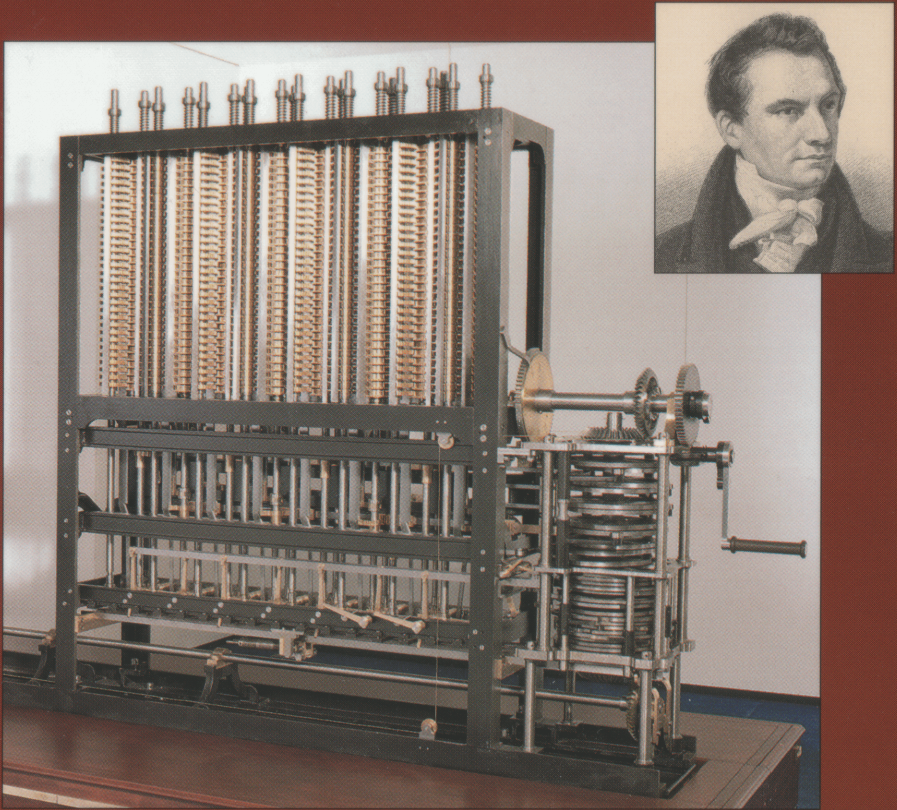
A Modern Difference Engine



Software Simulators for Charles Babbage's Difference Engine No. 2

James Donnelly

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Dedicated to all who follow Babbage's footsteps.

Contents

List of Illustrations

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My immense gratitude also goes to my wife Janet, who provided inspiration, support, ideas, and encouragement.

Introduction

The centerpiece of the Babbage exhibit at the Science Museum in London is the recently completed Difference Engine No. 2 – a modern realization of one of Charles Babbage's never-fulfilled designs. Indeed, Difference Engine No. 2 is a spectacular achievement that can be viewed as a technological masterpiece, kinematic art, or the intellectual vindication of Babbage's quest.

Inspiration

The author was one of a group of people attending *The Anniversary Conference* at Imperial College, London, in 1992. Organized by the Handheld & Portable Computer Club (HPCC), in association with the Imperial College Computing Department, this conference marked the 20th anniversary of the introduction of the HP–35 handheld calculator. The conference activities included a live demonstration of Difference Engine No. 2. Some attendees were left not only impressed, but wanting to explore further the types of problems that could be addressed with the Engine.

Except in limited instances, it is not possible for members of the general public to enter a problem into Difference Engine No. 2 and turn the crank. The two simulators provided on the disk, *The PC Difference Engine* and *The HP 48 Difference Engine* provide a chance to experiment with the method of differences in a manner related to the configuration of Difference Engine No. 2, with some added convenience from the modern computer age.

Organization

As the history of computation gets told to future generations, Charles Babbage (1791-1871) and his contributions deserve some mention, if at least to provide a historical context for those who followed. Babbage proposed designs for two basic types of machines for the mechanization of calculation: the *difference engine* and the *analytical engine*. Babbage is perhaps most famous for the concepts embodied in the design of his analytical engine.

This book begins with a discussion of Babbage's contributions, then follows with instructions for the use of *The HP PC Difference Engine* and *The HP 48 Difference Engine*.

Disk Contents

The disk contains two directories, *PC* and *HP48*. The files included are:

PC	README	Version number and notes.
	BABBAGE.EXE	The PC Difference Engine.
	BABSET.EXE	An initializing program.
HP48	README	Version number and notes.
	BABLIB	The Babbage Library.
	BABDIR	The Babbage Directory.

The PC programs may be run on any IBM compatible personal computer, and include special screen formatting code for the HP 95LX palmtop computer. The HP 48 programs may be run on any version of the HP 48 calculator.

Charles Babbage

Born into the upper middle class in 1791, Charles Babbage was the son of Benjamin Babbage and Elizabeth Teape. Benjamin Babbage was a goldsmith turned banker; his wife was the daughter of a prosperous family from Totnes, England. Benjamin moved to London from Totnes in 1791, where Charles was born. Educated mostly by private tutors, Charles Babbage received a classical education. In 1810 he entered Trinity College, Cambridge to become immersed in the center of intellectual life in England.

ENGRAVING OF CHARLES BABBAGE AT AGE 41 BY R. C. ROFFE

Upon arrival at Cambridge, Babbage quickly discovered the sorry state of mathematics therein, despite Cambridge's reputation in the subject. Babbage's interest in mathematics was wider than that of his tutor's, and thus he was forced to forge ahead on his own and in association with friends. Along with John Herschel (son of Sir William Herschel, the discoverer of Uranus), George Peacock, and others, Babbage founded the Analytical Society in 1812, in part to take sides in the debate between Newton's calculus which used "dot" notation and Leibniz's calculus with "d" notation. Only a few British mathematicians could read Continental publications, and most were staunchly loyal to Newton. Babbage and his friends, calling themselves "The Analyticals", took the risky political stance of supporting the adoption of Continental (particularly French) science and mathematics against the background of the French revolution and Britain's wars with France. The Analyticals provided a strong reforming influence on mathematics in Cambridge. The impact of the reform movement was felt throughout the worlds of science, technology and manufacturing.

Babbage graduated from Peterhouse in 1814, and received his MA in 1817. In 1814 he married Georgiana Whitmore and had eight children. Of these only three survived him. In 1827, his wife, father, and two children died, and in 1834 his only daughter, Georgiana, died. Babbage spent a year touring Europe after his wife's death and was thereafter a changed man – somewhat more bitter than before, but still able to host glittering soirees at his home for London society. His surviving children were Benjamin Herschel, Dugald Bromhead, and Henry Prevost. Of these, Henry showed the most interest in Babbage's work on Difference Engines and Analytical Engines. After his father's death, Henry built a printing calculator that could add, subtract, multiply, and divide. Henry's engine was based on designs for the Analytical Engine, but did not prove to work reliably. This engine may be seen at the Science Museum.

Babbage was elected a Fellow of the Royal Society in 1816, participated in the formation of the Astronomical Society of London in 1820 (later renamed the Royal Astronomical Society), and was elected Lucasian Professor of Mathematics at Cambridge (a chair once held by Newton) in 1828. Originally reluctant to accept the Lucasian chair, Babbage held the post until 1839.

Babbage didn't fit into any particular mold; this is indeed part of his appeal to historians. He published works on actuarial principles for life insurance, tables of logarithms, studies in manufacturing techniques, and various attacks on the scientific institutions of the day. Babbage contributed to the development of dynamometer railway cars, uniform postal rates, diving bells, and even made suggestions about submarine design.

A particularly interesting idea that Babbage developed was the use of occulting lights for communications between lighthouses and ships at sea. He suggested that specific patterns of occultations could identify a particular lighthouse, and that the pattern could be adjusted automatically to indicate tidal depths. Babbage even suggested that common questions and answers could be "preprogrammed" on interchangeable disks that could be read by the occulting mechanism.

Despite Babbage's achievements and acceptance in the best of social and intellectual circles, he seems to have been a fairly embittered man by late life. Having failed to receive sufficient government funding to complete his Difference Engine, he furthered his attacks upon the Establishment. In fact, Babbage was rather well known for his campaigns against organ grinders in his London neighborhood, and this leaves some students of history with an incorrect "aftertaste" for his genius and achievements. Babbage died at his home in London in 1871.

Genesis of an Idea

In the early nineteenth century, mathematicians and natural philosophers were producing mathematical tables as the principal aids to computation. Slide rules had limited accuracy, a few decimals at best, and were more frequently designed for specialized applications rather than general computation. Tables were being published for multiplication, division, squares, cubes, logarithms, sines, cosines, and so on.

The Accuracy Problem

The need for accurate astronomical and mathematical tables for navigation was most acute, and doubtless was the most significant influence leading to Babbage's quest for a machine to replace human travail. Navigation and commerce were dependent upon accurate tables – lives were even at stake. Errors in tables were compared to unseen shoals that could sink a ship.

The process of generating tables was not only tedious, but prone to three major sources of error. Errors could be generated in calculation, transcription, and finally when the results were typeset with loose type.

Babbage and Herschel are known to have invested considerable effort into verifying the accuracy of existing tables. One account suggests that during these efforts in 1821 Babbage exclaimed to Herschel, "I wish to God these calculations had been executed by steam!", to which Herschel replied, "It is quite possible." This was in fact a major milestone in the history of computing, as Babbage then began to consider what sort of machinery might applied to the problem.

There were two classes of calculating engines that Babbage ultimately proposed, the Difference Engine and the Analytical Engine. The Difference Engine was a machine dedicated to the production of tables, and the Analytical Engine was designed to tackle a more general set of problems. Both were vastly more complicated than their predecessors, and would have tested the limits of available manufacturing technology.

The Method of Differences

One of the common tools used in Babbage's time for the computation of tables was the method of differences. A section of a fairly well behaved smooth (continuous) function can be approximated by a polynomial, with accuracy improving with higher degree polynomials and shorter sections of the function in question. The process of sub-tabulation is the use of differences to tabulate the function over the interval for which the polynomial fits. To understand this process, consider the formula

$$T=x^2+x+41$$

Describe the differences between successive values for T for integral values of x as Δ (known as the first difference), and differences between Δ's as Δ^2 (known as second differences) , and write them in tabular form as follows:

x	T	Δ	Δ^2
0	41		
		2	
1	43		2
		4	
2	47		2
		6	
3	53		2
		8	
4	61		2
		10	
5	71		2
		12	
6	83		

Table of differences for the function $T=x^2+x+41$

Notice that the progression of Δ is steady, and that Δ^2 is constant. To calculate the value T for x=4, add Δ^2 to the previous Δ to get 8, then add the new Δ to the previous function value 53 to get 61. Armed with this pattern, one can build up a table for the function using only addition. This was the key to designing a machine to do the job.

In fact, the process generalizes – any polynomial of degree n will have a nth difference Δ^n which is constant, and n additions are required to compute each new value of the function.

The Difference Engines

There are really two stories to be told about the Difference Engine. The first is the intent and the design of the Engine; the second is the story of the project itself. Both are fascinating in their own right.

The First Model

By 1822 Babbage completed the first working model of his proposed Difference Engine with six figure wheels, which proved the principle of his design. The engine could produce values at a rate of about one every six seconds, and so was not dramatically faster than a human. Regrettably, this model and its drawings have been lost.

The model's impact, however, was dramatic. In an open letter to Sir Humphrey Davy, then president of the Royal Society, Babbage outlined the design of his model and the potential for further development. Babbage proposed not only a machine to generate values, but also to print them. This would eliminate the two principal sources of errors commonly found in tables: computing and transcription. Babbage also noted that if a machine were to be used for computing differences, then polynomial approximations of higher orders could be used to span larger portions of the function in question.

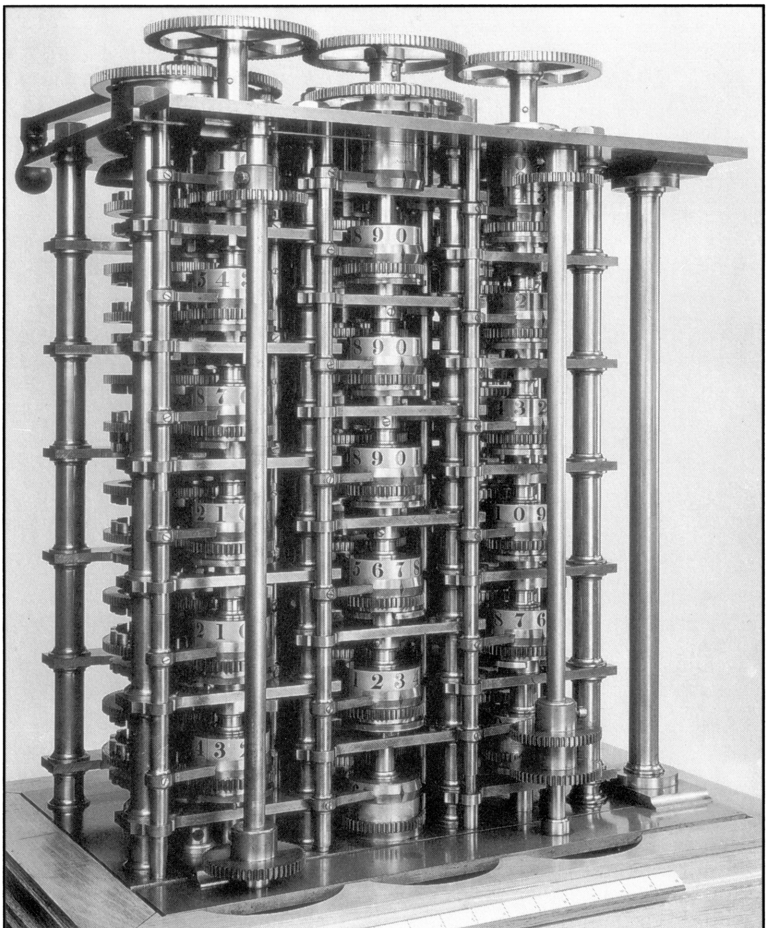
The Difference Engine

The scientific community recognized the importance of Babbage's achievements, and supported his proposal for government funding. Babbage was awarded the Gold Medal of the Astronomical Society of London.

The government funded an effort by Babbage to construct a full scale Difference Engine beginning in 1823, and so the world's first computer laboratory was born. The initial grant was for £1500, but over time the amount invested by the government grew to £17470. In contrast, the steam locomotive *John Bull,* completed in 1831 by Robert Stephenson and Co., cost about £784. It must be remembered that in the 1820s, complex mechanical mechanisms were essentially unknown beyond watches and the rare steam engine. Only in the 1830s did the spread of the railway bring the new engineering achievements into the popular domain.

Babbage hired an engineer named Joseph Clement to carry out the construction. Clement's shop became *the* engineering workshop in Britain. Many major contributions to the state of the art in light engineering in England can be traced to Clement's shop and the men who trained and worked within.

It was not until 1832 that a portion of the mechanism was actually assembled. Under pressure from the Royal Society to show that the project was proceeding properly, about one seventh of the Engine was completed. This portion may still be viewed in the Science Museum in London today. The machine was a success, and Babbage hosted many demonstrations.



CHARLES BABBAGE'S DIFFERENCE ENGINE NO. 1, PORTION, 1832

Work proceeded through September 1832, but a series of major factors intervened to stall the project short of completion:

- Goals had not been clearly set and agreed upon between Babbage and the government. Indeed, when the government ceased funding, Babbage felt betrayed.

- Despite his research and publications on manufacturing technology, Babbage was basically unable to lend practical project management skills to a research project of this complexity. In fact, it could be argued that neither Babbage nor Clement were up to the task of managing such a large project. One example of this problem was a pattern of demands for unneeded precision in areas that were not critical. Another problem was that Babbage had to reconcile the management aspects of projects in pure science and applied engineering, both appearing in the same project.

- The Reform movement, along with several changes in government, eliminated the political viability of further government support.

- Clement's shop attracted more and more business over the years, until the Difference Engine became a minor, rather than a major focus of effort. Clement wanted considerable compensation to move the project to Babbage's new workshops, and disputes over the amount were never resolved.

Work stopped in 1833, leaving Babbage's Difference Engine just short of completion. Babbage and Clement were quarreling over a variety of issues, and headstrong stands by many parties prevented any serious resumption of work. The consolation for the country was that the project contributed to a succession of important developments in machine tool technology and manufacturing techniques.

The Design

The Difference Engine consisted of a calculating mechanism and a printing mechanism, both driven by a single hand crank. The calculating mechanism was composed of seven axes of figure wheels, accommodating the tabular value (nearest the printing mechanism) and six axes of differences. A figure wheel is a horizontal gear wheel with the digits 0-9 inscribed about the circumference above the gear teeth, and a tab for triggering a carry lever. The elevation drawing for the Difference Engine in 1830 shows sixteen figure wheels for each axis. Each axis served as a number store and an adding mechanism. A mechanism within each figure wheel is designed to rotate the corresponding figure wheel on the next axis through as many steps as the value stored by the figure wheel.

The addition from one axis to the next consists of two basic steps: the addition of each figure wheel to the corresponding figure wheel on the next axis, then the propagation of carries from the least to the most significant figure wheels on the result axis. In later discussions of analytical engine design, Babbage suggested axes containing up to forty figures.

Rather than add the differences to subsequent axes sequentially, Babbage designed the Engine to first add the even differences to the odd differences simultaneously, then the odd differences to the even differences (and the first difference to the tabular value). This reduced the cycle time to four distinct phases (add, carry, add, carry) independent of the number of axes involved in the calculation. In modern computer parlance, this is known as an instruction pipeline. In practice, four full turns of the hand crank are required to complete a full cycle – one for each add and carry phase.

The complexity of the printing and control mechanism greatly exceeded that of the calculating mechanism. It has been suggested that the complexity of the printing mechanism was one of the major factors impeding the completion of the project. The printing mechanism was intended to imprint results directly to a soft metal printing plate. The control mechanism contained logic to print only one time those digits which changed in the more significant digits of a result. The design further provided for gaps after every fifth column or row to make the table easier to read.

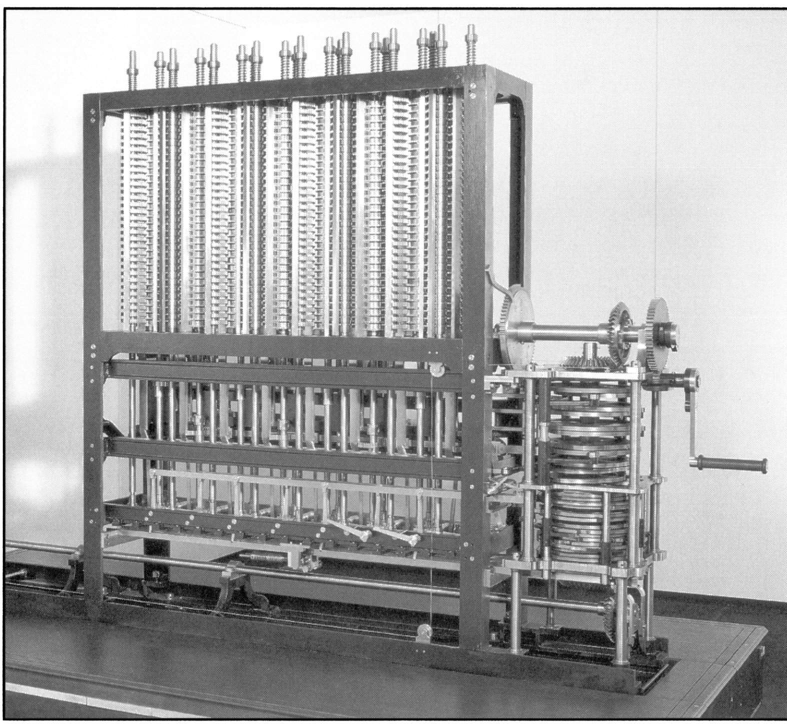
For instance, the following layout of a seven–place log table would have been possible:

N	0		1	2	3	4	5	6	7	8	9
2850	454	8449	8601	8753	8906	9058	9210	9363	9515	9668	9820
51		9972	0125	0277	0429	0581	0734	0886	1038	1191	1343
52	455	1495	1647	1800	1952	2104	2257	2409	2561	2713	2865
53		3018	3170	3322	3474	3627	3779	3931	4083	4235	4388
54		4540	4692	4844	4996	5148	5300	5453	5605	5757	5909
55		6061	6213	6365	6517	6670	6822	6974	7126	7278	7430
56		7582	7734	7886	8038	8190	8342	8494	8646	8798	8950
57		9102	9254	9406	9558	9710	9862	0014	0166	0318	0470
58	456	0622	0774	0926	1078	1230	1382	1534	1686	1838	1990
59		2142	2293	2445	2597	2749	2901	3053	3205	3357	3508
60		3660	3812	3964	4116	4268	4420	4571	4723	4875	5027
2861		5179	5330	5482	5634	5786	5938	6089	6241	6393	6545
62		6696	6848	7000	7152	7303	7455	7607	7758	7910	8062

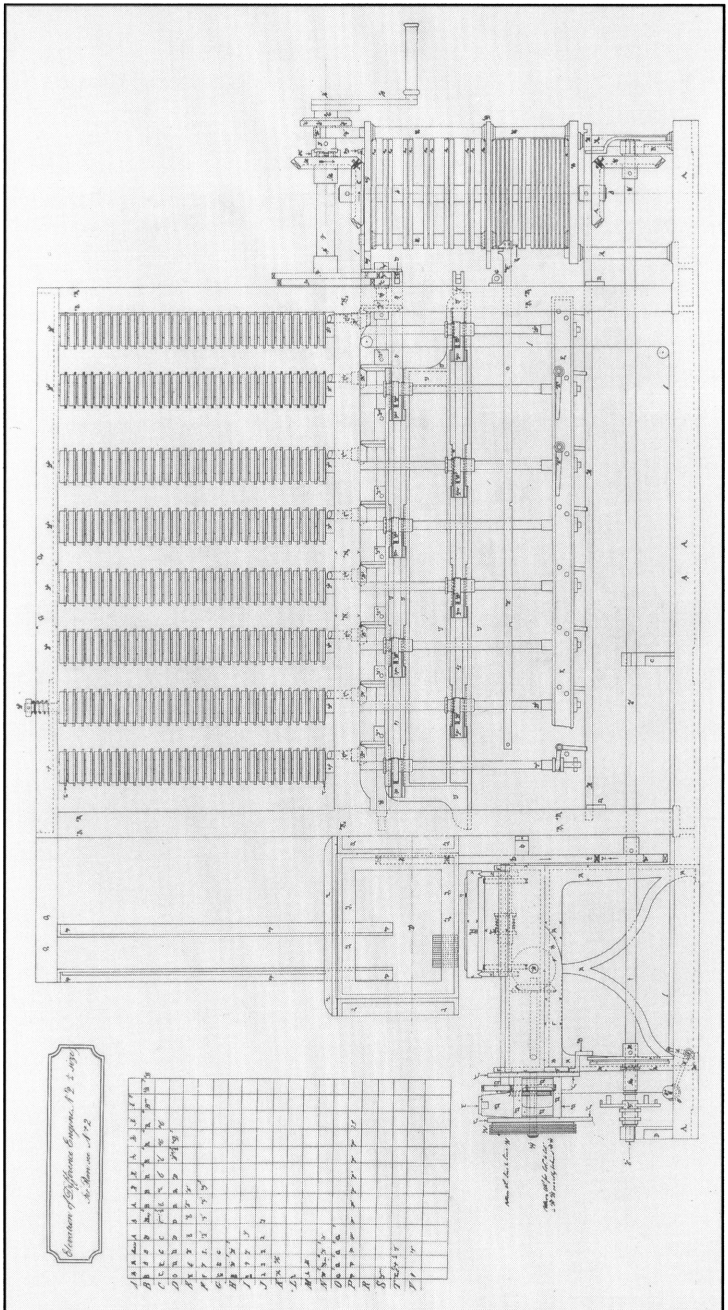
Difference Engine No. 2

Babbage designed a second machine, known as Difference Engine No. 2 between 1847 and 1849, based in part on new ideas developed while working on his Analytical Engine. This engine has 2–3 times fewer parts than the first design, and has a number of significant refinements in design layout and control.

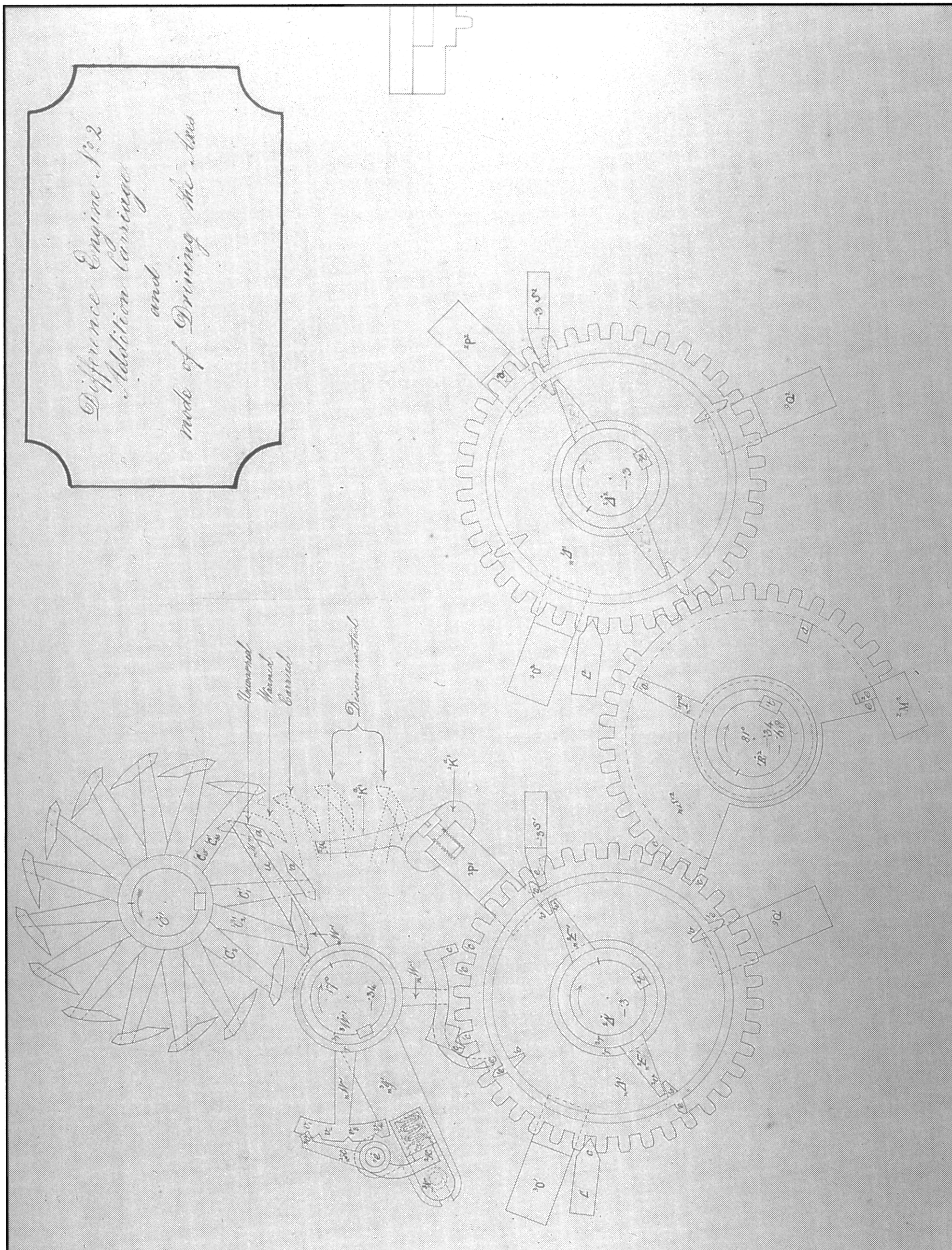
The calculating portion of Difference Engine No. 2 consists of eight axes of thirty one figure wheels. The bottom figure wheel represents the least significant digit. The tabular value is the axis on the left, and the highest order difference is on the right, nearest the driving crank. The Engine is controlled by a stack of fourteen conjugate cam systems. These cams control the timing of different actions within the calculating cycle by operating racks and levers below the axes. The Engine is 7 feet high, 11 feet long, and 18 inches deep. There are about 4000 moving parts and the Engine weighs about 3 tons. The Engine was completed in 1991 by a team at the Science Museum in London.



CHARLES BABBAGE'S DIFFERENCE ENGINE NO. 2, 1991



DESIGN DRAWING: DIFFERENCE ENGINE NO. 2, 1847



DESIGN DRAWING: DIFFERENCE ENGINE NO. 2 – DETAIL, 1847

The Scheutz Difference Engines

The first working difference engine to be completed was inspired by two publications: Babbage's *Economy of Machinery and Manufactures*, and a detailed description of the Difference Engine in the July 1834 edition of *Edinburgh Review* by Dionysius Lardner. Inspired by these accounts, the Swedish printer Georg Scheutz (1785–1873) and his son Edvard built a prototype which was completed in 1843. Based on the success of this prototype, Scheutz applied to the Swedish crown for funding to build a full-scale model. His initial request was denied, and it was not until 1851 that a resolution was passed to provide support.

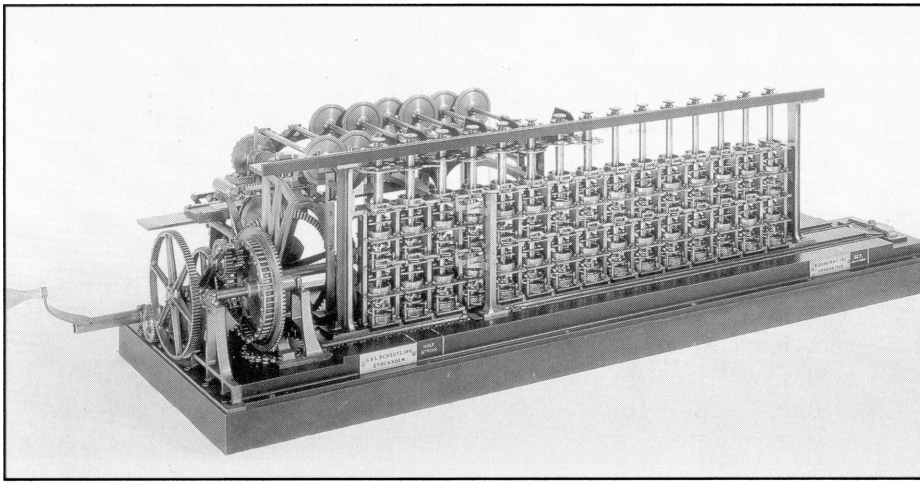
Georg Scheutz supervised the work on the first model, constructed with the assistance of J. W. Bergström. The first machine (complete with a working printing mechanism) was completed in late 1853. The arrangement of the axes is different from Babbage's designs – the axes are horizontal instead of vertical, although the figure wheels are still horizontal. The Scheutz engines had five axes of fifteen digits, accommodating the tabular value and four differences.

In 1854 the engine was taken to England under the auspices of the firm of Bryan Donkin and Company. Bryan Donkin (1768–1855) had extensive experience with special–purpose printing machines, and so the Scheutz engine fit right in with Donkin's experience. In fact, Donkin had supplied parts for Babbage's project. Bryan Donkin's second son Bryan was responsible for the engine's travel to England.

If Babbage had been disappointed in the lack of completion of his own designs, it did not dampen his enthusiastic support of the Scheutz engine beginning in 1855. The Scheutz engine won a gold medal at the Paris Exhibition in 1855, and broad acclaim followed. Georg Scheutz was elected a member of the Swedish Academy of Sciences in 1856 and shortly thereafter made a knight of the Order of Wasa.

In 1856 the engine was sold (with Babbage's help) to the Dudley Observatory in Albany, New York, after a string of failures to find a buyer in Europe. The engine was used to compute many sets of tables, including refraction tables and elliptic tables of the planet Mars.

Scheutz and Donkin built a second engine which was sold to the General Register Office in England in 1858. Shown below, this engine can now be seen in the Science Museum in London.



THE THIRD SCHEUTZ DIFFERENCE ENGINE, 1859

Neither engine was used for very long. Two major problems were evident. First, there was no positive locking for the positions of the figure wheels, allowing errors to creep in; secondly the carry mechanism was slow, causing the entire calculating process to be tedious. The print mechanism was not without its own difficulties.

Two additional engines were inspired by the Scheutz engines. One was built by Martin Wiberg (1826–1905) in Sweden, the other by George Grant (1849–1917) in the United States. Wiberg's engine was much more compact than the Scheutz engine, using metal disks instead of figure wheels, and was more reliable. By 1860 this engine was complete and had generated some interest rate tables. Grant's engine was completed and exhibited at the 1876 Centennial International Exhibition at Philadelphia. Like the Scheutz engines, both were ultimately relegated to history's curiosity collection.

Many years later, a modern version of these machines was installed at the Columbia University Statistical Bureau. Built by IBM in 1931, the machine was known as a "Difference Tabulator".

The Analytical Engine

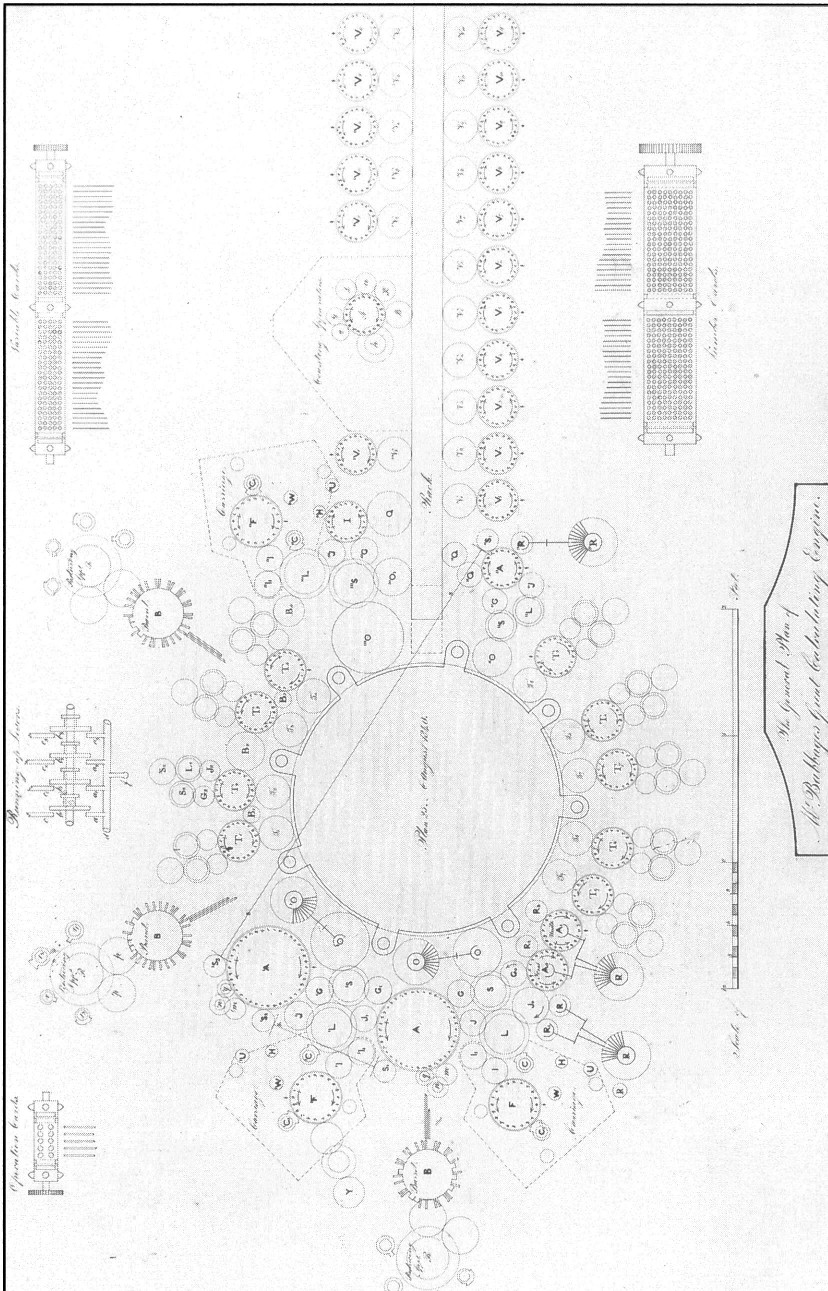
In 1833, after work stopped on the Difference Engine, Babbage began to further consider his earlier work on feedback functions and consider new mechanical methods for arithmetic operations. The design of his "Analytical Engine" was to evolve in many stages from 1836 to 1847, when Babbage turned back to begin the design of Difference Engine No. 2, based on some of his new ideas.

The Design

The design for the Analytical Engine was unlike that for the Difference Engine. Instead of being designed for a single purpose, calculating differences, the Analytical Engine was to be a universal calculating machine. Babbage designed a new system for managing carry propagation which he called the Anticipating Carriage, a step process for multiplication and division. Rather than storing numbers in accumulators, as had been done in the Difference Engine, Babbage separated the actions of calculation from storage. Thus the "mill" was the arithmetic unit, and the "store" was a memory for numbers. Numbers were represented in the mill and the store with forty digits and a separate sign wheel.

The process of multiplication and division was under the control of a series of "barrels" which activated control rods in a manner similar to those found in music boxes. Vertical columns of up to 200 studs on the barrel formed instruction sequences. Decisions could be made based on the results of a calculation. In fact, the design implemented what is now known in computer parlance as looping and relative addressing. The control logic was divided into several logical layers, with barrels at the simplest level, and overall control provided by punch cards (inspired by the Jacquard loom) at the highest level to take the place of barrels that would have been unmanageably large.

Programming by punch cards was divided into two independent streams of cards, one to specify a sequence of operations, the second to specify the source and destination of numbers being transferred between the mill and the store. Babbage realized that by combining programming cards and the ability to store and retrieve numbers to and from punch cards that the size and complexity of programs could be extended beyond the capacity of the mill and the store. Thus, he argued, the Analytical Engine should be able to carry out any calculation.



DESIGN DRAWING: THE ANALYTICAL ENGINE, 1840

Babbage never attempted to build the Analytical Engine in its entirety, possibly because of his disappointment that the Difference Engine was never completed. Several small mechanisms were constructed to validate portions of the design. After Charles Babbage died, his son Henry Prevost carried on the work through about 1910. The portion of the mill, arithmetic and printing mechanism remains in the Science Museum today. The only operations known to have been performed on it were apparently not without error.

The Legacy

From a practical point of view, Babbage's work probably had more influence on the British machine tool industry than the world of automatic calculation. Babbage's great recognition in the world of computing stems from his early conceptual breakthroughs in computer design which weren't fully realized for another century. Subsequent to Babbage's designs, several difference engines were built, notably the three by Scheutz, which were completed and promoted with Babbage's help.

Babbage's engines probably could have been built, given the resources, but they were large and very expensive. Even if the Analytical Engine had been built, it's unlikely that commercial production would have followed. Indeed, it is a loss that Babbage never published detailed designs for the Analytical Engine.

There is considerable danger that Babbage's efforts will be taken beyond their actual, lasting contributions to the world of computers. With the exception of the Scheutz engines, there are no computational devices that trace their roots to any of Babbage's designs. Doron Swade, Senior Curator of Computing and Control at the Science Museum, suggests that Babbage should be considered an *uncle*, not a father, of modern computing.

A favorite uncle, indeed!

The PC Difference Engine

The programs BABBAGE.EXE and BABSET.EXE provided on the disk can be executed on either an IBM–compatible personal computer or the Hewlett-Packard HP 95LX palmtop computer.

The Setup Program BABSET

The program BABSET.EXE is used to provide initial conditions for polynomials of the form:

$$C_7X^7+C_6X^6+C_5X^5+C_4X^4+C_3X^3+C_2X^2+C_1X+C_0$$

Enter the coefficients in ascending order. For example, set up the Engine for the problem X^2+X+41 at the value X=1.

In the dialog below, your answers are underlined:

```
C>BABSET
Enter order of polynomial (2-7)? 2

Enter constant: 41
Enter coefficient of 1st order term: 1
Enter coefficient of 2nd order term: 1

Y=X^2+X+41

Starting value for x? 1

d2: 2
d1: 2
 T: 43

C>_
```

The Engine Display

To run your PC Difference Engine, execute BABBAGE.EXE. For both the PC and the HP 95LX, the Engine is displayed as follows:

```
7: 0000000000000000000000000000000
6: 0000000000000000000000000000000
5: 0000000000000000000000000000000
4: 0000000000000000000000000000000
3: 0000000000000000000000000000000
2: 0000000000000000000000000000000
1: 0000000000000000000000000000000
T: 0000000000000000000000000000000
```

Each line of numbers represents a vertical axis on Difference Engine No. 2. The line at the bottom is the *tabular value*, or result axis. The second line from the bottom is the *first difference*, the line above is the *second difference*, and so on.

Active Keys

The cursor keys (◄►▲▼) are used to move the cursor between figures when the Engine is being set up.

The function keys have slightly different labels, depending on whether the PC Difference Engine is being run on a PC or an HP 95LX:

Key	PC	HP 95LX	Purpose
F1	CLALL	CLA	Sets all figures in the Engine to zero.
F2	CLROW	CLR	Sets all figures in an axis to zero.
F3	+–	+–	Takes the 10s complement of an axis.
F4	SET	SET	Switches to setup mode.
F5	RUN	RUN	Switches to run mode or "turns the crank".
⇑ F5	RUN	RUN	Shift-F5 switches to run mode or "turns the crank" continuously until a key is pressed.
F7	LOG	LOG	Logs tabular values to the file ENGINE.LOG.
F8	BELL	BELL	Allows the "bell" to ring upon overflow.
F10	QUIT	QUIT	Terminates the Engine.

Setting the Engine

The function keys SET and RUN switch the Engine between two basic modes: *setting up* the Engine, and *running* the Engine. Like the real Difference Engine No. 2, the PC Difference Engine cannot be operated while the figure wheels are being set. (The real Difference Engine No. 2 requires a very specific order of operations – the engineers who built the Engine discovered that missing a step in the setup can cause parts to break, which can be quite expensive.)

To set up the Engine, make sure the Engine is in setting mode, then use the cursor keys to move the cursor, and the digit keys to enter figures. To take the 10s complement of a number, press +– (F3).

The user interface for the PC Difference Engine makes a few concessions to the new computer age. On the real Engine, all figure wheels are set by hand, and there are no shortcut mechanisms. For this simulator, the clearing keys provide more functionality than exist on Difference Engine No. 2. Press CLALL to clear the entire Engine, or CLROW to clear the axis that the cursor is on.

Running the Engine

Once a problem has been set up, press RUN to switch to running mode, then RUN for each time you wish to "turn the crank".

Logging Results

The status of the Engine may be recorded in the data file ENGINE.LOG for each turn of the crank. When you press the function key LOG (F7), the current value of the tabular axis is appended to the end of the file. Thereafter, each time you press RUN, the newly calculated value for T will be appended to the end of the file. Each value is written to a new line in the log file, so the data is ready to be printed or perhaps imported into another application, such as a spreadsheet. Press LOG again to stop logging data.

Data is always logged to the same file name, so it is suggested that you rename any existing data file ENGINE.LOG to preserve results before starting a new experiment.

The Bell

A "bell" feature (not found on Difference Engine No. 2) has been added to sound a tone when an addition into any axis carries beyond the most significant figure wheel. This may be switched on and off by pressing the function key BELL (F8).

Terminating the Engine

To end the program, press QUIT (F10). If data logging was in progress, the file ENGINE.LOG will be ready for inspection.

The HP 48 Difference Engine

The HP 48 Difference Engine is an HP 48 program that has been designed to simulate the operation of Difference Engine No. 2. Three files are in the HP48 directory on the disk:

README Read this file to find the current version of the Difference Engine and any update information.

BABLIB The HP 48 Difference Engine Library.

BABDIR A directory containing programs to help set up problems for The HP 48 Difference Engine.

Installing the Library

The HP 48 Difference Engine has been implemented as a library object with library ID #36Bh. To install the library, perform the following:

- Download the library to the HP 48 in binary mode.
- Recall the library to the stack.
- Purge the variable that the library was stored in, leaving the library object in level one of the stack..
- Store the library object in a port, such as port 0. For instance, when the library object is in level one of the stack, execute 0 STO.
- Turn the calculator off, then on again. The calculator will perform a system halt, which updates the system configuration to recognize the new library. The library automatically attaches itself to the HOME directory.

To display the library commands, press [↰] [LIBRARY] BABLI. The command DE48 runs the simulator, TITLE displays the title picture, and BLVER displays the version of the software.

Removing the Library

To remove the library, switch to the HOME directory, enter `:0:875` (where `0` refers to the port in which the library is stored), duplicate the entry, and execute **DETACH** and then **PURGE**. The HP 48 display will flicker briefly as the library is purged.

The Setup Directory BABDIR

The directory BABDIR contains a program to help set up problems for the Engine. Set the HP 48 to the directory in which you wish to store BABDIR, and download the directory in ASCII mode. To view the variables in BABDIR, press `BABDI` in the VAR menu.

To determine the initial values for a problem, enter a polynomial expressed in terms of X, the value for X, and evaluate **BABSET**. For example, determine the Engine settings for the polynomial *X^4-2*X^3+4*X^2-5*X+3* at initial value -6:

```
2: 'X^4-2*X^3+4*X^2-5*X+3'
1:                      -6
```

BABSET ⇒

Axis	Label	Value
Fourth difference	d4:	24
Third difference	d3:	-192
Second difference	d2:	682
First difference	d1:	-1416
Tabular value	T:	1905

To remove the BABDIR directory, move to its parent directory, enter `'BABDIR'` and execute **PGDIR**.

The Engine Display

To run your HP 48 Difference Engine, press [↰] [LIBRARY], BABLIB then DE48. The title graphic appears, then the HP 48 Difference Engine is displayed:

```
7:0000000000000000000000000000000
6:0000000000000000000000000000000
5:0000000000000000000000000000000
4:0000000000000000000000000000000
3:0000000000000000000000000000000
2:0000000000000000000000000000000
1:0000000000000000000000000000000
T:0000000000000000000000000000000
CLALL|CLRW|SET ■|RUN|0|QUIT
```

Each line of numbers represents a vertical axis on Difference Engine No. 2. The line at the bottom is the *tabular value*, or result axis. The second line from the bottom is the *first difference*, the line above is the *second difference*, and so on.

The function keys SET and RUN switch the Engine between two basic modes: *setting up* the Engine, and *running* the Engine. Like the real Difference Engine No. 2, the HP 48 Difference Engine cannot be operated while the figure wheels are being set. (The real Difference Engine No. 2 requires a very specific order of operations – the engineers who built the Engine discovered that missing a step in the setup can cause parts to break, which can be quite expensive.)

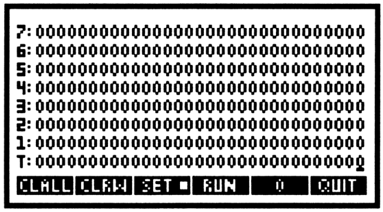
To set up the Engine, make sure the Engine is in setting mode, then use the cursor keys to move the cursor, and the digit keys to enter figures. To take the 10s complement of a number, press +– (F3).

The user interface for the HP 48 Difference Engine makes a few concessions to the new computer age. On the real Engine, all figure wheels are set by hand, and there are no shortcut mechanisms. For this simulator, the clearing keys provide more functionality than exist on Difference Engine No. 2. Press CLALL to clear the entire Engine, or CLRW to clear the axis that the cursor is on.

A printing mechanism is provided in the HP 48 Difference Engine. The Science Museum has not yet obtained the funds to build the printing portion of Difference Engine No. 2. That engine would be able to set tables in the manner described previously. In the case of the HP 48 Difference Engine, printing functionality is limited to listing either the entire state of the Engine or just the tabular value. Also, a counter for the number of turns is provided.

Setting the Engine

When the Engine is being set up for a new problem, a cursor appears at the least significant digit of the T axis:



The following keys are active:

General Operations

QUIT — Terminates the Engine.

[ON] — Terminates the Engine.

[NXT] — Displays the next page of menu keys.

BELL — Toggles the overflow "bell" on and off.

Cursor Movement

[▲][▼] — These keys move the cursor between axes.

[◀][▶] — These keys move the cursor between digits.

[↰][◀] [↰][▶] — The left-shifted arrow keys move the cursor eight figure wheels at a time.

[↱][◀] [↱][▶] — The right-shifted arrow keys move to the left or right end of the axis.

Setting Figures

[0] - [9]	The digit keys enter a digit for the figure wheel indicated by the cursor.
[+/-]	Sets the ten's complement of the number on the axis.
CLALL	Sets all figure wheels on all axes to zero.
CLRW	Sets all the figures on the current axis to zero.
[←] CLRW	Sets all the figures on the current axis to nine.
SET	Switches the Engine between set and run mode.
RUN	If the Engine is being set, switches to run mode. If the Engine is in run mode, "turns the crank once".
0	The fifth menu key label is the counter for the number of turns. Press the fifth menu key to reset this counter to zero.

Printer Control

Printed reports may be generated by selecting one of the available print options on the second page of menu keys. Printer output may be directed to either the IR port or the serial port, depending on the status of flag –34. If flag –34 is clear (default), printer output will be directed to the IR port.

PRINT	Prints the current status of the Engine. The FULL and TOT menu keys control whether all axes are printed or just the total axis.
TRC	When set, prints the status of the Engine according to the FULL and TOT keys every time the crank is turned.
FULL	Selects a full printout of the engine.
TOT	Selects the abbreviated printout of the engine (number of turns of the crank and total axis).

Running the Engine

The Engine may be run either once or continuously. To "turn the crank once", press RUN . If the trace function (TRC) has been selected, the status of the Engine will be printed according to the FULL or TOT selection.

To let the Engine run continuously, press ↰ RUN , then press any key to stop the Engine. If the trace function has been selected, the status of the Engine will be printed for each cycle.

When the "crank is turned" an overflow condition in any axis will generate a beep (simulating the bell in Babbage's early designs). This beep may be suppressed or restored by toggling the BELL menu key in the second row of menu keys.

Sample Problems

This chapter presents just a few examples of problems that you can set up on your own Difference Engine. Tables in Babbage's time were prepared with many approximations, so after you've run these examples, go on to reconstruct parts of your favorite function!

A Table of Squares

A simple table that can be generated with the Difference Engine is a table of squares. To set up the problem, place the differences d2=2, d1=1, and the tabular value T=1 into the Engine. This sets up the Engine for the X=1 condition:

7:	0000000000000000000000000000000
6:	0000000000000000000000000000000
5:	0000000000000000000000000000000
4:	0000000000000000000000000000000
3:	0000000000000000000000000000000
2:	0000000000000000000000000000002
1:	0000000000000000000000000000001
T:	0000000000000000000000000000001

Now each "turn of the crank" generates a successive square for the natural numbers. The Engine can be set up for any starting position, once the differences have been worked out. Here's the Engine position for X=6315:

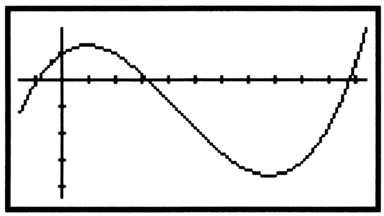
7:	0000000000000000000000000000000
6:	0000000000000000000000000000000
5:	0000000000000000000000000000000
4:	0000000000000000000000000000000
3:	0000000000000000000000000000000
2:	0000000000000000000000000000002
1:	0000000000000000000000000012269
T:	0000000000000000000000037638225

Traversing Roots

Consider the function

$$x^3 - 4x^2 + 2x + 1.$$

Plotted on the HP 48 in the range $(-.5 < x < 3.5)$ $(-4.2 < y < 1.875)$ the function looks like this:



The roots are at approximately –.3, 1, and 3.3.

To set up the Engine, follow either the *PC Preparation* or the *HP 48 Preparation* directions on the next pages, then proceed to *Running the Example.*

PC Preparation

To prepare the initial values for the PC Engine, use the BABSET.EXE program. Enter the coefficients in ascending order. In the following example, your answers are underlined:

```
C>BABSET
Enter order of polynomial (2-7)? 3

Enter constant: 1
Enter coefficient of 1st order term: 2
Enter coefficient of 2nd order term: -4
Enter coefficient of 3rd order term: 1

Y=X^3-4*X^2+2*X+1

Starting value for x? -2

d3: 6
d2: -26
d1: 41
 T: -27

C>_
```

When you're ready, the Engine should look like this:

7:	000000000000000000000000000000000
6:	000000000000000000000000000000000
5:	000000000000000000000000000000000
4:	000000000000000000000000000000000
3:	000000000000000000000000000000006
2:	999999999999999999999999999999974
1:	000000000000000000000000000000041
T:	999999999999999999999999999999973

HP 48 Preparation

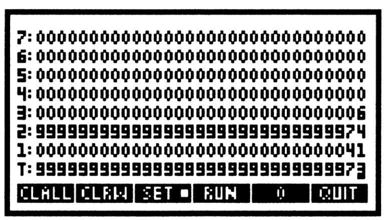
To prepare the initial values for the HP 48 Engine, use the BABSET function in BABDIR. Enter the function 'X^3-4*X^2+2*X+1' in stack level 2, and the integer value -2 for x.

The function BABSET returns the following initial values for the Engine:

Axis	Label	Value
Third difference	d3:	6
Second difference	d2:	-26
First difference	d1:	41
Tabular value	T:	-27

Set up the Engine by entering the values onto the axes. For negative numbers, you can enter the absolute value, then press [+/-] to take the 10s complement.

When you're ready, the Engine should look like this:



Running the Example

Now press SET (F4) to switch out of setup mode, and RUN (F5) to "turn the crank". Notice that the first root is passed after two turns:

7:	0000000000000000000000000000000
6:	0000000000000000000000000000000
5:	0000000000000000000000000000000
4:	0000000000000000000000000000000
3:	0000000000000000000000000000006
2:	9999999999999999999999999999986
1:	0000000000000000000000000000007
T:	0000000000000000000000000000001

The next root is found on the third turn:

7:	0000000000000000000000000000000
6:	0000000000000000000000000000000
5:	0000000000000000000000000000000
4:	0000000000000000000000000000000
3:	0000000000000000000000000000006
2:	9999999999999999999999999999992
1:	9999999999999999999999999999999
T:	0000000000000000000000000000000

The final root is passed during the sixth turn:.

7:	0000000000000000000000000000000
6:	0000000000000000000000000000000
5:	0000000000000000000000000000000
4:	0000000000000000000000000000000
3:	0000000000000000000000000000006
2:	0000000000000000000000000000010
1:	0000000000000000000000000000011
T:	0000000000000000000000000000009

From here on, the value of the function increases– we have passed the area of interest for finding roots.

Tabulating the Logarithm Function

The number of successive values of a function that can be tabulated depends on the quality of the approximation, the number of differences calculated, the number of significant figures in the digits, and the step size of the interval.

The Engine setup shown below contains differences calculated by an ordinary PC spreadsheet, using a limited number of significant digits. The differences were calculated for the initial value x=1000, stepping forward by x=1.

d7:	-7.54952E-15
d6:	-4.44089E-15
d5:	7.99361E-15
d4:	-2.62812E-12
d3:	8.72510E-10
d2:	-4.35165E-07
d1:	4.34512E-04
T:	3

Here is the initial setup for the Engine. Remember that negative numbers are entered as 10s complements:

7:	9999999999999999999999999245048
6:	9999999999999999999999999555911
5:	0000000000000000000000000799361
4:	9999999999999999999999737188000
3:	0000000000000000000087251000000
2:	9999999999999999564835000000000
1:	0000000000000043451200000000000
T:	0000000000300000000000000000000

As the crank is turned, values for log(x) are produced at first with many digits of precision, but with fewer as the approximation diverges from the actual function. By the value 1029, the last value good to 7 digits has been generated. By the value 1053, the last value good to 5 digits has been generated.

The engineer William Gravatt (1806–1866) worked closely with the Scheutz engine, and wrote about the operation of the engine in the publication *Specimen Tables Calculated and Stereomoulded by The Swedish Calculating Machine.* An extract of these notes has been published by Uta Merzbach in *Georg Scheutz* *and the First Printing Calculator.* In these notes, Gravatt suggests that the production of a table of Logarithms of the natural numbers from 1 to 10,000 to five places can be accomplished with "ten easy calculations, and by ten forward and ten backward settings of the machine". He estimates that the time required to produce the table would be 80 hours, given the time required to set up the engine and the engine's rate of production of 120 numbers per hour.

A Large Polynomial

One of the example problems used to test Difference Engine No. 2 at the Science Museum in London is a seventh order polynomial:

$$Y(X)=8X^7+2X^6+9X^5+5X^4+X^3+7X^2+4X+41$$

Calculating setup values for this problem produces quantities that approach the numerical limits of the setup programs provided on the disk. The calculated differences for the polynomial shown above at X=–27 are:

d7:	40320
d6:	-1208160
d5:	17511240
d4:	-163514400
d3:	1105301664
d2:	-5761960484
d1:	24097001804
T:	-83034482251

The precision of the setup programs are limited to fourteen figures for the PC program BABSET, and twelve figures for the HP 48 program. Large values of |X| can yield erroneous results.

The Powers of 7

The Difference Engine can rapidly generate values that exceed the precision of contemporary handheld calculators when applied to a series of powers. One example of this is the generation of a table of the powers of 7. The table below shows the differences for the powers of 7 from –7 to 10:

X	T	d1	d2	d3	d4	d5	d6	d7
-7	-823543							
		543607						
-6	-279936		-341796					
		201811		201726				
-5	-78125		-140070		-109200			
		61741		92526		52080		
-4	-16384		-47544		-57120		-20160	
		14197		35406		31920		5040
-3	-2187		-12138		-25200		-15120	
		2059		10206		16800		5040
-2	-128		-1932		-8400		-10080	
		127		1806		6720		5040
-1	-1		-126		-1680		-5040	
		1		126		1680		5040
0	0		0		0		0	
		1		126		1680		5040
1	1		126		1680		5040	
		127		1806		6720		5040
2	128		1932		8400		10080	
		2059		10206		16800		5040
3	2187		12138		25200		15120	
		14197		35406		31920		5040
4	16384		47544		57120		20160	
		61741		92526		52080		5040
5	78125		140070		109200		25200	
		201811		201726		77280		5040
6	279936		341796		186480		30240	
		543607		388206		107520		5040
7	823543		730002		294000		35280	
		1273609		682206		142800		
8	2097152		1412208		436800			
		2685817		1119006				
9	4782969		2531214					
		5217031						
10	10000000							

To calculate your own table of the powers of 7, set the Engine with values from the table on the previous page. For X=0, the differences are:

d7:	5040
d6:	-15120
d5:	16800
d4:	-8400
d3:	1806
d2:	-126
d1:	1
T:	0

The Engine should be set as follows:

7:	0000000000000000000000000005040
6:	9999999999999999999999999984880
5:	0000000000000000000000000016800
4:	9999999999999999999999999991600
3:	0000000000000000000000000001806
2:	9999999999999999999999999999874
1:	0000000000000000000000000000001
T:	0000000000000000000000000000000

When X reaches 26826, the largest value in the series that can be represented on the Difference Engine has been produced. At this point, the Engine will contain the following values:

7:	0000000000000000000000000005040
6:	0000000000000000000000135187920
5:	0000000000000000001813140384720
4:	0000000000000016212496970080320
3:	0000000000108729111232044996126
2:	0000000583375175566201366625550
1:	0002608464880430254827829662151
T:	9997500666573762420947214427776

The next "turn of the crank" will overflow the tabular value axis.

Personal computers require an extended or "infinite precision" math library to produce equal results showing all significant figures.

Bibliography

The following is a short list of books that the interested reader may wish to consider. Some are less central to the story of Babbage, but are included for their potential interest to the student of computer history.

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Morrison, Philip, and Emily Morrison., ed. *Charles Babbage On the Principles and Development of the Calculator* (New York: Dover Publications, 1961)

Swade, Doron *Building Babbage's Dream Machine* New Scientist: June 29, 1991

Swade, Doron *Charles Babbage and his Calculating Engines* (London: Science Museum, 1991)

Charles Babbage (1791–1871) was a man of many talents. He is best remembered for his designs of mechanical computing engines.

Of the two classes of calculating engines that Babbage proposed, the Difference Engine and the Analytical Engine, only the first Difference Engine came close to completion.

In 1991 the Science Museum in London completed the construction of the calculating portion of Babbage's Difference Engine No. 2. The success of this project proves that Babbage's design was sound.

This book includes a description of Babbage's designs and a disk containing two software simulators for Difference Engine No. 2. The simulators run on any IBM-compatible personal computer, Hewlett-Packard HP 95LX palmtop computer, or Hewlett-Packard HP 48 calculator.

These simulators provide you the means to relive and explore the uses of difference engines in the mid-nineteenth century in a manner never before possible.

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