment): annual (NOI ÷ V; NOI ÷ SP)
r:	Basic Rate; annual
f:	Mortgage Constant: annual (ADS ÷ P)
i:	Mortgage Interest Rate (contract): annual
R_e:	Equity Dividend Rate: annual (CTO ÷ V_e)
y:	Equity Yield Rate; Rate of Return on Equity Investment: annual
IRR:	Internal Rate of Return: annual
CRR:	Capital Recovery Rate (on improvements): annual
CR:	Capitalization Rate (for investment in improvements): annual (CR = r + CRR)
n:	Income Projection Period; Investment Holding Period

5. MORTGAGE-EQUITY (Ellwood) ANALYSIS SYMBOLS

NOI:	Net Operating Income (annual)
CF:	Cash Flow (annual)
ADS:	Annual Debt Service
CTO:	Cash Throw-Off to Equity; Equity Dividend (CTO = NOI − ADS)
P:	Mortgage Principal (original)
b:	Mortgage Balance Outstanding at End of Income Projection Period (n): p = P − b
f:	Mortgage Constant (annual)
I:	Mortgage Interest Rate (nominal; annual)
i:	Effective Interest Rate (period of conversion)

m:	Loan-to-Value Ratio; Mortgage Principal as a Percentage of Value
e:	Equity Dividend Rate (annual)
y:	Equity Yield Rate (annual)
(l − m):	Equity as a Percentage of Value
c:	Mortgage Coefficient ($c = y + p\ 1/s_n - f$)
$1/s_n$:	Sinking Fund Factor at Equity Yield Rate (y) over Income Projection Period (n)
r:	Basic Rate; (annual)
R:	Overall Rate (annual)
dep.:	"Depreciation", Capital Loss on Resale as a Percentage of Value
app.:	"Appreciation", Capital Gain on Resale as a Percentage of Value
CPR:	Cash Proceeds of Resale; Reversion at End of Income Projection Period (n)
NCPR:	Net Cash Proceeds of Resale to Equity; Equity Reversion at End of Income Projection Period (n): NCPR = CPR − b

6. CASH FLOW AND INVESTMENT ANALYSIS SYMBOLS

CO:	Capital Outlay; Investment in Time Period 0
CF_t:	Cash Flow (positive or negative) in Time Period "t"
PW:	Present Worth; Present Value
NPV:	Net Present Value (NPV = PW − CO)
PI:	Profitability Index (PI = PW ÷ CO)
IRR:	Internal Rate of Return

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