## Sparcom

## Pocket Professional ${ }^{\text {TM }}$ OWNER'S MANUAL



# The Pocket Professional'" 

## Electrical Engineering Application Pac

## Owner's Manual

## SPARCOM ${ }^{\circledR}$

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## Chapter 1

## Getting Started

## In This Chapter

Sparcom's Pocket Professional- software is the first of its kind, developed to provide speed, efficiency and portability to students and professionals in the technical fields. When you slide the Pocket Professional- Electrical Engineering Application Pac into your HP 48SX, your calculator is instantly transformed into an electronic "textbook," ready to efficiently solve your electrical engineering problems. The Pac is organized into seven sections: Equation Library, AC Circuits, Fourier/Laplace Transforms, Ladder Network Analysis, Transmission Lines, Two-port Networks, and Constants Library. . . all available in an efficient, menu-driven format.
This chapter covers:

| $\square$ | Installing and Removing the Card |
| :--- | :--- |
| $\square$ | Using the Main Menu |
| $\square$ | Using the Equation Library |
| $\square$ | What You Should Know About the Solver |
| $\square$ | Summary of Functions |
| $\square$ | Summary of Softkeys |

## Installing and Removing the Card

The HP 48SX has two ports for installing plug-in cards. You can install your Electrical Engineering Application Pac card in either port. Be sure to turn off the HP 48SX while installing or removing the card. Otherwise, user memory may be erased.

## To Install the Application Card

1. Turn off the HP 48SX. Do not press ©0N until you have completed the installation procedure.
2. Remove the port cover. Press against the grip lines and push forward. Lift the cover to expose the two plug-in ports, as shown below:

## Getting Started


3. Select either empty port for the Pocket Professional card, and position the card just outside the slot. Point the triangular arrow on the card toward the calculator port opening, as shown above.
4. Slide the card firmly into the slot. After you first feel resistance, push the card about $1 / 4$ inch further, until it is fully seated.

5. Replace the port cover.

## Memory Requirements

The EE Application Pac requires some RAM memory in order to work. This memory is used for temporary storage, and for saving variables such as equations to be plotted later. Errors may be encountered if the available memory is less than about 4000 bytes. For more information, see Chapter 5 of the HP 48SX Owner's Manual.

## To Remove an Application Card

1. Turn the HP 48SX off. Do not press $\operatorname{ON}$ until you have completed the removal procedure.
2. Remove the port cover. Press against the card's grip lines and push forward. Lift the cover to expose the two plug-in ports, as shown below:

3. Press against the card's grip lines and slide the card out of the port, as shown above.
4. Replace the port cover.

## Accessing the Electrical Engineering Application Pac

After you turn on your HP 48SX by pressing ON, there are three ways to start the application.

First Method: Press 国 LBRARYy to display all libraries available to the HP 48SX. Find and press 药EAPP to enter the Electrical Engineering Application Pac library directory. The screen displays new menu keys or "softkeys" along the bottom, as shown:


Press the EEAPR softkey again to start the application.

The \#eemay softkey accesses the Constants Library function, described in Chapter 3. \#EEREW and \#EREDE are functions required by the software, but are not available to the user. बF\& and eneme are two programs available to the user to plot the gain and phase of a transfer function, and are explained in Chapter 4.

Additional softkeys are accessed by pressing the $\times x$ key. One of these keys is the AEOEF softkey. Pressing this key displays a screeen containing the revision number of the Electrical Engineering Application Pac. (Press $\pi$ 四 to exit the revision screen).

Second Method: Type in EEAPP (using alpha entry mode) and press ENTER.

Third Method: Add the command "EEAPP" to the CST (custom) menu (for more information, refer to Chapter 15 of the HP-48SX Owner's Manual, "Customizing the Calculator"). After the command has been added, press [GIT, then press $E A P$ P\% to start the software.

## Using the Main Menu

After you start the application, the main menu screen appears:


The main menu lists the seven major areas of application in a menu-driven format. Menu-driven means that the information is selected by moving the pointer to an item in the menu and pressing ENER.

## Applications in the Main Menu

Each application in the main menu is briefly described below and is discussed in detail in the next three chapters of this manual.

| Equation Library | Allows you to solve, plot and analyze over 300 <br> equations commonly used by electrical engi- <br> neers |
| :--- | :--- |
| Constants Library | Lists over 20 universal and physical contants, <br> plus 22 silicon properties and 5 magnetic prop- <br> erties |
| AC Circuits | Solves problems in 9 topics including imped- <br> ance, admittance, power factor, and star-to- <br> delta circuit transformation |
| Fourier/Laplace Trans- <br> forms | Lists tables of transforms; includes a pole zero- <br> analysis section, and FFT and inverse FFT <br> computation |
| Ladder Network Analy- <br> sis | Computes performance parameters for a <br> loaded ladder network |
| Transmission Lines | Allows you to compute propagation character- <br> istics, impedance, and VSWR for a transmis- <br> sion line |
| Two-port Networks | Computes circuit performance parameters for <br> a given source and load impedance, converts <br> between z, y, a and h parameters; and com- <br> bines two-ports into equivalent networks |

The "softkeys" located along the bottom of each screen give you options that relate to the information displayed on any given screen. The following softkeys appear along the bottom of the main menu. A summary of common softkeys used throughout the Pac is given at the end of this chapter.

## Forvi Toggle between the large and small font for easy viewing of results

ब1ㅆ․ Exits the Electrical Engineering Application Pac

## Moving Around the Screen

Use the $\square$ and $\square$ keys to move the pointer up and down in the menu list. Pressing $\square$ moves the pointer to the bottom of the screen, or pages down (one screen at a time) if the pointer is already at the bottom of the screen. Pressing moves the pointer to the top of the screen, or pages

## Getting Started

up. Pressing $\boldsymbol{\square} \boldsymbol{\square}$ moves the pointer to the bottom of the list and moves the pointer to the begining of the menu.

## Viewing Items Too Wide for the Display

If the text of a menu item is too wide to fit within the display, an ellipsis (...) appears at the end of the line. Press $⿴ 囗 \rightarrow$ nsin to display the rest of the text. Press $\pi$ or ENER to return to the original display of the item.

## Changing the Font Size

The default font for the Electrical Engineering Application Pac displays
 information in a larger, medium-sized font, which is case-sensitive. The font size stays medium (shown below) until you press


## Using the Search Mode

When menu lists are long, it is faster to locate an item using the search mode. To initiate a search, press $\alpha$ to display the following screen:

| \{ HIME EEAPPD \} | PRG |
| :--- | :--- |
| Search for: |  |
|  |  |

The calculator is now in alpha entry mode, as indicated by the alpha ( $\alpha$ ) annunciator at the very top of the screen. Alpha entry mode overrides the function of the standard keyboard. This means that each key that has a white capital letter printed to its lower right loses its original function and types that letter onto the command line when pressed. (See Chapter 2 of the HP 48SX Owner's Manual for a complete description of how the alpha mode operates). Type the first letter or letters of the name you want to search for, to create a search string, and press ENIER. The search function is
case-sensitive. To enter a lower case letter in the alpha entry mode, preceed the letter with the 8 key.

Pressing WTW returns you to the main menu.

## Editing Text Entries

The search mode softkeys, shown on the screen above, are command line editing keys. They are built into the HP 48SX and allow you to edit the search string. Their functions are summarized below:

## ॠSIR Moves the cursor to the beginning of the current word.

SIIf Moves the cursor to the beginning of the next word.
बE= Deletes all the characters in the current word to the left of the cursor.

EI. Deletes all the characters from the cursor's current position to the first character of the next word.
\#s: Toggles between insert and typeover modes.

## Using the Equation Library

The Equation Library contains over 300 equations commonly used by today's electrical engineering professionals and students. The Equation Library enables you to:

- Select the equation category and topic from the main menu.
- List all the equations in a topic.
- Solve a specific equation or set of equations.
- View a description of the variables.
- View a figure that illustrates the problem, when available.
- Plot the equation.

The next few pages show you how to solve a single equation. Solving multiple equations systematically is discussed later in the chapter. For this example,
let's suppose you want to calculate the resistance of a wire 1.569 _cm long and $0.00245 \mathrm{~cm}^{2}$ in area with a resistivity of $1.5 \mu \Omega \cdot \mathrm{~cm}$.

## Accessing Equations

The first step in solving this problem is to locate the necessary equation in the Equation Library. At the main menu, move the pointer to EQUATION LIBRARY and press ENER. This displays the list of 11 main categories:


Move the pointer to CIRCUIT ELEMENTS, and press ENEE to display the list of topics in this category:

| Circuit Elements $\rightarrow$ RESSITANCE CYLNOER CDAX CABLE <br>  PAKALLEL PLATE CAP |
| :---: |
|  |

## Selecting and Displaying Equations

Move the pointer to the topic RESISTANCE and press ENER, or $\overline{E C E N}$, to display the equation set for resistance:

|  |
| :---: |
|  |

This screen lists all the equations in the current topic. In this case, there are four. You may choose to solve all the equations systematically or solve any one equation. Solving multiple equations will be discussed later in this chapter.

For this example, the resistance of the wire is given by the first equation in the set:

$$
R=\frac{\rho \cdot l e n}{A}
$$

where $\rho$ is the resistivity, len is the length, A is the area of cross section, and $\mathbf{R}$ is the resistance. Any equation may be selected by moving the pointer to the desired equation and pressing sumesm. If no equation set is selected, then all equations will be solved systematically. When an equation is selected, a triangular tag is placed in front of the equation:


If you want to view the equation in its full "textbook" form, place the pointer at the equation and press ENER. This displays the equation on the screen:

| $\begin{aligned} & \text { HP 4aSX ERUATION WKIITER } \\ & R=\frac{\rho \cdot \operatorname{len}}{A} \end{aligned}$ |
| :---: |
| press tenteri to return to lis |

Press EENER or Win to return to the list of equations.

When displaying a lengthy equation from the Equation Library, pressing $\square$ or $\square$ scrolls the screen to the left or to the right revealing the entire equation. Pressing $\square$ moves the display window to the end of the equation, and pressing $\square \square$ moves the display window to the beginning of the equation.

## Viewing Variable Definitions

You can view a list that defines all the variables in the selected equation, or set of equations, by pressing the $\delta$ \%/mes softkey at the equations screen. The screen below displays the definitions screen for the first equation of the RESISTANCE topic:

|  |  |
| :---: | :---: |
|  | IP |

To continue solving the problem, you need to invoke the solver function.

## Using the Solver Function

The Sparcom "solver" is a software function that simplifies the job of setting up equations to be calculated by the HP 48SX. The solver function is discussed in more detail later in this chapter, under "What You Need to Know About the Solver."

Enter the solver function of the Pac by pressing S**) screen, the units key becomes available. To work with units for this example,
 selected equation(s) now appear on the screen, with units, waiting for you to enter values:


To enter the resistivity, move the pointer to $\rho$ and press ENER. This displays the following screen:


Enter the resistivity value at the prompt:


After the entering a value, there are two ways to assign units to your entry. The easiest way is by selecting one of the unit softkeys provided on the menu line, or typing in your own choice of units.

If you choose not to add units, just press ENEE at the prompt, and the software will assign SI units. In some cases, more units are available than the six softkeys displayed in the first screen. In these cases, press $\times \mathrm{xT}$ to display the additional units. For a complete description of units supported by the HP 48SX and their respective symbols, see the HP 48SX Owner's Manual. For this example, press "\#s:

| \{ HIME EEAPPD \} |  |
| :---: | :---: |
| Set f, resi |  |
| 1.5_ $\mu \Omega \times \mathrm{cm}$ |  |
|  |  |

Now press ENTER to store this value into the variable $\rho$. This returns you to the solver screen with $1.5 \_\mu \Omega \cdot \mathrm{cm}$ stored into the variable, $\rho$ :

## Getting Started



The triangular tag indicates that $\rho$ is a known variable. Repeat this procedure for the other known variables, $A$ and len. This results in the following screen:


With three of the four variables known in this equation, you may now solve the equation for the resistance by pressing Emim. After a few moments, the calculator returns to this screen with the calculated value of R :

| Resistance 7R: 0009414 and $^{2}$ HEN: '1.569.cm' |
| :---: |
|  |

The * by R indicates that this value was calculated and was not initially specified.

## Converting Data to Different Units.

Suppose you want to convert the resistance (R) from ohms to kilo ohms. First press the kxi key to reveal the next page of softkeys available for this display:


Move the pointer to the variable R and press ©an \% . This lists all of the possible units for R :


Now move the pointer to _ $k \Omega$ and press ENTE:


This converts the resistance in $\Omega$ to $\mathrm{k} \Omega$. If you want to use the data for further calculations, move the pointer to the data item and press $\alpha \times \mathrm{VIT}$ and then press \#STM . o place it on the calculator stack.

## Options After Solving the Equation

Pressing $\pi$ 四 exits the Electrical Engineering Application Pac and places you in the calculator operating environment. Pressing ©menk resets all entries in the current topic to zero. Pressing TPBina deletes the global copies of each variable in the currently selected set of equations that reside in the EEAPPD directory.
 menu, a new RESUME SOLVING... entry will have been added to the list, as shown:


Selecting the RESUME SOLVING function returns you directly to the equation set you were working with, with all previous entries still intact.

## Managing Units

When solving an equation，億綡絃（a toggle key）controls whether the calculations are performed in your choice of units，or in Systeme Internationale d＇Unites（SI）units．When the 算絰緮 softkey appears，it means that all entries are converted to SI units and the unit designations are
 values will contain the unit designations that you specify．

Using designated units usually increases the processing time．

## Solving Multiple Equations

For many problems，the result of one calculation acts as the input to another． The Electrical Engineering Application Pac is capable of solving multiple equations，systematically．

## Selecting the Equation Set

Suppose you want to calculate the performance characteristics of an ideal transformer．From the Equation Library menu screen，move the pointer to TRANSFORMERS and press ENTER．This category contains only one topic， IDEAL TRANSFORMERS．

The equations for this topic are displayed on the screen when you move the pointer to IDEAL TRANSFORMERS and press ENTER：

|  |
| :---: |
|  |

These are the five equations in their written form：
1）$\frac{V 1}{V 2}=\frac{n 1}{n 2}$
2）$/ 1 \cdot n 1=/ 2 \cdot n 2$
3） $\operatorname{Rin}=\frac{R 2}{a^{2}}$
4）$a=\frac{n 2}{n 1}$
5) $\mathrm{V} 2=12 \cdot \mathrm{R} 2$

To view the variables for this equation set, press URS. All the variables for the IDEAL TRANFORMERS topic are listed in the following table:

| Variable | Description | Default Units |
| :--- | :--- | :--- |
| V1 | primary voltage | $1 \_V$ |
| V2 | secondary voltage | $1 \_V$ |
| n1 | number of turns in primary | 1 |
| n2 | number of turns in secondary | 1 |
| I1 | current in primary | $1 \_A$ |
| I2 | current in secondary | $1-A$ |
| R2 | secondary load resistance | $1-\Omega$ |
| Rin | resistance at primary | $1-\Omega$ |
| a | turns ratio | $1-2$ |

## Solving the Equation Set

Press SEEEEM to select the desired equation to be solved. In the following, the top four equations have been selected as indicated by the triangular tags to the left of the equations.


Press SOM © to enter the solver function for these four equations. Enter all the information pertaining to the problem, using the procedure described previously. Press ©file to start the solver. The solver then steps through each equation in the list, solving those equations that contain sufficient data to calculate an unknown variable. When all known variables are found, or all remaining equations have more than one unknown variable, the solver stops. It then lists the variables it can't find, and returns to the solver screen. The given variables and calculated results for all four selected equations are shown below:

## Given

V2 = 10_V

Result
V1 $=5.7143 \_\mathrm{V}$

$$
\begin{array}{ll}
\mathrm{n} 1=100 & 12=7.1429 \mathrm{E}-2 \_A \\
\mathrm{n} 2=175 & \text { Rin }=0 \\
\mathrm{I}=125 \_\mathrm{mA} & \mathrm{a}=1.75 \\
& \text { R2 }=0 \_\Omega
\end{array}
$$

With the information given, the solver finds all the variables except Rin and R2. The calculator beeps and indicates that all the variables cannot be calculated. Then, all the known and calculated variables are shown on the solver screen. Notice that Rin and R2 are not marked by an asterisk *:


## Tagging Variables

If you want to solve for only one variable in the list, you can tag is as "wanted." Move the pointer to the variable you want to tag, press 狪 to display the additional softkeys for this screen, and press WANIT. This places a "?" tag in front of the variable you want to solve for:


If you tag Rin and press © A \& , the solver stops when it finds a value for Rin, rather than solving for the entire set. It is possible to tag more than one variable in the list as wanted.

## Plotting One Equation

Any equation in the Equation Library that is of the form $y=f(a, b, c \ldots)$ can be easily plotted using the Electrical Engineering Application Pac. To plot an equation, the dependent variable on the left ( y ) and the desired independent variable (a or b or c...) on the right side must be unknown (no triangular tag). However, all other variables must be known.

Finding and Selecting the Equation

As an example, plot the variation of capacitance of a pn junction as a function of applied voltage. The equations that describe the capacitance of pn junctions are filed in the SOLID STATE DEVICES category, under the topic PN JUNCTIONS. The equation screen for this topic is shown below:


Move the pointer the third equation in the list and press Scmeme Press ENER to view the written out form of the equation, or VARS: to view the subset of variables for this equation. The equation and a table of its variables are shown below:

$$
C J=\left(\frac{q \cdot \varepsilon o \cdot \varepsilon S i}{2\left(\frac{1}{N A}+\frac{1}{N D}\right) \cdot(V a-V b i)}\right)^{V_{2}}
$$

Variable
ND
NA
Va
Vbi
CJ
$\varepsilon O$
$\varepsilon S i$
q

Description
donor density acceptor density appliced voltage built-in voltage junction capacitance per unit area *permittivity of free space (no user entry) *relative permittivity of Si (no user entry) *charge on the electron (no user entry) 1_C
*These variables are not visible on the screen and are automatically extracted by the software from the Constants Library. No user entry is needed.

## Tagging and Entering the Variables

To plot the capacitance curve (CJ versus Va), NA, ND, and Vbi must be tagged as known variables. Move the pointer to the third equation and press
 variables:

## Getting Started

$$
\begin{aligned}
& \mathrm{ND}=1 \mathrm{E} 15 \_\mathrm{cm}^{-3} \\
& \mathrm{NA}=1 \mathrm{E} 18 \_\mathrm{cm}^{-3} \\
& \mathrm{Vbi}=0.7583 \_\mathrm{V}
\end{aligned}
$$

With these three variables entered, return to the equations screen by pressing
 equation is of the proper form, and all but Va and CJ have been specified on the right hand side, it may be plotted.

## Entering the $X$ and $Y$ Coordinates

The first prompt asks whether you want to erase the previous plot and reset the axes $\begin{aligned} & \text { Xusing , or whether you want the new plot drawn over any existing }\end{aligned}$
 prompt enter ${ }^{\boldsymbol{W}} \mathrm{B} \mathrm{E} \mathrm{S}$ to clear all previous plots from the screen.

Now enter the minimum and maximum values for the x coordinate for the graph. Type the coordinates for the plot on the same line, separated by a space (use the key). For this example plot, select the no units option ( Im N: ), then enter 110 for Va ; (the assumed units are _V). This results in the following screen:

| E EEAPPD 3 PKG |  |
| :---: | :---: |
| $\begin{aligned} & \text { Enter hori } \\ & \text { for T KK) } \\ & \text { <Min〉 }\langle\text { Max } \end{aligned}$ |  |
| 1104 |  |
| +xkIP |  |

The plot function now prompts for the limits of the y -axis (in this case, CJ , the capacitance in $\mathrm{pF} / \mathrm{m}^{2}$ units). You can either enter the lower and upper limits for $y$, or allow the system to auto range when ENTE is pressed. For this example, press ENTER to auto range a plot of CJ versus Va over the range of 1 to $10 \_\mathrm{V}$, shown below:


## Plotting Speed

If units are on (the Manm key is displayed at the solver screen) a plot can take up to 10 minutes to display. If you turn the units off (i.e., toggle the units key to remove the box) the plot function performs in approximately one tenth of the time.

However, as described earlier in this chapter under "Managing Units," when you turn off units, all values are converted to SI units. Therefore, when you enter the x -axis coordinates, you need to enter them as low limit and upper limit. The plot will also be displayed in the default SI units.

## Softkeys for the Plot Function

The softkeys shown in the above plot are plot function keys in the HP 48SX. For example, pressing EबORE displays the ( $\mathrm{x}, \mathrm{y}$ ) coordinates of any point on the screen indicated by the cursor. For a description of the behavior of the plot function softkeys, see Chapter 18 of the HP 48SX Owner's Manual.
 are supported by the Electrical Engineering Application Pac only when units are off. You can remove the softkeys from the plot to expose more of the
 an equation or to return to the equation screen.

## Making Multiple Plots of an Equation

In some cases, you may want to graph an equation on the same axes several times. To do this, simply answer \#®e\% to the "Clear plot first?" prompt after you have pressed peot.

For example, suppose you're interested in plotting a new capacitance curve for a higher doping (e.g., ND $=1 E 16 \_1 / \mathrm{cm}^{3}$ ). Return to the solver screen by pressing S©ME and enter the new value for ND. Then go to the equations
 softkey. At the prompt, press \#\#®. The new graph will plot over the previous one, as shown:

## Getting Started



There is no limit to the number of graphs that may be plotted on a given axis. However, the HP 48SX plot/graphics function keys support only the most recent plot.

## What You Should Know About the Solver

As you have seen in the examples in this chapter, the Sparcom solver allows you to easily specify the values and units of your equation or set of equations before sending the data to the HP 48SX numerical root-finder. For the selected equations(s), the solver screen lists all the variables, shows whether they are known (triangular tag), unknown (no tag), wanted ("?" tag), or just calculated ( ${ }^{*}$ ), and whether units are on or off.

Once you set these parameters, pressing ©emme. activates the HP 48SX root-finder to calculate the solution(s). The root-finder requires an initial value on which to base its search. You can provide a "guess" for the calculator to use, or the solver will provide a "guess" value of 1 . The root-finder then generates pairs of intermediate values and interpolates between them to find the solution. The time required to find the root depends on how close the initial guess is to the actual solution.

## Speeding Up Computing Time

You can speed up computing time by providing the calculator a "guess" value close to the expected solution. At the variables screen, enter your guess value into the "unknown" variable. The variable will then be tagged as "known" (triangle). Press the K Ime softkey to toggle the variable back to "unknown" (no tag). Now press 『बàe.

## "Bad Guess" Message

If the calculator displays the message, "Bad Guess(es)," it indicates an error has been made in setting up the problem. Go back through the set up process and check for errors in specifying data.

For more information, refer to Chapter 17 of the HP 48SX Owner's Manual .

## Loading Values from the Stack

There are two methods of entering a value into the Sparcom solver directly from the calculator stack:

First Method: Make sure the value you want is on the stack. Press \#\#mer, then choose an equation set to solve, or select RESUME SOLVING to return to the equation set you're last working with. At the variables screen, move the pointer to the variable that will incorporate the value currently on the stack and press ENER. A prompt message asks you to enter the value. Press tor to reveal the command line editing keys. Press the 新前 softkey to invoke a limited version of the HP 48SX Interactive Stack. Move the pointer to the appropriate stack level and press ECHO then ENTER. This takes you back to the "Enter value" prompt message. Press ENER again to store the echoed value into the current variable and return to the solver screen.

Second Method: Alternatively, store the desired value into a global variable in the EEAPPD directory under the same name as the equation variable. When the solver is entered, it will automatically recall the value and load it into the selected equation variable.

## Sparcom's "EEAPPD" Directory

When you plug in the Electrical Engineering Application Pac for the first time, the software creates its own directory, EEAPPD, in the HOME directory of the HP 48SX. ALL operations performed by the software take place in the EEAPPD directory. It is, therefore, the only place where global variables are created or purged by the solver. If you purge this directory by mistake, it will be recreated in its entirety, but all the values that you previously stored will be lost.

The variable created in the EEAPPD directory and its functions is described below:

EEpar The parameter EEpar is utilized to provide a direct path from the main menu to the solver level. EEpar is created (or rewritten) whenever the equation, solver, or variable levels of the the Equation Library is exited. The three possible exit routes that trigger an EEpar update are: 1) Pressing quit the software and exit to the calculator stack, 2) Pressing UP to return to the topic level, or 3) pressing to return to the main menu level.

## Summary of Functions

The following figure diagrams the basic flow and function of each level of the equation library and solver. On the following page, the softkeys available at each level are explained in more detail.


## Summary of Softkeys



Stores all variable values and iterates through the set of selected equations in an attempt to find values for all wanted variables．After completion of the solver process， the user is returned to the solver level，where newly found variables are marked with＂＂＂．

ल＝ 2
Resets the values of the current variable set to zero．
羔emes Enters the equation level of the current topic．
EGETE Displays a figure for the currently selected topic or displays ＂No figure＂．
－्बाI\＃Toggles between small and medium fonts．
esem Toggles the currently selected variable between known and unknown，adding or removing the triangular tag．

青白会 Returns to the main menu．
Plots the selected equation，prompting the user for $x$－axis
and $y$－axis values．This feature works only for equations of
the form $y=f(a, b, \ldots$.$) where y$ and one variable on the right
are unknown．
－ares

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Purges the global copies（in the EEAPPD directory）of the current variable set displayed in the solver level．

Exits the Electrical Engineering Application Pac．
Marks or unmarks the currently selected equation with the triangular tag．Only variables in the marked equations will appear in the solver and variable levels（with the exception of constants）．If no equations are selected，all will be used．

Enters the solver level of the currrent topic．
Copies selected entry to calculator stack．
Toggle key．Indicates that units are on．
unrs

## up

## vans

view
want

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## Chapter 2

## Equation Library

## In This Chapter

The Equation Library contains over 300 equations organized into 11 main categories. Each category contains several topics. Each topic includes an equation set, a complete list of variables, sometimes a figure illustrating the equation set, and a set of units for all variables. This chapter describes these topics and provides one or more examples using the equation set. The computed results for all examples have been rounded off to the fourth decimal place.
$\square \quad$ Circuit Elements
$\square$ Simple DC Circuits
$\square \quad$ RL and RC Circuits
$\square$ RLC Circuits
$\square$ Electrical Resonance
$\square$ OpAmp Circuits
$\square$ Simple AC Circuits
$\square$ Transformers
$\square$ Transmission Lines
$\square$ Motors and Generators
$\square$ Solid State Devices

## Circuit Elements

The following topics calculate values of electrical circuit elements from first principles.

- Resistance
- Cylinder/Coaxial Cable
- Spherical Shell, R/C
- Solenoid/ Toroid/ Loops
- Parallel Plate Capacitor


## Resistance

The four equations in this set compute the resistance or conductance of a rectangular bar, and show the reciprocal relationship betweeen resistivity and conductivity, and calculate the effect of temperature on resistance.

1) $R=\frac{\rho \cdot l e n}{A}$
2) $G=\frac{\sigma \cdot A}{l e n}$
3) $\rho=\frac{1}{\sigma}$
4) $R=R o \cdot(1+\alpha \cdot(T-T o))$

| Variable | Description | Units |
| :---: | :---: | :---: |
| R | resistance of a bar (at temperature T ) | 1_S |
| G | conductance of a bar | 1 S |
| A | uniform area of cross section | 1_cm ${ }^{\wedge} 2$ |
| Ien | length of the bar | 1_cm |
| $\rho$ | resistivity | $1 \_\Omega \cdot \mathrm{m}$ |
| $\sigma$ | conductivity | 1_S/m |
| Ro | resistance at temperature To | $1 \_\Omega$ |
| $\alpha$ | temperature coefficient of resistance | 1_1/K |
| T | temperature | $1{ }^{\circ} \mathrm{C}$ |
| To | reference temperature | $1{ }_{-}{ }^{\circ} \mathrm{C}$ |



Example 1: A rectangular bar 1.41 _cm long and 0.00425 _ $\mathrm{cm}^{2}$ in area has a conductivity of 10.5 S $/ \mathrm{cm}$. What is its resistance, resistivity in $\Omega \cdot \mathrm{cm}$ and conductance? Use equations 1,2 and 3.

## Given

$\mathrm{A}=0.00425 \mathrm{~cm}^{2}$
len $=1.41 \_\mathrm{cm}$
$\sigma=10.5 \_\overline{\mathrm{S}} / \mathrm{cm}$

## Result

$\mathrm{R}=31.5966 \_\Omega$
$\mathrm{G}=3.1649 \mathrm{E}-2 \_\mathrm{S}$
$\rho=9.5238 \mathrm{E}-\mathrm{L}_{-} \Omega \cdot \mathrm{cm}$

Example 2: A filament resistor measures $58.5 \_\Omega$ at $100_{-}^{\circ} \mathrm{C}$ and $50 \Omega$ at a reference temperature. Its temperature coefficient of resistance is $0.0025 \_1{ }^{\circ} \mathrm{C}$. What is the reference temperature in _${ }^{\circ} \mathrm{F}$ ? Use equation 4.

## Given

$R=58.5 \_\Omega$
Ro $=50 \_\Omega$
$\alpha=0.0025-1 /{ }^{\circ} \mathrm{C}$
$\mathrm{T}=100{ }^{\circ} \mathrm{C}$

Result
To $=89.6_{-}{ }^{\circ} \mathrm{F}$

## Cylinder/Coaxial Cable

The equations in this topic calculate the radial resistance of a thin cylindrical tube and the inductance or capacitance of a coaxial cable.

1) $R=\frac{r 2-r 1}{2 \cdot \pi \cdot r 2 \cdot l e n} \cdot \rho$
2) $L=\frac{\mu o \cdot \mu r \cdot l e n}{2 \pi} \cdot L N\left(\frac{r 2}{r 1}\right)$
3) $C=\frac{2 \cdot \pi \cdot \varepsilon o \cdot \varepsilon r \cdot l e n}{L N\left(\frac{r 2}{r 1}\right)}$

| Variable | Description | Units |
| :--- | :--- | :--- |
| R | radial resistance of a cylinder | $1 \_\Omega$ |
| r 1 | inner conductor radius | $1 \_\mathrm{cm}$ |
| r 2 | outer conductor radius | $1 \_\mathrm{cm}$ |
| len | length of cable | $1 \_\mathrm{cm}$ |
| $\rho$ | resistivity | $1 \_\Omega \cdot \mathrm{cm}$ |
| L | inductance | $1 \_\mathrm{mH}$ |
| $\mu \mathrm{r}$ | relative permeability | 1 |
| C | capacitance | $1 \_\mu \mathrm{F}$ |
| $\varepsilon \mathrm{rr}$ | relative permittivity | 1 |



Example 1: A thin 20 _ft long, cylindrical tube has an inner radius of 0.15 _cm and an outer radius of 0.75 _cm. The cylindlindrical tube is made of material with a resistivity of $0.75 \Omega \Omega \cdot \mathrm{~cm}$. Find the radial resistance using the first equation.

| Given | Result |
| :--- | :--- |
| $\mathrm{r} 1=0.15 \_\mathrm{cm}$ | $\mathrm{R}=1.5665 \mathrm{E}-4 \_\Omega$ |
| $\mathrm{r} 2=0.75 \_\mathrm{cm}$ |  |
| len $=20-\mathrm{ft}$ |  |
| $\rho=0.75 \_\Omega \cdot \mathrm{cm}$ |  |

Example 2: A 20 _ft cable with an inner conductor radius of 0.15 _cm and an outer conductor radius of 0.75 _cm is filled with either a magnetic material with a relative permeability of 1.25 or a dielectric material with a relative permittivity of 4.9. Find the total inductance and capacitance for these cables in mH and $\mu \mathrm{F}$ respectively. Use equations 2 and 3 .
Given
$\mathrm{r} 1=0.15 \_\mathrm{cm}$
$\mathrm{r} 2=0.75 \_\mathrm{cm}$
$\mathrm{len}=20 \_\mathrm{ft}$
$\varepsilon \mathrm{r}=4.9$
$\mu \mathrm{r}=1.25$

## Result

$\mathrm{L}=2.4528 \mathrm{E}-3 \_\mathrm{mH}$
$\mathrm{C}=1.0325 \mathrm{E}-3 \mu \mathrm{~F}$
$\mathrm{r} 2=0.75 \_\mathrm{cm}$

## Spherical Shell, R/C

These formulas cover the calculation of the radial resistance and capacitance of a thin spherical shell.

$$
R=\frac{r 2-r 1}{4 \cdot \pi \cdot r 1 \cdot r 2} \cdot \rho \quad C=\frac{4 \cdot \pi \cdot \varepsilon o \cdot \varepsilon r}{\frac{1}{r 1}-\frac{1}{r 2}}
$$

| Variable | Description | Units |
| :--- | :--- | :--- |
| R | radial resistance | $1 \_\Omega$ |
| r1 | inner spherical radius | $1 \_\mathrm{m}$ |
| r2 | outer spherical radius | $1 \_\mathrm{m}$ |
| $\rho$ | resistivity | $1 \_\Omega \cdot \mathrm{m}$ |
| C | capacitance | $1 \_\mathrm{F}$ |
| $\varepsilon \mathrm{\varepsilon r}$ | relative permittivity | 1 |

Example: A spherical shell has a resistance of $0.0125 \_\Omega$. The inner radius of the shell is 0.85 _cm and the outer radius is 0.985 _cm. Find the resistivity of the material of the shell. If the shell is replaced by a dielectric material with a permittivity of 11.7 , what is the capacitance of the shell?

$$
\begin{array}{ll}
\text { Given } & \text { Result } \\
\mathrm{R}=0.0125 \_\Omega & \rho=0.9742 \_\Omega \cdot \mathrm{cm} \\
\mathrm{r} 1=0.85 \_\mathrm{cm} & \mathrm{C}=8.0736 \mathrm{E}-5 \_\mu \mathrm{F} \\
\mathrm{r} 2=0.985 \_\mathrm{cm} & \\
\varepsilon \mathrm{r}=11.7 &
\end{array}
$$

## Solenoid/Toroid/Loops

The first equation calculates the inductance of a solenoid of length, len, and a cross-sectional area A, with a core whose relative permeability is $\mu$ r. For completeness, equations which compute the inductance of toroid and the self-inductance of a wire loop are also included in this subtopic.

1) $L s=\mu o \cdot \mu r \cdot n^{2} \cdot l e n \cdot A$
2) $L t=\frac{\mu \mathrm{O} \cdot \mu r \cdot N t^{2} \cdot h}{2 \cdot \pi} \cdot L N\left(\frac{r o}{r i}\right)$
3) $L I=\mu o \cdot a \cdot\left(L N\left(\frac{8 \cdot r o}{a}\right)-1.75\right)$

| Variable | Description | Units |
| :---: | :---: | :---: |
| Ls | solenoid inductance | 1_H |
| Lt | toroid inductance | 1 H |
| L | loop inductance | 1-H |
| n | number of turns per unit length | 1_1/m |
| Ien | length of solenoid | 1_m |
| A | area of cross section | $1 \_m{ }^{\wedge} 2$ |
| $\mu \mathrm{r}$ | relative permeability | 1 |
| Nt | number of turns | 1 |
| ri | inside toroid radius | 1_m |
| ro | outside toroid radius/mean loop radius | 1_m |
| h | thickness of the toroid | 1_m |
| a | wire radius | 1_m |



Example 1: A 25 _cm long solenoid has a coil of 15 turns $/ \mathrm{cm}$. The core of the solenoid has an area of cross-section of 3.25 in $^{2}$ and is filled with a magnetic material with a relative permeability of 1000 . Find the inductance of the solenoid, using equation 1.

```
Given
len \(=25 \_\mathrm{cm}\)
```

Result
$\mathrm{L}=1482.1194 \_\mathrm{mH}$

$$
\begin{aligned}
& \mathrm{n}=15-1 / \mathrm{cm} \\
& \mathrm{~A}=3.25 \mathrm{in}^{2} \\
& \mu \mathrm{r}=1000
\end{aligned}
$$

Example 2: A 150 twin toroid has an inner radius of 1.00 in and an outer radius of 1.25 in. The toroid has a relative permeability of 650 and a thickness of 0.15 in. Using equation 2 , find the inductance of the toroid.

$$
\begin{aligned}
& \text { Given } \\
& \mu \mathrm{r}=650 \\
& \mathrm{Nt}=150 \\
& \mathrm{ri}=1 \_\mathrm{in} \\
& \mathrm{ro}=1.25 \_\mathrm{in} \\
& \mathrm{~h}=5.7 \_\mathrm{cm}
\end{aligned}
$$

## Result

Lt $=37.2036 \_\mathrm{mH}$

## Parallel Plate Capacitor

This formula computes the capacitance between two parallel plates separated by a small spacing, d (ignoring fringing field effects).

$$
C=\frac{\varepsilon o \cdot \varepsilon r \cdot A}{d}
$$



Example: A parallel plate capacitor is built using a dielectric with a relative permittivity of 3.9 and a plate separation of $1.56 \mathrm{E}-6 \_\mathrm{cm}$. The plate area is $2.8 \_\mathrm{cm}^{2}$. What is the capacitance in $\mu \mathrm{F}$ ?

## Given

$\varepsilon \mathrm{r}=3.9$
$\mathrm{A}=2.8 \_\mathrm{cm}^{2}$
$\mathrm{d}=1.56 \mathrm{E}-6 \_\mathrm{cm}$

Result
$\mathrm{C}=0.6198 \mu \mathrm{~F}$

## Simple DC Circuits

This category covers circuit fundamentals, including Ohm's law, combining two circuit elements of the same type in series or parallel, energy stored in reactive elements, circuit performance parameters, and the Wheatstone's bridge. These topics focus on basic circuit principles of equivalence, energy storage, and power delivered to a load.

- Ohm's Law and Power
- Combination of 2 R's, 2 C's or 2 L's
- Energy Stored in L or C
- DC Circuit Properties
- Wheatstone's Bridge


## Ohm's Law and Power

The relationship between current, voltage, resistance, and power is based on Ohm's law. The equations in this set show the interrelationship between these four variables.

1) $V=I \cdot R$
2) $P=V \cdot I$
3) $P=I^{2} \cdot R$
4) $P=\frac{V^{2}}{R}$

| Variable | Description | Units |
| :--- | :--- | :--- |
| V | voltage | $1 \_V$ |
| I | current | $1 \_A$ |
| R | resistance | $1 \_\Omega$ |
| P | power dissipated | $1 \_W$ |

Example: A 5_V battery with no internal resistance has a load of 1250_ $\Omega$. Calculate the current in the load and the power dissipated in the load.

$$
\begin{aligned}
& \text { Given } \\
& V=5 \mathrm{~V} \\
& \mathrm{R}=1250 \_\Omega
\end{aligned}
$$

Result
I = 0.004 A
$P=0.02 \_W$

## Combination of 2 R's,2 C's or 2 L's

This equation set covers the effects of combining two resistors, two inductors or two capacitors in either series or parallel.

1) $R s=R 1+R 2$
2) $\frac{1}{R p}=\frac{1}{R 1}+\frac{1}{R 2}$
3) $L s=L 1+L 2$
4) $\frac{1}{L p}=\frac{1}{L 1}+\frac{1}{L 2}$
5) $\frac{1}{C s}=\frac{1}{C 1}+\frac{1}{C 2}$
6) $\mathrm{Cp}=\mathrm{C} 1+\mathrm{C} 2$

| Variable | Description | Units |
| :--- | :--- | :--- |
| Rs | equivalence of 2 resistors in series | $1 \_\Omega$ |
| Rp | equivalence of 2 resistors in parallel | $1-\Omega$ |
| R1 | resistance 1 | $1 \_\Omega$ |
| R2 | resistance 2 | equivalence of 2 inductors in series |
| Ls | equivalence of 2 inductors in parallel | $1-H$ |
| Lp | inductor 1 | inductor 2 |



Example: Calculate the effect of two $1250 \_\Omega$ and $28050 \_\Omega$ resistors, two $275 \mu \mathrm{H}$ and $1.225 \_\mathrm{mH}$ inductors, and two $0.65 \mu \mathrm{~F}$ and $0.52 \mu \mathrm{~F}$ capacitors connected in series and parallel.

Given Resistor
R1 $=1250 \_\Omega$
R2 $=2850 \_\Omega$
$\mathrm{L} 1=275 \mu \mathrm{H}$
L2 $=1.225 \mathrm{mH}$
$\mathrm{C} 1=0.68 \mu \mathrm{~F}$
$\mathrm{C} 2=0.52 \mu \mathrm{~F}$

Result
Rs $=4100 \_\Omega$
$\mathrm{Rp}=868.9024 \_\Omega$
Ls $=1.5 \mathrm{mH}$
$\mathrm{Lp}=0.2246 \mathrm{mH}$.
$\mathrm{Cs}=0.2947 \mu \mathrm{~F}$
$\mathrm{Cp}=1.2 \mu \mathrm{~F}$

## Energy Stored in L or C

An inductor carrying current stores magnetic potential energy in the magnetic field surrounding the conductor. Equations 2 and 3 describe the relationship between charge, energy stored in the electric field, and voltage across the capacitor.

1) $E=\frac{L \cdot I^{2}}{2}$
2) $Q=C \cdot V$
3) $E=\frac{C \cdot V^{2}}{2}$

| Variable | Description | Units |
| :--- | :--- | :--- |
| E | stored energy | $1 \_J$ |
| L | inductance | $1 \_H$ |
| I | current | $1 \_A$ |
| Q | charge on C | $1 \_C$ |
| C | capacitance | $1-F$ |
| V | capacitor voltage | $1 \_V$ |

Example 1: A 4.2_mH inductor carries a current of 1.89_A. What is the energy stored in the inductor? Use equation 1.

$$
\begin{aligned}
& \text { Given } \\
& \mathrm{L}=4.2 \_\mathrm{mH} \\
& \mathrm{I}=1.89 \_\mathrm{A}
\end{aligned}
$$

Result

Example 2: A $6.8 \mu \mathrm{~F}$ capacitor is charged to a level of 2_V. Using equations 2 and 3 , find the charge on the capacitor and the energy stored.

$$
\begin{array}{ll}
\text { Given } & \text { Result } \\
\mathrm{C}=6.8 \_\mu \mathrm{F} & \mathrm{Q}=0.0000136 \_\mathrm{C} \\
\mathrm{~V}=2 \_\mathrm{V} & \mathrm{E}=0.0000136 \_\mathrm{J}
\end{array}
$$

## DC Circuit Properties

These equations describe two valuable parameters in circuit analysis that complement each other: Thevenin's voltage source and Norton's current source. The equations compute load current, load voltage, power dissipated in the load, and maximum power available to the load from the source.

1) $V s=I s \cdot R s$
2) $L L=\frac{V s}{R s+R l}$
3) $V L=I L \cdot R I$
4) $P L=V L \cdot I L$
5) $P L=I L^{2} \cdot R I$
6) $P L \max =\frac{V s^{2}}{4 \cdot R s}$

| Variable | Description | Units |
| :--- | :--- | :--- |
| Vs | source voltage | $1 \_V$ |
| Is | short circuit current | $1-A$ |
| Rs | source resistance | $1-\Omega$ |
| VL | load voltage | $1 \_V$ |
| IL | load current | $1-A$ |
| RI | load resistance | $1-\Omega$ |
| PL | power in load | $1-W$ |
| PLmax | maximum power available in RI | $1 \_W$ |



Example: A $10 \_V$ battery with a $50 \_\Omega$ internal resistance is supplying power to a load of $125 \_\Omega$. Find the circuit performance parameters for this circuit.

## Given

Vs $=10 \_\mathrm{V}$
Rs $=50 \_\Omega$
$\mathrm{RI}=125 \_\Omega$

Result
Is $=0.2 \mathrm{~A}$
$\mathrm{IL}=5.7143 \mathrm{E}-2 \mathrm{~A}$
$\mathrm{VL}=7.1429 \mathrm{~V}$
$\mathrm{PL}=0.4082 \mathrm{~W}$
PLmax $=0.5$ W

## Wheatstone's Bridge

These equations describe the relationship between current and voltage in branches of a Wheatstone's bridge circuit. The equations for $\mathrm{Ra}, \mathrm{Rb}$ and Rc describe intermediate equivalent values to handle equations for current in the galvanometer circuit. They do not represent any physical resistors.

1) $\frac{R 1}{R 2}=\frac{R 3}{R 4}$
2) $R a=\frac{R 1 \cdot R g}{R 1+R 3+R g}$
3) $R b=\frac{R 3 \cdot R g}{R 1+R 3+R g}$
4) $R c=\frac{R 1 \cdot R 3}{R 1+R 3+R g}$
5) $V g=\frac{V s \cdot(R a \cdot R 4-R b \cdot R 2)}{R c \cdot(R a+R b+R 2+R 4)+(R b+R 4) \cdot(R a+R 2)}$
6) $I g=\frac{V g}{R g}$

| Variable | Description | Units |
| :---: | :---: | :---: |
| Vg | Thevenin voltage | 1_V |
| Ig | galvanic I | 1 A |
| R1 | resistance of arm 1 of the bridge | 1 - $\Omega$ |
| R2 | resistance of arm 2 of the bridge | 1 - $\Omega$ |
| R3 | resistance of arm 3 of the bridge | $1 \_\Omega$ |
| R4 | resistance of arm 4 of the bridge | 1 _ $\Omega$ |
| Rg | resistance in galvanic kg | 1 - $\Omega$ |
| Ra | equivalent resistance | 1 - $\Omega$ |
| Rb | equivalent resistance | 1 _ $\Omega$ |
| Rc | equivalent resistance | 1 - $\Omega$ |
| Vs | source voltage | 1_V |
|  |  |  |

Example 1: Four resistors, $1200 \_\Omega, 2500 \_\Omega, 2000 \_\Omega$ and $4000 \_\Omega$ form the four branches of a Wheatstone's bridge. The bridge is driven by a 10_V source. The galvanometer in the bridge link has a series resistance of $10000 \_\Omega$. Find the galvanometer current and the bridge voltage.

Given
R1 $=1200 \_\Omega$
R2 $=2500 \_\Omega$
$R 3=2000 \_\Omega$
R4 $=4000 \_\Omega$
Vs $=10 \_\mathrm{V}$
$\mathrm{Rg}=10000 \_\Omega$
Note: Ra, Rb and Rc have been calculated, but have no physical significance.
Example 2: In the Wheatstone's bridge in Example 1, replace R1 by 1250_ $\Omega$ to make a balanced bridge. Find the new galvanometer current and bridge voltage.

Given
R1 $=1250 \_\Omega$

Result
$\mathrm{Vg}=-7.4184 \mathrm{E}-2 \mathrm{~V}$
$\mathrm{Ig}=-7.4184 \mathrm{E}-6 \_\mathrm{A}$
$\mathrm{Ra}=909.0909 \_\Omega$
$\mathrm{Rb}=1515.1515 \Omega$
$R c=181.8182_{-} \Omega$

Result
$\mathrm{Vg}=4.8401 \mathrm{E}-12 \_\mathrm{V}$

$$
\begin{array}{ll}
\mathrm{R} 2=2500 \_\Omega & \mathrm{Ig}=4.84018 \mathrm{E}-16 \_\mathrm{A} \\
\mathrm{R} 3=2000-\Omega & \mathrm{Ra}=943.3962 \_\Omega \\
\mathrm{R} 4=4000 \_\Omega & \mathrm{Rb}=1509.4340 \_\Omega \\
\mathrm{Vs}=10 \_\mathrm{V} & \mathrm{Rc}=188.6792_{-} \Omega \\
\mathrm{Rg}=10000 \_\Omega &
\end{array}
$$

Note: Ra, Rb and Rc have been calculated, but have no physical significance.

The result for Vg given in the example above represents a calculation within the accuracy of the HP 48SX and should, for practical purposes, be interpreted as $0 \_\mathrm{V}$.

## RL and RC Circuits

This category covers the response of RC and RL circuits to a step function and converting from series to parallel equivalents.

- RL Circuit Response
- RC Circuit Response
- RL Series $\longleftrightarrow$ Parallel Conversion
- RC Series $\longleftrightarrow$ Parallel Conversion


## RL Circuit Response

These equations describe inductor current and voltage in response to a step function input stimulus. The first two equations characterize an RL series circuit, while the last pair describe a parallel RL circuit.

1) $v I=(V s-l o \cdot R) \cdot e^{-R \cdot t / L}$
2) $i l=\frac{V s}{R}+\left(10-\frac{V s}{R}\right) \cdot e^{-R \cdot t / L}$
3) $v l=R \cdot(I s-l o) \cdot e^{-R \cdot t / L}$
4) $i l=I s+(l o-I s) \cdot e^{-R \cdot t / L}$

| Variable | Description | Units |
| :--- | :--- | :--- |
| vl | inductor voltage | $1 \_V$ |
| L | inductance | $1 \_\mathrm{H}$ |


| il | inductor current | $1 \_A$ |
| :--- | :--- | ---: |
| Vs | source voltage | $1 \_V$ |
| $R$ | resistance | $1-\Omega$ |
| lo | current at $t=0$ | $1 \_A$ |
| t | time | $1-s$ |
| Is | source current | $1 \_A$ |



Example 1: A $120 \_\Omega$ resistor and a 0.18 mH inductor are connected in series and are subjected to a 5 V step at $\mathrm{t}=0$. The inductor carries no current before the voltage stimulus. Calculate the current through the inductor and the voltage across it after a time lapse of $1.75 \mu \mathrm{~s}$. Use equations 1 and 2 to solve this problem.

$$
\begin{aligned}
& \text { Given } \\
& \mathrm{L}=0.18 \mathrm{mH} \\
& \mathrm{Vs}=5 \_\mathrm{V} \\
& \mathrm{R}=120 \_\Omega \\
& \mathrm{Io}=0 \_\mathrm{A} \\
& \mathrm{t}=1.75 \mu \mathrm{~s}
\end{aligned}
$$

## Result

$\mathrm{vl}=1.5570 \mathrm{~V}$
il $=2.8692 \mathrm{E}-2 \_\mathrm{A}$

Example 2: A $1500 \_\Omega$ resistor and a 0.15 mH inductor are connected in parallel at $\mathrm{t}=0$ to a current source delivering 176 mA . The inductor carries no current at $\mathrm{t}=0$. Find the current in the inductor and the voltage across it, $0.75 \mu \mathrm{~s}$ after the current stimulus has been applied. Use the last two equations in this set to solve this problem.

## Given

$\mathrm{L}=0.15 \mathrm{mH}$
$R=1500_{-} \Omega$
$\mathrm{lo}=0 \mathrm{~A}$
Is $=176$ mA
$\mathrm{t}=0.75 \mu \mathrm{~s}$

## Result

$$
\begin{aligned}
& \mathrm{vl}=0.1460 \_\mathrm{V} \\
& \mathrm{il}=0.1759 \_\mathrm{A}
\end{aligned}
$$

## RC Circuit Response

These four equations describe the current and voltage response in a series RC and parallel RC circuit to an input voltage step.

## Equation Library

1) $i c=\frac{V s-V o}{R} \cdot e^{-t /(R \cdot C)}$
2) $v c=V s+(V o-V s) \cdot e^{-t /(R \cdot C)}$
3) $v c=I s \cdot R+(V o-I s \cdot R) \cdot e^{-t /(R \cdot C)}$
4) $i c=\frac{I s \cdot R-V o}{R} \cdot e^{-t /(R \cdot C)}$

| Variable | Description | Units |
| :---: | :---: | :---: |
| vc | capacitor voltage | 1_V |
| C | capacitance | 1_F |
| ic | capacitor I | 1_A |
| Vs | source voltage | 1 V |
| R | resistance | 1_ $\Omega$ |
| Vo | capacitor voltage at $\mathrm{t}=0$ | 1_V |
| t | time | 1 s |
| Is | source current | 1_A |



Example 1: A $5 \_\Omega$ resistor and a $0.18 \_\mu \mathrm{F}$ capacitor are connected in series and are stimulated by a $5 \_\mathrm{V}$ step function. The capacitor has an initial voltage of $-0.5 \_\mathrm{V}$. Find the capacitor current and voltage across the capacitor $0.75 \mu$ s after the input stimulus has been applied. Use equations 1 and 2.

```
Given
\(\mathrm{C}=0.18 \mu \mathrm{~F}\)
\(\mathrm{Vs}=5 \_\mathrm{V}\)
\(\mathrm{R}=5\) _ \(\bar{\Omega}\)
\(\mathrm{Vo}=-0.5 \mathrm{~V}\)
\(\mathrm{t}=0.75 \mu \mathrm{~s}\)
```

Example 2: A parallel RC circuit, using a $10 \_\mathrm{k} \Omega$ resistor and a $0.33 \mu \mathrm{~F}$ capacitor with an initial voltage of 0.25 _ V , is stimulated by a 5.8 _ mA step
current source. Find the capacitor current after a time lapse of 0.075 _ms. What is the voltage across the capacitor?

## Given

$$
\begin{aligned}
& \mathrm{C}=0.33 \_\mathrm{F} \\
& \mathrm{R}=10-\mathrm{k} \Omega \\
& \mathrm{Vo}=0.25 \mathrm{~V} \\
& \mathrm{t}=0.075 \_\mathrm{ms} \\
& \mathrm{Is}=5.8 \_\mathrm{mA}
\end{aligned}
$$

Result
$\mathrm{vc}=1.5477 \mathrm{~V}$
ic $=5.6452 \mathrm{E}-3 \mathrm{~A}$

## RL Series $\longleftrightarrow$ Parallel Conversion

These five equations convert a series RL circuit to its parallel equivalent, and vice versa. Equations 1, 2 and 3 help convert a series RL circuit to a parallel equivalent circuit. Equations 3,4 and 5 convert a parallel RL circuit to its series equivalent.

1) $R 2=\frac{R 1^{2}+\omega^{2} \cdot L 1^{2}}{R 1}$
2) $L 2=\frac{R 1^{2}+\omega^{2} \cdot L 1^{2}}{\omega^{2} \cdot L 1}$
3) $\omega=2 \cdot \pi \cdot f$
4) $L 1=\frac{R 2^{2} \cdot L 2}{R 2^{2}+\omega^{2} \cdot L 2^{2}}$
5) $R 1=\frac{\omega^{2} \cdot L 2^{2} \cdot R 2}{R 2^{2}+\omega^{2} \cdot L 2^{2}}$

| Variable | Description | Units |
| :--- | :--- | :--- |
| R1 | series resistance | $1 \_\Omega$ |
| L1 | series inductance | $1 \_\mathrm{H}$ |
| R2 | parallel resistance | $1 \_\Omega$ |
| L2 | parallel inductance | $1 \_\mathrm{H}$ |
| $\omega$ | radian frequency | $1 \_r / s$ |
| f | frequency | $1 \_\mathrm{Hz}$ |



Example 1: An inductor has a series resistance of $0.1 \_\Omega$ and an inductance of 0.015 mH . At 1.25 MHz . Calculate its parallel equivalent using equations 1-3.

Given
$\mathrm{R} 1=0.1 \_\Omega$
$\mathrm{L} 1=0.015 \mathrm{mH}$
$\mathrm{f}=1.25 \_\mathrm{MHz}$

## Result

R2 $=3515.725 \_\Omega$
L2 $=1.5000 \mathrm{E}-5 \mathrm{H}$
$\omega=7853981.634_{-} \mathrm{r} / \mathrm{s}$

Example 2: A $1000 \_\Omega$ resistor and an inductor of $0.015 \_\mathrm{mH}$ are connected in parallel. At $1_{-} \mathrm{MHz}$, what is its series equivalent? Use equations 3,4 , and 5.

## Given

$R 2=1000 \_\Omega$
L2 $=0.015-\mathrm{mH}$
$f=1.0 \mathrm{MHz}$

## Result

R1 $=0.2249 \_$
$\mathrm{L} 1=1.4997 \overline{\mathrm{E}}-5 \mathrm{H}$
$\omega=6283185.3072_{-} \mathrm{r} / \mathrm{s}$

## RC Series $\leftrightarrow$ Parallel Conversion

Equations 1, 2 and 3 convert a series RC circuit to its parallel equivalent.
Equations 3, 4 and 5 convert an RL parallel circuit to its series equivalent.

1) $R 2=\frac{1+\omega^{2} \cdot R 1^{2} \cdot C 2^{2}}{\omega^{2} \cdot R 1 \cdot C 1^{2}}$
2) $\mathrm{C} 2=\frac{C 1}{1+\omega^{2} \cdot R 1^{2} \cdot C 1^{2}}$
3) $\omega=2 \cdot \pi \cdot f$
4) $R 1=\frac{R 2}{1+\omega^{2} \cdot C 2^{2} \cdot R 2^{2}}$
5) $C 1=\frac{1+\omega^{2} \cdot R 2^{2} \cdot C 2^{2}}{\omega^{2} \cdot R 2^{2} \cdot C 2}$

| Variable | Description | Units |
| :--- | :--- | :--- |
| R1 | series resistance | $1 \_\Omega$ |
| C1 | series capacitance | $1 \_F$ |
| R2 | parallel resistance | $1 \_\Omega$ |
| C2 | parallel capacitance | $1 \_\mathrm{F}$ |
| $\omega$ | radian frequency | $1 \_r / s$ |
| f | frequency | $1 \_\mathrm{Hz}$ |



Example 1: A $10 \_\Omega$ resistor and a $0.015 \mu \mathrm{~F}$ capacitor are connected in series. At $1.0 \_\mathrm{MHz}$, find its parallel equivalent using equations 1-3.

## Given

R1 $=10 \_\Omega$
$\mathrm{C} 1=0.015 \mu \mathrm{~F}$
$f=1.0 \_M H z$

## Result

R2 $=454.0092 \_$_ $\Omega$
$\mathrm{C} 2=1.4670 \mathrm{E}-\overline{8}^{-} \mathrm{F}$
$\omega=6283185.3072_{-} \mathrm{r} / \mathrm{s}$

Example 2: A $10 \_\mathrm{k} \Omega$ resistor and a $0.005 \mu \mathrm{~F}$ capacitor are connected in parallel. At $1.00 \_\mathrm{MHz}$, what is its series equivalent? Use equations 3-5.

## Given

R2 $=10 \_k \Omega$
$\mathrm{C} 2=0.005 \mu \mathrm{~F}$
$\mathrm{f}=1.0 \_\mathrm{MHz}$

## Result

R1 $=3.9984 \_\Omega$
$\mathrm{C} 1=0.0050^{-} \mu \mathrm{F}$
$\omega=6283185.3072 \_$r/s

## RLC Circuits

This category includes descriptions of steady-state and transient behavior of RLC circuits.

- Impedance Series for RLC Circuit
- Admittance Parallel for RLC Circuit
- Overdamped RLC Circuit
- Critically Damped RLC Circuit
- Underdamped RLC Circuit


## Impedance Series for RLC Circuit

The equations below calculate the magnitude and phase of impedance for a series RLC circuit.

1) $\omega=2 \cdot \pi \cdot f$
2) $X I=\omega \cdot L$
3) $X_{c}=\frac{1}{\omega \cdot c}$
4) $Z r=R$
5) $Z i=X I-X c$
6) $Z=\sqrt{Z r^{2}+Z i^{2}}$
7) $\varphi=\operatorname{ASIN}\left(\frac{Z i}{Z}\right)$

| Variable | Description | Units |
| :--- | :--- | :--- |
| $\omega$ | radian frequency | $1 \_r / s$ |
| f | frequency | $1 \_\mathrm{Hz}$ |
| R | series resistance | $1 \_\Omega$ |
| L | series inductance | $1 \_\mathrm{H}$ |
| C | series capacitance | $1 \_\mathrm{F}$ |
| Zr | real part of impedance | $1 \_\Omega$ |
| Zi | imaginary part of impedance | $1 \_\Omega$ |
| Z | total impedance | $1 \_\Omega$ |
| $\phi$ | phase angle of impedance | $1-\_$ |
| XI | inductive reactance | $1 \_\Omega$ |
| XC | capacitive reactance | $1 \_\Omega$ |



Example: A series RLC circuit consists of a $10 \_\Omega$ resistor, a $0.25 \_\mu \mathrm{H}$ inductor and a $0.0033 \mu \mathrm{~F}$ capacitor. What is its impedance and phase angle at 1_MHz?

## Given

$f=1 \_M H z$
$R=10 \Omega$
$L=0.25 \mu \mathrm{H}$
$\mathrm{C}=0.0033 \mu \mathrm{~F}$

## Result

$\mathrm{Zr}=10 \_\Omega$
$\mathrm{Zi}=-46.6580 \Omega$
$\phi=-77.9031^{-}{ }^{-}$
$Z=47.7176 \_\bar{\Omega}$
$\mathrm{XI}=1.5708 \_$
$X c=48.2288 \Omega$
$\omega=6283185.30718$ r $/ \mathrm{s}$

Note: XI, Xc, and $\omega$ are listed for reference. Solutions for $\phi$ often result in numbers that may seem strange at first; the extraction of angles show results that may be offset in integer multiples of $2 \pi$ (or $360^{\circ}$ ).

## Admittance Parallel for RLC Circuit

These equations calculate the impedance and admittance of a parallel RLC circuit.

1) $\omega=2 \cdot \pi \cdot f$
2) $X I=\omega \cdot L$
3) $X_{c}=\frac{1}{(\omega \cdot C)}$
4) $Y r=\frac{1}{R}$
5) $Y i=\frac{1}{X c}-\frac{1}{X I}$
6) $Y=\sqrt{Y I^{2}+Y i^{2}}$
7) $\varphi y=\operatorname{ATAN}\left(\frac{Y i}{Y_{r}}\right)$
8) $Z=\frac{1}{Y}$
9) $\varphi z=-\varphi y$

| Variable | Description | Units |
| :--- | :--- | :--- |
| $\omega$ | radian frequency | $1 \_r / s$ |
| f | frequency | $1 \_\mathrm{Hz}$ |
| R | parallel resistance | $1 \_\Omega$ |
| L | parallel inductance | $1 \_\mathrm{H}$ |
| C | parallel capacitance | $1-\mathrm{F}$ |
| XI | inductive reactance | $1 \_\Omega$ |
| Xc | capacitive reactance | $1 \_\Omega$ |
| Y | total admittance | $1-\mathrm{S}$ |
| Yr | real part of admittance | $1-\mathrm{S}$ |
| Yi | imaginary part of admittance | $1 \_\mathrm{S}$ |
| Z | total impedance | $1 \_\Omega$ |
| $\phi \mathrm{y}$ | admittance phase angle | $1-0$ |
| $\phi \mathrm{Z}$ | impedance phase angle | $1-0$ |



Example: A parallel RLC circuit has a $10 \mathrm{~K} \Omega$ resistor, a 0.25 mH inductor and a $0.033 \_\mu \mathrm{F}$ capacitor in parallel. Find its admittance and impedance at 1_MHz.

## Given

$\mathrm{f}=1 \_\mathrm{MHz}$
$R=10 \_k \Omega$
$\mathrm{L}=0.025 \mathrm{mH}$
$\mathrm{C}=0.033 \mu \mathrm{~F}$

Result
$\omega=6283185.3071 \_$r/s
$X I=157.0746 \Omega$
$\mathrm{Xc}=4.8288 \_\bar{\Omega}$
$Y=0.20098$ _S
$\mathrm{Yr}=0.0001$ S
$\mathrm{Yi}=0.20098 \mathrm{~S}$
$Z=4.9756 \Omega$
$\phi \mathrm{y}=89.9715^{\circ}$
$\phi \mathrm{z}=-89.9715^{\circ}{ }^{\circ}$

## Overdamped RLC Circuit

These equations describe the response of an RLC circuit to a step function DC input voltage stimulus.

1) $\alpha p=\frac{1}{2 \cdot R \cdot C}$
2) $\alpha s=\frac{R}{2 \cdot L}$
3) $\alpha=\alpha p$
4) $\alpha=\alpha s$
5) $\omega o=\frac{1}{\sqrt{L \cdot C}}$
6) $\omega o=2 \cdot \pi \cdot f o$
7) $s 1=-\alpha+\sqrt{\alpha^{2}-\omega o^{2}}$
8) $s 2=-\alpha-\sqrt{\alpha^{2}-\omega o^{2}}$
9) $A 1=\frac{\left(V o \cdot s 2+\frac{1}{C} \cdot\left(\frac{V o}{R}+10\right)\right)}{(s 2-s 1)}$
10) $A 2=\frac{\left(V o \cdot s 1+\frac{1}{C} \cdot\left(\frac{V_{0}}{R}+10\right)\right)}{(s 2-s 1)}$
11) $v=A 1 \cdot e^{s 1 \cdot t}+A 2 \cdot e^{s 2 \cdot t}$

| Variable | Description | Units |
| :--- | :--- | :--- |
| $R$ | resistance | $1 \_\Omega$ |
| L | inductance | $1-H$ |
| $C$ | capacitance | $1-F$ |
| $\alpha$ s | Napiere's frequency for a series circuit | $1-1 / \mathrm{s}$ |
| $\alpha p$ | Napiere's frequency for a parallel circuit | $1-1 / \mathrm{s}$ |
| $\omega 0$ | natural radian frequency | $1-r / s$ |
| fo | natural frequency | $1 \_\mathrm{Hz}$ |

s1
s2
A1
A2
Vo
lo
v
t
$\alpha$
first natural root
second natural root
constant 1
constant 2
DC stimulus voltage
current in inductor at $t=0$
time dependent voltage
time
Napiere's constant

1_1/s
1_1/s
1-V
$1^{-} V$
1-V
1-A
1-V
1_s
1_1/s

Example 1: A parallel RLC circuit has a $100 \_\Omega$ resistance, a $40 \_\mathrm{mH}$ inductance and a $0.25 \mu \mathrm{~F}$ capacitance. Find its natural frequencies. Use equations $1,3,5,6,7$, and 8 .

Given
$R=100 \_\Omega$
$\mathrm{L}=40 \_\overline{\mathrm{m}} \mathrm{H}$
$\mathrm{C}=0.25 \mu \mathrm{~F}$

## Result

$$
\begin{aligned}
& \alpha \mathrm{p}=20000 \_1 / \mathrm{s} \\
& \omega \mathrm{o}=10000 \_\mathrm{r} / \mathrm{s} \\
& \mathrm{fo}=1591.5494 \_\mathrm{Hz} \\
& \mathrm{~s} 1=-2679.49191 / \mathrm{s} \\
& \mathrm{~s} 2=-37320.5081 \_1 / \mathrm{s}
\end{aligned}
$$

## Critically Damped RLC Circuit

These equations describe the response to a DC input step function for a critically damped RLC circuit.

1) $\alpha p=\frac{1}{2 \cdot R \cdot C}$
2) $\alpha s=\frac{R}{2 \cdot L}$
3) $\alpha=\alpha p$
4) $\alpha=\alpha s$
5) $\omega_{o}=\frac{1}{\sqrt{L} \cdot C}$
6) $\omega o=2 \cdot \pi \cdot f o$
7) $D 1=\alpha \cdot V o-\frac{1}{C} \cdot\left(\frac{V o}{R}+10\right)$
8) $D 2=V_{0}$
9) $v=D 1 \cdot t \cdot e^{-\alpha \cdot t}+D 2 \cdot e^{-\alpha \cdot t}$

| Variable | Description | Units |
| :--- | :--- | :--- |
| $R$ | resistance | $1 \_\Omega$ |
| $L$ | inductance | $1 \_H$ |
| $C$ | capacitance | $1 \_F$ |
| $\alpha$ | Napiere's constant | $1 \_1 / s$ |

## Equation Library

| $\omega 0$ | natural radian frequency | 1_r/s |
| :---: | :---: | :---: |
| fo | natural frequency | 1_Hz |
| D1 | constant 1 | $1 \mathrm{~V} / \mathrm{S}$ |
| D2 | constant 2 | 1-V |
| V | time dependent voltage | 1_V |
| t | time | 1_s |
| $\alpha p$ | Napiere's constant, parallel | 1_1/s |
| $\alpha \mathrm{s}$ | Napiere's constant, series | 1_1/s |
| 10 | inductor current at time t=0 | 1_A |
| Vo | source voltage | 1-V |

Example 1: A series RLC circuit has a 200 ' $\Omega$ resistance, a 40 mH inductance and a $0.25 \mu \mathrm{~F}$ capacitor. Is the circuit critically damped?

Given
$R=200 \_\Omega$
$\mathrm{L}=40 \mathrm{~m} \mathrm{H}$
$\mathrm{C}=0.25 \mu \mathrm{~F}$

## Result

$\omega \mathrm{o}=10000 \mathrm{r} / \mathrm{s}$
fo $=1591.5494 \mathrm{~Hz}$
$\alpha \mathrm{s}=10000 \_1 / \mathrm{s}$
$\alpha=10000 \_\overline{1} / \mathrm{s}$

## Underdamped RLC Circuit

These equations describe the transient response of an RLC circuit to an input DC stimulus when the circuit is underdamped.

1) $\alpha p=\frac{1}{2 \cdot R \cdot C}$
2) $\alpha s=\frac{R}{2 \cdot L}$
3) $\alpha=\alpha p$
4) $\alpha=\alpha s$
5) $\omega_{o}=\frac{1}{\sqrt{L \cdot C}}$
6) $\omega o=2 \cdot \pi \cdot f o$
7) $\omega d=\sqrt{\omega o^{2}-\alpha^{2}}$
8) $v=B 1 \cdot\left(e^{-\alpha \cdot t} \cdot \operatorname{COS}(\omega d \cdot t)\right)+B 2 \cdot\left(e^{-\alpha \cdot t} \cdot \operatorname{SIN}(\omega d \cdot t)\right)$
9) $B 1=V_{0} \quad$ 10) $B 2=-\left(\frac{\alpha}{\omega d}\right) \cdot(V o+2 \cdot 10 \cdot R)$

| Variable | Description | Units |
| :--- | :--- | :--- |
| R | resistance | $1 \_\Omega$ |
| L | inductance | $1 \_H$ |


| C | capacitance | $1 \_\mathrm{F}$ |
| :--- | :--- | :--- |
| $\alpha \mathrm{s}$ | Napiere's constant for a series circuit | $1-1 / \mathrm{s}$ |
| $\alpha \mathrm{p}$ | Napiere's constant for a parallel circuit | $1-1 / \mathrm{s}$ |
| $\alpha$ | Napiere's constant | $1-1 / \mathrm{s}$ |
| $\omega 0$ | natural radian frequency | $1-\mathrm{r} / \mathrm{s}$ |
| $\omega \mathrm{d}$ | damped frequency | $1-r / \mathrm{s}$ |
| fo | natural frequency | $1-\mathrm{Hz}$ |
| B1 | constant 1 | $1-\mathrm{V}$ |
| B2 | constant 2 | $1-\mathrm{V}$ |
| Vo | DC voltage stimulus | $1-\mathrm{V}$ |
| lo | current in inductance at $\mathrm{t}=0$ | $1 \_\mathrm{A}$ |
| v | time dependent voltage | $1-\mathrm{V}$ |
| t | time | $1 \_\mathrm{s}$ |

Example 1: A parallel RLC circuit has a $400 \_\Omega$ resistor, a 40 _mH inductor and a $0.25 \mu \mathrm{~F}$ capacitor. Find the natural frequency and the damped frequency for this circuit.

## Given

$R=400 \_\Omega$
$\mathrm{L}=40 \_\overline{\mathrm{m}} \mathrm{H}$
$\mathrm{C}=0 . \overline{25} \mu \mathrm{~F}$

## Result

$$
\begin{aligned}
& \alpha \mathrm{p}=5000 \_1 / \mathrm{s} \\
& \alpha=5000 \_1 / \mathrm{s} \\
& \omega \mathrm{o}=10000 \_\mathrm{r} / \mathrm{s} \\
& \omega \mathrm{~d}=8660.2540 \mathrm{r} / \mathrm{s} \\
& \mathrm{fo}=1591.5494 \_\mathrm{Hz}
\end{aligned}
$$

## Electrical Resonance

This category includes bandwidth and quality factor calculations for series or parallel resonance circuits.

- RLC Resonance
- Q of a Series RLC
- Q of a Parallel RLC


## RLC Resonance

This equation set characterizes properties of an RLC circuit at resonance.

1) $\omega_{o}=\frac{1}{\sqrt{L} \cdot C}$
2) $\omega o=2 \cdot \pi \cdot f o$
3) $Z s=R$
4) $Z p=\omega o \cdot C \cdot R^{2}$

| Variable | Description | Units |
| :--- | :--- | :--- |
| R | resistance |  |
| L | inductance |  |
| C | capacitance |  |
| Wo | natural radian frequency |  |
| fo | frequency |  |
| Zs | impedance of a series RLC |  |
| Zp | impedance of a parallel RLC |  |

Example: A tank circuit used for an IF transformer in a super heterodyne receiver has a capacitance of $0.1392 \mu \mathrm{~F}$, an inductance of $0.8782 \mu \mathrm{H}$, and has a resistance of $100 \_k \Omega$. Find the frequency of resonance and the impedance at resonance.

Given
$\mathrm{R}=100 \mathrm{k} \Omega$
$L=0.8782 \mu \mathrm{H}$
$\mathrm{C}=0.1392 \mu \mathrm{~F}$

## Result

$$
\begin{aligned}
& \omega \mathrm{o}=2860116.1216 \_\mathrm{r} / \mathrm{s} \\
& \mathrm{fo}=455201.6186 \_\mathrm{Hz} \\
& \mathrm{Zs}=100000 \_\Omega \\
& \mathrm{Zp}=3981.2816 \_\mathrm{M} \Omega
\end{aligned}
$$

## Q of a Series RLC

This equation set describes the series resonant circuit in terms of the quality factor.

1) $\omega o=\frac{1}{\sqrt{L} \cdot \mathrm{C}}$
2) $Q=\frac{\omega O \cdot L}{R}$
3) $Q=\frac{1}{\omega o \cdot R \cdot C}$
4) $Q=\frac{1}{R} \cdot \sqrt{L / C}$
5) $\omega 1=\omega 0 \cdot\left(\frac{-1}{2 \cdot Q}+\sqrt{1+\frac{1}{(2 \cdot Q)^{2}}}\right)$
6) $\omega 2=\omega 0 \cdot\left(\frac{1}{2 \cdot Q}+\sqrt{1+\frac{1}{(2 \cdot Q)^{2}}}\right)$
7) $\beta=\omega 2-\omega 1$
8) $\omega o=\sqrt{\omega 1 \cdot \omega 2}$
9) $\omega o=2 \cdot \pi \cdot f o$
Variable
$\omega 0$
fo
Q
$\omega 1$
$\omega 2$
$\beta$
L
C
R

Description
natural radian frequency natural frequency
quality factor
lower 3 dB cutoff radian frequency upper 3 dB cutoff radian frequency
3dB bandwidth radian frequency inductance
capacitance resistance

Units
1_r/s
1_Hz
1
1_r/s
1_r/s
1_r/s
1_H
1_F
$1 \_\Omega$


Example : Suppose you have a series RLC circuit with an inductance of $0.8782 \_\mu \mathrm{H}$, a capacitance of $0.1392 \mu \mathrm{~F}$ and a resistance of $0.3 \_\Omega$. Find its resonance frequency, 3 dB bandwidth, and lower and upper cutoff radian frequencies.

## Given

$\mathrm{R}=0.3 \_\Omega$
$\mathrm{L}=0.8782 \mu \mathrm{H}$
$\mathrm{C}=0.1392 \mu \mathrm{~F}$

## Result

$$
\begin{aligned}
& \omega \mathrm{o}=2.8601 \mathrm{Mr} / \mathrm{s} \\
& \mathrm{fo}=455.2016 \_\mathrm{kHz} \\
& \mathrm{Q}=8.3725 \\
& \omega 1=2.6944-\mathrm{Mr} / \mathrm{s} \\
& \omega 2=3.0360-\mathrm{Mr} / \mathrm{s} \\
& \beta=0.3416 \_\mathrm{Mr} / \mathrm{s}
\end{aligned}
$$

## Q of a Parallel RLC Circuit

This equation set describes a parallel resonant circuit in terms of quality factor.

1) $\omega_{o}=\frac{1}{\sqrt{L} \cdot C}$
2) $Q=\omega o \cdot R \cdot C$
3) $Q=R \cdot \sqrt{C / L}$
4) $\omega 1=\omega o \cdot\left(\frac{-1}{2 \cdot Q}+\sqrt{1+\frac{1}{(2 \cdot Q)^{2}}}\right)$
5) $\omega 2=\omega 0 \cdot\left(\frac{1}{2 \cdot Q}+\sqrt{1+\frac{1}{(2 \cdot Q)^{2}}}\right)$
6) $\beta=\omega 2-\omega 1 \quad$ 7) $\omega o=\sqrt{\omega 1 \cdot \omega 2}$
7) $\omega o=2 \cdot \pi \cdot f o$

| Variable | Description | Units |
| :--- | :--- | :--- |
| $\omega 0$ | natural radian frequency | $1 \_r / s$ |
| fo | natural frequency | $1 \_\mathrm{Hz}$ |
| Q | quality factor | $1-$ |
| $\omega 1$ | lower 3dB cutoff radian frequency | $1 \_r / s$ |
| $\omega 2$ | upper 3dB cutoff radian frequency | $1-r / s$ |
| $\beta$ | 3dB bandwidth radian frequency | $1 \_r / s$ |
| L | inductance | $1 \_\mathrm{H}$ |
| C | capacitance | $1 \_\mathrm{F}$ |
| R | resistance | $1 \_\Omega$ |



Example: A parallel RLC circuit is constructed with a 100 k $\Omega$ resistor, a $40 \_\mathrm{mH}$ inductor and a $0.15 \_\mathrm{F}$ capacitor. Find the electrical characteristics of this tank circuit.

## Given

$R=100 \_k \Omega$
$\mathrm{L}=40 \mathrm{mH}$
$\mathrm{C}=0.15 \mu \mathrm{~F}$

## Result

$$
\begin{aligned}
& \omega 0=12909.9445 \_\mathrm{r} / \mathrm{s} \\
& \mathrm{fo}=2054.6815 \_\mathrm{Hz} \\
& \mathrm{Q}=193.6492 \\
& \omega 1=12876.6542 \_\mathrm{r} / \mathrm{s} \\
& \omega 2=12943.3209 \_\mathrm{r} / \mathrm{s} \\
& \beta=66.6667 \_\mathrm{r} / \mathrm{s}
\end{aligned}
$$

## OpAmp Circuits

This category consists of OpAmp circuits that focus on five specific configurations: A basic inverting amplifier, a non-inverting amplifier, a current amplifier, a current to voltage converter, and a voltage to current converter. You can use these OpAmp equations in designing OpAmp building blocks.

- Inverting OpAmp
- Non-Inverting OpAmp
- Current OpAmp
- Current to Voltage Converter
- Voltage to Current Converter


## Inverting OpAmp

The equations below represent design equations for an inverting OpAmp. The equations cover ideal and actual OpAmp cases. The impact of non-ideal opamp parameters on Avc is evident from the equations below.

$$
\begin{array}{ll}
\text { 1) } A v c=\frac{\frac{-R f}{R 1}}{1+\frac{1}{\beta \cdot A v}} & \text { 2) } \beta=\frac{R 1}{R 1+R f} \\
\text { 3) } A v c=\frac{-\left(\frac{R f}{R 1}\right)}{1+\frac{R f+R o}{\beta \cdot A v \cdot R f}} & \text { 4) } R f o p t=\sqrt{\frac{R i d \cdot R o}{2 \cdot \beta}}
\end{array}
$$

5) $R i n=R 1 \cdot\left(1+\frac{R f}{A v o \cdot R 1}\right)$
6) Rout $=\frac{R o}{1+\beta \cdot A v}$
7) $f c p=\frac{f o p \cdot A v o \cdot R 1}{R f}$
8) $\operatorname{tr}=\frac{0.35 \cdot R f}{f o p \cdot A v o \cdot R 1}$
9) $R p=\frac{R 1 \cdot R f}{R 1+R f}$

| Variable | Description | Units |
| :--- | :--- | :--- |
| Av | open loop voltage gain | 1 |


| Avo | open loop DC voltage gain | 1 |
| :--- | :--- | :--- |
| Avc | closed loop voltage gain | 1 |
| Avco | closed loop DC voltage gain | 1 |
| $\beta$ | feedback ratio | 1 |
| fcp | 3dB bandwidth | $1 \_\mathrm{Hz}$ |
| fop | first pole of OpAmp | $1-\mathrm{Hz}$ |
| R1 | input resistor | $1-\Omega$ |
| Rf | feedback resistor | $1-\Omega$ |
| Rfopt | optimum Rf for minimum gain error | $1-\Omega$ |
| Rid | differential input resistance | $1-\Omega$ |
| Rin | load resistance of circuit | $1-\Omega$ |
| Ro | output resistance of OpAmp | $1-\Omega$ |
| Rout | output resistance | $1-\Omega$ |
| tr | rise time 10-90\% | $1-\mathrm{s}$ |
| Rp | optimum resistance | $1 \_\Omega$ |



Example 1: For an OpAmp with an input resistance of $10 \_k \Omega$, a feedback resistance of $50 \_\mathrm{k} \Omega$ and an openloop gain of 100,000 , find the closed loop voltage gain and feedback ratio. Use equations 1 and 2.

Given
$A v=100000$
$R 1=10 \_k \Omega$
$\mathrm{Rf}=50 \_\mathrm{k} \Omega$

## Result

Avc $=-4.9997$
$\beta=0.1667$

Example 2: Continuing the example above, if you include a $150 \_\Omega$ output resistance, you get the following results, using equations 2 and 3.

## Given

$A v=100000$
$\mathrm{R} 1=10 \_\mathrm{k} \Omega$
$\mathrm{Rf}=50 \_\mathrm{k} \Omega$
$R \mathrm{Ro}=150 \_\Omega$

## Result

Avc $=-4.9997$
$\beta=0.1667$

## Non-Inverting OpAmp

This equation set provides the key design equations for a non-inverting amplifier. As in the inverting OpAmp case, ideal and practical cases are included.

1) $A v c=\frac{1+\frac{R f}{R 1}}{1+\frac{1}{\beta \cdot A v}}$
2) $\beta=\frac{R 1}{R 1+R f}$
3) $A v c=\frac{1+\frac{R f}{R 1}}{1+\frac{1}{\beta \cdot A v}+\frac{2 \cdot R f}{A v \cdot R i d}}$
4) $A v c=\frac{1+\frac{R f}{R 1}}{1+\frac{R 1+R f+R o}{A v \cdot R f}}$
5) Rfopt $=\sqrt{\frac{R i d \cdot R o \cdot R f}{2 \cdot R 1}}$
6) $\operatorname{Rin}=\frac{\beta \cdot A v \cdot R i d^{2} \cdot R f}{(R f+R o) \cdot(R i d+2 \cdot \beta \cdot R f)}$
7) Rout $=\frac{R o \cdot(R f+R o) \cdot(R i d+2 \cdot \beta \cdot R f)}{\beta \cdot A v \cdot R f \cdot R i d}$
8) $f c p=\frac{f o p \cdot A v o \cdot R 1}{R f+R 1}$
9) $t r=\frac{0.35 \cdot(R 1+R f)}{f o p \cdot A v o \cdot R 1}$
10) $R p=\frac{R 1 \cdot R f}{R 1+R f}-R s$

| Variable | Description | Units |
| :--- | :--- | :--- |
| Av | open loop voltage gain | 1 |
| Avo | open loop DC voltage gain | 1 |


| Avc | closed loop voltage gain | 1 |
| :--- | :--- | :--- |
| $\beta$ | feedback ratio | 1 |
| fcp | 3 dB bandwidth | $1 \_\mathrm{Hz}$ |
| fop | first pole of OpAmp | $1 \_\mathrm{Hz}$ |
| R1 | input resistor | $1 \_\Omega$ |
| Rf | feedback resistor | $1 \_\Omega$ |
| Rfopt | optimum Rf for minimum gain error | $1 \_\Omega$ |
| Rid | differential input resistance | $1 \_\Omega$ |
| Rin | input resistance of circuit | $1 \_\Omega$ |
| Ro | output resistance of OpAmp | $1 \_\Omega$ |
| Rout | output resistance | $1 \_\Omega$ |
| tr | rise time 10-90\% | $1 \_s$ |
| Rp | optimum value of Rp | $1 \_\Omega$ |
| Rs | source resistance | $1 \_\Omega$ |



Example 1: An ideal non-inverting OpAmp has an open loop voltage gain of 1000 , a $15 \_\mathrm{k} \Omega$ feedback resistor and a $1 \_\mathrm{k} \Omega$ input resistor. Calculate the feedback ratio and the closed loop voltage gain. Use equations 1 and 2.

## Given

$A v=1000$
R1 $=1 \_k \Omega$
$\mathrm{Rf}=15 \_\mathrm{k} \Omega$

Result
$\mathrm{Avc}=15.7480$
$\beta=0.0625$

Example 2: The above amplifier has a differential input resistance of $12 \_\mathrm{k} \Omega$. Calculate the revised value of closed loop.gain, using equation 3.

Given
Rid $=12 \mathrm{k} \Omega$
R1 $=1 \mathrm{k} \Omega$
$A v=1000$
$\beta=0.0625$
$\mathrm{Rf}=15 \mathrm{k} \Omega$

## Current Amplifier

This equation set describes the behavior of a current amplifier.

1) $A i c=\frac{1+\frac{R f}{R s}}{1+\frac{1}{\beta \cdot A V}}$
2) $\beta=\frac{R s}{R s+R 1}$
3) $A i c=\frac{(R s+R f) \cdot A v}{R I+R o+R s \cdot(1+A v)}$
4) $\quad \mathrm{Aic}=\frac{\mathrm{Rid} \cdot((\mathrm{Rf}+\mathrm{Rs}) \cdot \mathrm{Av}+\mathrm{Rs})}{(\mathrm{Rid}+\mathrm{Rf}) \cdot(\mathrm{Rs}+\mathrm{RI}+\mathrm{Ro})+\mathrm{Rs} \cdot(\mathrm{RI}+\mathrm{Ro})+(\mathrm{Rs} \cdot \mathrm{Rid} \cdot \mathrm{Av})}$
5) $\operatorname{Rin}=R f \cdot(1+A v)$
6) $\operatorname{Rin}=\frac{R i d \cdot(R f \cdot(R s+R I+R o)+R s \cdot(R I+R o))}{(R i d+R f) \cdot(R s+R I+R o)+R s \cdot(R I+R o)+R s \cdot R i d \cdot A v}$
7) Rout $=$ Rs $\cdot(1+A v)$ 8) (below)
$R o u t=\frac{(R f+R s) \cdot(R o+R s) \cdot((R i d+R f) \cdot(R s+R o)+R s \cdot R o+R s \cdot R i d \cdot A v)}{(R i d+R f+R s) \cdot(R f \cdot R s+R f \cdot R o+R s \cdot R o)}$

| Variable | Description | Units |
| :--- | :--- | :--- |
| Aic | closed loop current gain | 1 |
| Av | closed loop voltage gain | 1 |
| $\beta$ | feedback ratio | 1 |
| R1 | resistance | $1 \Omega \Omega$ |
| Rf | feedback resistance | $1-\Omega \Omega$ |
| Rid | differential input resistance | $1-\Omega \Omega$ |
| Rin | input resistance | $1-\Omega$ |
| RI | load resistance | $1-\Omega$ |
| Ro | output resistance of OpAmp | $1-\Omega$ |
| Rout | output resistance | $1-\Omega$ |
| Rs | resistance | $1 \_\Omega$ |



Example 1: A current amplifier has a $25 \_k \Omega$ feedback resistor, a load resistance of $1500 \_\Omega$ and a source resistance of $50 \_\Omega$. If the open loop gain is 50 , find the feedback ratio and current gain. Use equations 1 and 2.

## Given

$A v=1000$
$\mathrm{RI}=1500 \Omega$
$\mathrm{Rf}=25 \mathrm{k} \bar{\Omega}$
Rs $=50 \_\Omega$

Example 2: The amplifier described above has an output resistance of $10 \_\Omega$. Find the closed loop current gain, in this case. Use equation 3.

## Given

Ro $=10 \_\Omega$

## Result

Aic $=485.9360$
$\beta=3.2258 \mathrm{E}-2$

Result
Aic $=485.8417$

## Current to Voltage Converter

These equations model a current-voltage converter that provides an output voltage proportional to input current. The circuit is characterized by zero input resistance and zero output resistance for an ideal circuit.

1) $A r c=\frac{-(R f \cdot R i d \cdot A v)}{R f+R i d \cdot(1+A v)}$
2) $R$ in $=\frac{R f}{1+A v}$
3) $\operatorname{Rin}=\frac{R i d \cdot(R o+R f)}{R o+R f+R i d \cdot(1+A v)}$
4) Rout $=\frac{R o \cdot(R f+R i d)}{R o+R f+R i d \cdot(1+A v)}$

| Variable | Description | Units |
| :--- | :--- | :--- |
| Arc | closed loop transresistance | $1 \_\Omega$ |
| Av | open loop voltage gain | $1 \_$ |
| Rf | feedback resistor | $1 \_\Omega$ |
| Rin | input resistance | $1 \_\Omega$ |
| Ro | output resistance of OpAmp | $1 \_\Omega$ |
| Rout | output resistance | $1 \_\Omega$ |



Example 1: A current to voltage converter is being designed using a $100 \_\mathrm{k} \Omega$ feedback resistor, a $12 \_k \Omega$ differential input resistor, with an output resistance of $250 \_\Omega$ and an open-loop voltage gain of 10000 . Find the input and output resistances and transfer resistance for an ideal converter. Use equations 1,2 and 4.

## Given

$A v=10000$
$\mathrm{Rf}=100 \mathrm{k} \Omega$
Rid $=12 \mathrm{k} \Omega$
$R \mathrm{Ro}=250 \_\Omega$

## Result

Arc $=-99906.7537 \_\Omega$
Rin $=9.9990 \_\Omega$
Rout $=0.2331 \_\Omega$

Example 2: Using the same example, calculate the output and input resistance for the non-ideal converter. Use equation 3 instead of equation 2.

Given
$A v=10000$
$\mathrm{Rf}=100 \mathrm{k} \Omega$
Rid $=12 k \Omega$
$R o=250 \_\Omega$

Result
Rin $=10.0156 \_\Omega$
Rout $=0.2331 \_\Omega$

## Voltage to Current Converter

This topic describes design equations for a voltage to current amplifier. This circuit is characterized by an output current proportional to input voltage.

1) $A g c=\frac{\frac{1}{R s}}{1+\frac{1}{\beta \cdot A v}}$
2) $\beta=\frac{R s}{R s+R I+R o}$
3) $\mathrm{Agc}=\frac{\mathrm{Rid} \cdot \mathrm{Av}-\mathrm{Rg}}{(\mathrm{Ro}+\mathrm{R} 1) \cdot(\mathrm{Rs}+\mathrm{Rid}+\mathrm{Rg})+\mathrm{Rs} \cdot(\mathrm{Rid}+\mathrm{Rg})+\mathrm{Rs} \cdot \mathrm{Rid} \cdot \mathrm{Av}}$

## Equation Library

4) $\operatorname{Rin}=\operatorname{Rid} \cdot(1+A v \cdot \beta)$
5) $R i n=R i d+R g+\frac{R s \cdot(R o+R I+R i d \cdot A v)}{R s+R I+R o}$
6) Rout $=R o+\frac{R s \cdot(R g+R i d \cdot(1+A v))}{R s+R i d+R s}$

| Variable | Description <br> Agc | Units |
| :--- | :--- | :--- |
| transconductance | $1 \_S$ |  |
| $\beta$ | general resistance | $1 \_\Omega$ |
| Av | factor | 1 |
| RI | open loop gain | 1 |
| Ro | current sensing R | $1 \_\Omega$ |
| Rid | output resistance of OpAmp | $1 \_\Omega$ |
| Rin | differential input resistance | $1 \_\Omega$ |
| Rg | input resistance | $1 \_\Omega$ |
| Rout | resistor | $1 \_\Omega$ |
|  | output resistance | $1 \_\Omega$ |



Example: A voltage to current converter needs to be designed with a general resistance of $100 \_\Omega$, an open loop voltage gain of 1000, a generator resistance of $50 \_\Omega$ a differential input resistance of $500 \_\Omega$, a load of $320 \_\Omega$ and an OpAmp output resistance of 725 . $\Omega$. Find the transconductance, transfer factor, and input and output resistance for an ideal converter.

## Given

$A v=1000$
Rs $=100 \Omega$
$\mathrm{Rg}=50 \Omega$
Rid $=500 \_\Omega$
Ro $=725 \Omega$
$\mathrm{RI}=320 \_\Omega$

Result
Agc $=9.8868 \mathrm{E}-3$ _S
$\beta=8.7336 \mathrm{E}-2$
Rin $=44168.1223 \Omega$
Rout $=72232.1429 \_\Omega$

## Simple AC Circuits

The simple AC circuit equations cover rules for combining two AC impedance elements. Series and parallel combinations, power dissipation in a load, and power factor calculations are included.

- Impedance ( Z ) to Admittance ( Y ) Conversion
- Admittance ( Y ) to Impedance ( Z ) Conversion
- Two Impedances in Series
- Two Impedances in Parallel
- AC Circuit Calculations (Current in Load)


## Impedance $(Z)$ to Admittance $(\mathrm{Y})$ Conversion

1) $Z=\sqrt{Z I^{2}+Z i^{2}}$
2) $\varphi z=\operatorname{ATAN}\left(\frac{Z i}{Z r}\right)$
3) $\varphi y=-\varphi z$
4) $Y=\frac{1}{Z}$
5) $Y r=Y \cdot \operatorname{COS}(\varphi y)$
6) $Y i=Y \cdot \operatorname{SIN}(\varphi y)$

| Variable | Description | Units |
| :--- | :--- | :--- |
| Z | impedance | $1 \_\Omega$ |
| Zr | real part of impedance | $1-\Omega$ |
| Zi | imaginary part of impedance | $1-\Omega$ |
| $\phi \mathrm{Z}$ | phase angle of impedance | $1-0$ |
| Y | admittance | $1-\mathrm{S}$ |
| Yr | real part of admittance | $1-\mathrm{S}$ |
| Yi | imaginary part of admittance | $1-\mathrm{S}$ |
| $\phi \mathbf{y}$ | phase angle of admittance | $1-0$ |



Example: Convert an impedance with a real part and imaginary part of $100 \_\Omega$ and $125 \_\Omega$ to an admittance. Find the phase angle.

Given
$\mathrm{Zi}=125 \_\Omega$
$\mathrm{Zr}=100 \_\Omega$

Result

$$
\begin{aligned}
& \mathrm{Z}=160.0781 \_\Omega \\
& \mathrm{Y}=6.2470 \mathrm{E}-3 \_\mathrm{S} \\
& \mathrm{Yr}=3.9024 \mathrm{E}-3-\mathrm{S} \\
& \mathrm{Yi}=-4.8780 \mathrm{E}-3-\mathrm{S} \\
& \phi \mathrm{Z}=51.3402- \\
& \phi \mathrm{y}=-51.3402 .
\end{aligned}
$$

Note: $\phi \mathrm{z}$ and $\phi \mathrm{y}$ are reduced to $-180^{\circ}+180^{\circ}$ range only.

## Admittance to Impedance Conversion

Conversion of admittance to impedance is covered in this set of equations.

1) $Y=\sqrt{Y Y^{2}+Y i^{2}}$
2) $\varphi y=\operatorname{ATAN}\left(\frac{Y i}{Y r}\right)$
3) $\varphi z=-\varphi y$
4) $Z=\frac{1}{Y}$
5) $Z r=Z \cdot \operatorname{COS}(\varphi z)$
6) $Z i=Z \cdot \operatorname{SIN}(\varphi z)$

| Variable | Description | Units |
| :--- | :--- | :--- |
| Z | impedance | $1 \_\Omega$ |
| Zr | real part of impedance | $1 \_\Omega$ |
| Zi | imaginary part of impedance | $1 \_\Omega$ |
| $\phi \mathrm{Z}$ | phase angle of impedance | $1-0$ |
| Y | admittance | $1-S$ |
| Yr | real part of admittance | $1-\mathrm{S}$ |
| Yi | imaginary part of admittance | $1-\mathrm{S}$ |
| $\phi \mathrm{y}$ | phase angle of admittance | $1-0$ |



Example: Using the values for real and imaginary parts of admittance from the previous example, calculate the impedance.

> Given
> $\mathrm{Yr}=3.9024 \mathrm{E}-3$ S
> $\mathrm{Yi}=-4.8780 \mathrm{E}-3 \_\mathrm{S}$

## Result

$Y=6.2469 E-3 \_$
$\phi \mathrm{y}=-51.3402^{-}$

$$
\begin{aligned}
& \mathrm{Z}=160.0781 \_\Omega \\
& \mathrm{Zr}=100 \_\Omega \\
& \mathrm{Zi}=125-\Omega \\
& \phi \mathrm{Z}=51.3402_{-}{ }^{\circ}
\end{aligned}
$$

## Two Impedances in Series

The equations in this topic cover combining two impedances algebraically.

1) $Z r=Z 1 r+Z 2 r$
2) $Z i=Z 1 i+Z 2 i$
3) $Z s=\sqrt{Z r^{2}+Z i^{2}}$
4) $\varphi s=\operatorname{ATAN}\left(\frac{Z i}{Z r}\right)$
5) $Z 1=\sqrt{Z 1 r^{2}+Z 1 i^{2}}$
6) $\varphi 1=\operatorname{ATAN}\left(\frac{Z 1 i}{Z 1 r}\right)$
7) $Z 2=\sqrt{Z 2 r^{2}+Z 2 i^{2}}$
8) $\varphi 2=\operatorname{ATAN}\left(\frac{Z 2 i}{Z 2 r}\right)$
Variable
Z 1
Z 1 r
$\mathrm{Z1i}$
$\phi 1$
Z 2
Z 2 r
Z 2 i
$\phi 2$
Zs
Zr
Zi
$\phi \mathrm{s}$

Description
impedance of ac element 1
real part of Z1
imaginary part of $Z 1$
phase angle of $\mathrm{Z1}$
impedance of ac element 2
real part of Z2
imaginary part of Z2
phase angle of Z2
equivalent impedance
real part of Zs
imaginary part of Zs
phase angle of Zs

Units
1 _ $\Omega$
$1-\Omega$
1 _ $\Omega$
$1^{-}$
1- $\Omega$
$1-\Omega$
$1-\Omega$
$1{ }^{\circ}$
1- $\Omega$
$1 \_\Omega$
$1 \_\Omega$
1_。


Example: An impedance consisting of $100 \Omega$ resistor and $125 \_\Omega$ inductive reactance is connected in series with an impedance with $125 \Omega \Omega$ resistance
and $180 \_\Omega$ capacitive reactance. Find the resulting impedance and phase angle.
Given
$\mathrm{Z1r}=100 \_\Omega$
$\mathrm{Z1i}=125 \_\Omega$
$\mathrm{Z} 2 \mathrm{r}=125 \_\Omega$
$\mathrm{Z} 2 \mathrm{i}=-180 \_\Omega$

## Result

$$
\begin{aligned}
& \mathrm{Z1}=160.0781_{1} \Omega \\
& \phi 1=51.3402^{\circ} \\
& \mathrm{Z2}=219.1461_{-}^{-} \Omega \\
& \phi 2=-55.2222^{-} \\
& \mathrm{Zs}=231.6247_{-} \Omega \\
& \mathrm{Zr}=225 \_\Omega \\
& \mathrm{Zi}=-55 \Omega \\
& \phi \mathrm{~s}=-13.7363_{-}^{\circ}
\end{aligned}
$$

## Two Impedances in Parallel

This equation set finds the result of two impedances connected in parallel.

1) $Z 1=\sqrt{Z 1 r^{2}+Z 1 i^{2}}$
2) $\varphi 1=\operatorname{ATAN}\left(\frac{Z 1 i}{Z 1 r}\right)$
3) $Z 2=\sqrt{Z 2 r^{2}+Z 2 i^{2}}$
4) $\varphi 2=\operatorname{ATAN}\left(\frac{Z 2 i}{Z 2 r}\right)$
5) $Z p=\left(\frac{(Z 1 r \cdot Z 2 r-Z 1 i \cdot Z 2 i)^{2}+(Z 1 r \cdot Z 2 i+Z 2 r \cdot Z 1 i)^{2}}{(Z 1 r+Z 2 r)^{2}+(Z 1 i+Z 2 i)^{2}}\right)^{V_{2}}$
6) $\varphi p=\operatorname{ATAN}\left(\frac{Z 1 r \cdot Z 2 i+Z 2 r \cdot Z 1 i}{Z 1 r \cdot Z 2 r-Z 1 i \cdot Z 2 i}\right)-\operatorname{ATAN}\left(\frac{Z 1 i+Z 2 i}{Z 1 r+Z 2 r}\right)$
7) $Z r=Z p \cdot \operatorname{COS}(\varphi p)$
8) $Z i=Z p \cdot \operatorname{SIN}(\varphi p)$

| Variable | Description | Units |
| :---: | :---: | :---: |
| Z1 | impedance of ac element 1 | 1_ $\Omega$ |
| Z1r | real part of Z1 | $1 \_\Omega$ |
| Z1i | imaginary part of $\mathrm{Z1}$ | $1 \_\Omega$ |
| $\phi 1$ | phase angle of $\mathrm{Z1}$ | $1{ }^{\circ}$ |
| Z2 | impedance of ac element 2 | $1 \_\Omega$ |
| Z2r | real part of Z2 | $1-\Omega$ |
| Z2i | imaginary part of Z2 | $1 \_\Omega$ |
| $\phi 2$ | phase angle of Z2 | $1{ }^{-}$ |
| Zp | equivalent impedance | $1-\Omega$ |
| Zr | real part of Zp | $1 \_\Omega$ |

Zi
$\phi p$
imaginary part of Zp
1 _ $\Omega$
phase angle of Zp
$1^{-}$


Example: Two impedances $(212,185)$ and $(475,-874)$ are connected in parallel. Find the combined impedances.

## Given

$\mathrm{Z} 1 \mathrm{r}=212 \Omega$
$\mathrm{Z1i}=185 \Omega$
$\mathrm{Z} 2 \mathrm{r}=475 \Omega$
$Z 2 i=-874 \_\Omega$

## Result

$$
\begin{aligned}
& \mathrm{Z1}=281.3699 \_\Omega \\
& \phi 1=41.1093^{-}- \\
& \mathrm{Z} 2=994.7366-\Omega \\
& \phi 2=-61.4769^{-}- \\
& \mathrm{Zp}=287.6615_{-}^{-} \Omega \\
& \phi \mathrm{p}=24.7157_{-}^{-} \\
& \mathrm{Zr}=261.3099_{-} \Omega \\
& \mathrm{Zi}=120.2759 \_\Omega
\end{aligned}
$$

## Current in Load

These equations calculate the current in a load ZL from a voltage source with internal impedance Zg .

1) $Z r=Z g r+Z L r$
2) $Z i=Z g i+Z L i$
3) $Z s=\sqrt{Z r^{2}+Z L i^{2}}$
4) $\varphi s=\operatorname{ATAN}\left(\frac{Z i}{z r}\right)$
5) $Z g=\sqrt{Z g r^{2}+Z g i^{2}}$
6) $\varphi g=\operatorname{ATAN}\left(\frac{Z g i}{Z g r}\right)$
7) $Z L=\sqrt{Z L r^{2}+Z L i^{2}}$
8) $\varphi L=A T A N\left(\frac{Z L i}{Z L r}\right)$
9) $L L=\frac{V g}{Z s}$
10) $\varphi i=-\varphi s$
11) $V L=I L \cdot Z L$
12) $\varphi V=\varphi L+\varphi i$
13) $P L=V L \cdot I L \cdot \operatorname{COS}(\varphi V+\varphi i)$
14) $V=V L \cdot I L$
15) $p f=\operatorname{CoS}(\varphi V+\varphi i)$

| Variable | Description | Units |
| :---: | :---: | :---: |
| Zg | impedance of ac voltage source | 1_S |
| Zgr | real part of Zg | 1_ $\Omega$ |
| Zgi | imaginary part of Zg | 1 - $\Omega$ |
| $\phi \mathrm{g}$ | phase angle of Zg | $1{ }^{-}$ |
| ZL | load impedance | 1- $\Omega$ |
| ZLr | real part of ZL | 1 - $\Omega$ |
| ZLi | imaginary part of ZL | 1_ $\Omega$ |
| $\phi$ L | phase angle of ZL | $1{ }^{-}$ |
| Zs | combined series impedance | $1 \_\Omega$ |
| Zr | real part of Zs | 1- $\Omega$ |
| Zi | imaginary part of Zs | 1 - $\Omega$ |
| $\phi$ s | phase angle of Zs | $1{ }^{-}$ |
| IL | current in load | 1 -A |
| $\phi$ | phase angle of current | $1{ }^{-}$ |
| VL | voltage across the load | 1-V |
| $\phi \mathrm{V}$ | phase angle of load voltage | $1^{-}$ |
| PL | power in the load | 1_W |
| VI | volt-amps | 1-W |
| pf | power factor | 1 |
| Vg | source voltage | 1_V |



Example: A $100 \_$V voltage source, with an impedance of $(10,25)$, drives a load of ( $30,-40$ ). Calculate voltage across the load, power in the load, and power factor.

## Given

$\mathrm{Vg}=100 \mathrm{~V}$
$\mathrm{Zgr}=10 \_\Omega$
$\mathrm{Zgi}=25 \_\Omega$
ZLr $=30 \_\Omega$
$Z \mathrm{Li}=-40 \_\Omega$

## Result

$\mathrm{IL}=2.3408$ A
$\mathrm{VL}=117.0 \overline{4} 11 \mathrm{~V}$
PL $=267.9677 \_W$
pf $=0.9781$

## Transformers

The equations in this category describe ideal transformers.

1) $\frac{V 1}{V 2}=\frac{n 1}{n 2}$
2) $11 \cdot n 1=12 \cdot n 2$
3) $\operatorname{Rin}=\frac{R 2}{a^{2}}$
4) $a=\frac{n 2}{n 1}$
5) $V 2=12 \cdot R 2$

| Variable | Description | Units |
| :--- | :--- | :--- |
| V1 | primary voltage | $1-V$ |
| V2 | secondary voltage | $1-V$ |
| n1 | number of turns in primary | 1 |
| n2 | number of turns in secondary | 1 |
| I1 | current in primary | $1-A$ |
| I2 | current in secondary | $1-A$ |
| R2 | secondary load resistance | $1-\Omega$ |
| Rin | resistance at primary from R2 | $1-\Omega$ |
| a | turns ratio | 1 |



Example: An ideal transformer has 20 primary turns and 40 turns in the secondary winding. The input voltage is $5 \_\mathrm{V}$, the load secondary resistance is $15 \_\Omega$, and the primary current is 0.75 _A. Find the secondary current and voltage.

## Given

V1 = 5_V
$\mathrm{n} 1=20$
$\mathrm{n} 2=40$
$11=0.75$ A
$R 2=15 \_\Omega$

## Result

V2 $=10 \_V$
12 $=0.375$ A
$\operatorname{Rin}=3.75 \_\Omega$
$\mathrm{a}=2.0$

## Transmission Lines

The transmission line category includes skin effect and ideal transmission line calculations.

■ Skin Effect

- Ideal Transmission Line


## Skin Effect

The resistance of a conductor carrying a current is distributed uniformly over the cross sectional area at low frequencies. However, at higher frequencies, the self inductance forces the current to crowd toward the surface. Skin depth $\delta$ represents effective depth of penetration of the RF signal.

1) $\delta=\frac{1}{\left(\frac{\pi \cdot f \cdot \mu o}{\rho}\right)^{v_{2}}}$
2) $R f=\frac{R d c}{1-\left(1-\frac{\delta}{r}\right)^{2}}$

| Variable | Description | Units |
| :--- | :--- | :--- |
| $\delta$ | skin depth | $1-\mathrm{m}$ |
| f | frequency | $1-\mathrm{Hz}$ |
| $\rho$ | resistivity | $1 \_\Omega \cdot \mathrm{m}$ |
| Rf | resistance at frequency f | $1 \_\Omega$ |
| Rdc | resistance at dc | $1 \_\Omega$ |
| r | radius of wire | $1 \_\mathrm{m}$ |

Example 1: A conductor with a radius of 0.1 cm carries a $50 \_\mathrm{MHz}$ signal in a material with a resistivity of $0.0000025 \_\Omega \cdot \mathrm{cm}$. Find the skin depth of this material in $\mu$.

$$
\begin{array}{ll}
\text { Given } & \text { Result } \\
\mathrm{f}=50 \_\mathrm{MHz} & \delta=11.2539 \_\mu \\
\rho=0.0000025 \_\Omega \cdot \mathrm{cm} &
\end{array}
$$

## Equation Library

Ideal Transmission Line
Assumes that the transmission lines are ideal, allowing you to calculate various parameters, such as characteristic impedance, VSWR.

1) $Z o=\sqrt{1 / c}$
2) $\beta=\omega \cdot \sqrt{1 \cdot c}$
3) $V S W R=\frac{1-\rho}{1+\rho}$
4) Zinqrt $=\frac{Z o^{2}}{Z I}$
5) $\rho=\frac{Z I-Z o}{Z I+Z o}$
6) $\omega=2 \cdot \pi \cdot f$

| Variable | Description | Units |
| :--- | :--- | :--- |
| Zo | characteristic impedance | $1 \_\Omega$ |
| ZI | load impedance | $1-\Omega$ |
| I | inductance/unit length | $1-\mathrm{H} / \mathrm{m}$ |
| c | capacitance/unit length | $1-\mathrm{F} / \mathrm{m}$ |
| Zingrt | input impedance at quarter wave length | $1-\Omega$ |
| $\beta$ | phase constant | $1-\mathrm{r} / \mathrm{m}$ |
| $\omega$ | radian frequency | $1-\mathrm{r} / \mathrm{s}$ |
| VSWR | Voltage Standing Wave Radio | 1 |
| $\rho$ | reflection coefficient | 1 |
| f | frequency | $1 \_\mathrm{Hz}$ |

Example: An ideal transmission line has a series inductance of $1 \mathrm{E}-8$ _H/m, a shunt capacitance of $7.0359 \mathrm{E}-14 \_\mathrm{F} / \mathrm{m}$, and a load impedance of $1000 \_\Omega$.
Calculate the transmission line parameters at $100 \_\mathrm{kHz}$.

## Given

$\mathrm{I}=1 \mathrm{E}-8 \_\mathrm{H} / \mathrm{m}$
$\mathrm{c}=7.0359 \mathrm{E}-14 \_\mathrm{F} / \mathrm{m}$
$\mathrm{ZI}=1000 \_\Omega$
$\mathrm{f}=100 \_\mathrm{kHz}$

## Result

$$
\begin{aligned}
& \mathrm{Zo}=376.999 \_\Omega \\
& \beta=1.6666 \mathrm{E}-5 \_\mathrm{r} / \mathrm{m} \\
& \text { Zinqrt }=142.1282 \_\Omega \\
& \rho=0.4524 \\
& \text { VSWR }=0.37699
\end{aligned}
$$

$$
\omega=628318.5307 \_\mathrm{r} / \mathrm{s}
$$

## Motors and Generators

This category covers basic properties of motors and generators.

## Transmission Lines

The transmission line category includes skin effect and ideal transmission line calculations.

■ Skin Effect

- Ideal Transmission Line


## Skin Effect

The resistance of a conductor carrying a current is distributed uniformly over the cross sectional area at low frequencies. However, at higher frequencies, the self inductance forces the current to crowd toward the surface. Skin depth $\delta$ represents effective depth of penetration of the RF signal.

1) $\delta=\frac{1}{\left(\frac{\pi \cdot f \cdot \mu o}{\rho}\right)^{v_{2}}}$
2) $R f=\frac{R d c}{1-\left(1-\frac{\delta}{r}\right)^{2}}$

| Variable | Description | Units |
| :--- | :--- | :--- |
| $\delta$ | skin depth | $1 \_\mathrm{m}$ |
| f | frequency | $1 \_\mathrm{Hz}$ |
| $\rho$ | resistivity | $1 \_\Omega \cdot \mathrm{m}$ |
| Rf | resistance at frequency f | $1 \_\Omega$ |
| Rdc | resistance at dc | $1 \_\Omega$ |
| r | radius of wire | $1 \_\mathrm{m}$ |

Example 1: A conductor with a radius of 0.1_cm carries a 50 _MHz signal in a material with a resistivity of $0.0000025 \_\Omega \cdot \mathrm{cm}$. Find the skin depth of this material in $\mu$.

$$
\begin{array}{ll}
\text { Given } & \text { Result } \\
\mathrm{f}=50 \_\mathrm{MHz} & \delta=11.2539 \_\mu \\
\rho=0.0000025 \_\Omega \cdot \mathrm{cm} &
\end{array}
$$

## Ideal Transmission Line

Assumes that the transmission lines are ideal, allowing you to calculate various parameters, such as characteristic impedance, VSWR.

1) $Z o=\sqrt{1 / c}$
2) $\beta=\omega \cdot \sqrt{1 \cdot c}$
3) $V S W R=\frac{1-\rho}{1+\rho}$
4) Zinqrt $=\frac{Z o^{2}}{Z I}$
5) $\rho=\frac{Z I-Z o}{Z I+Z o}$
6) $\omega=2 \cdot \pi \cdot f$

| Variable | Description | Units |
| :--- | :--- | :--- |
| Zo | characteristic impedance | $1-\Omega$ |
| ZI | load impedance | $1-\Omega$ |
| I | inductance/unit length | $1-\mathrm{H} / \mathrm{m}$ |
| c | capacitance/unit length | $1-\mathrm{F} / \mathrm{m}$ |
| Zingrt | input impedance at quarter wave length | $1-\Omega$ |
| $\beta$ | phase constant | $1-\mathrm{r} / \mathrm{m}$ |
| $\omega$ | radian frequency | $1 \_\mathrm{r} / \mathrm{s}$ |
| VSWR | Voltage Standing Wave Radio | 1 |
| $\rho$ | reflection coefficient | 1 |
| f | frequency | $1 \_\mathrm{Hz}$ |

Example: An ideal transmission line has a series inductance of $1 \mathrm{E}-8$ _H/m, a shunt capacitance of $7.0359 \mathrm{E}-14 \_\mathrm{F} / \mathrm{m}$, and a load impedance of $1000 \_\Omega$.
Calculate the transmission line parameters at 100 kHz .

$$
\begin{aligned}
& \text { Given } \\
& \mathrm{I}=1 \mathrm{E}-8 \_\mathrm{H} / \mathrm{m} \\
& \mathrm{c}=7.0359 \mathrm{E}-14 \_\mathrm{F} / \mathrm{m} \\
& \mathrm{ZI}=1000 \_\Omega \\
& \mathrm{f}=100 \_\mathrm{kHz}
\end{aligned}
$$

## Result

Zo $=376.999 \_\Omega$
$\beta=1.6666 \mathrm{E}-5 \mathrm{r} / \mathrm{m}$
Zinqrt $=142.1282 \Omega$
$\rho=0.4524$
VSWR $=0.37699$

$$
\omega=628318.5307 \_\mathrm{r} / \mathrm{s}
$$

## Motors and Generators

This category covers basic properties of motors and generators.

- DC Generators
- DC Motors
- Induction Motors
- Synchonous Machines


## DC Generators

These equations govern voltage generation in a DC generator and its relationships to the mechanical energy input.

1) $E g=K v \cdot \varphi \cdot n$
2) $E g=V a \cdot \frac{I L}{l a}+I f^{2} \cdot \frac{R f}{l a}+l a \cdot R a$
3) $T=K T \cdot \varphi \cdot l a$
4) $K T=\frac{p \cdot Z c}{2 \cdot \pi \cdot a p}$
5) $K T=\frac{60 \cdot K v}{2 \cdot \pi}$
6) $E g=\frac{p \cdot Z c}{60 \cdot a p} \cdot \varphi \cdot n$
7) $K v=\frac{p \cdot Z c}{60 \cdot a p}$
8) $P m=P r+T \cdot \Omega$
9) $\Omega=\frac{n}{60} \cdot 2 \pi$

| Variable | Description | Units |
| :--- | :--- | :--- |
| Eg | generated voltage | $1 \_\mathrm{V}$ |
| KV | voltage constant | $1-$ |
| $\phi$ | magnetic flux | $1 \_\mathrm{Wb}$ |
| $\Omega$ | mech angular velocity | $1 \_\mathrm{r} / \mathrm{s}$ |
| Va | terminal voltage | $1 \_\mathrm{V}$ |
| Ra | armature resistance | $1 \_\Omega$ |
| IL | load current | $1 \_\mathrm{A}$ |
| If | field current | $1 \_\mathrm{A}$ |
| la | armature current | $1 \_\mathrm{A}$ |
| Rf | field resistance | $1 \_\Omega$ |
| T | torque | $1 \_\mathrm{N} \cdot \mathrm{m}$ |
| KT | torque constant | 1 |
| p | number of poles | 1 |


| Zc | number of armature wires | 1 |
| :--- | :--- | :--- |
| ap | number of parallel paths | 1 |
| n | rotational speed (rpm) | 1 |
| Pm | mech power | $1-W$ |
| Pr | mech power loss | $1 \_W$ |

Example 1: A DC generator has four poles rotating with an angular velocity of 150 _rpm. If the flux at each pole is $0.5 \_\mathrm{Wb}$, calculate the generated voltage if the voltage constant is 2.25 . Use equation 1.
Given
$K v=2.25$
$\phi=0.5 \mathrm{~Wb}$
$\omega=150$ _rpm

Result
Eg = 168.75_V

Example 2: For the generator in Example 1, if there are 148 armature wires with four parallel paths, calculate the torque constant and torque with an armature current of 10_A. Use equations 1,3 and 4.

## Given

$p=4$
$Z c=148$
ap $=4$
$l a=10 \_A$

Result
$K T=23.5549$
$T=117.7747 \_N \cdot m$

## DC Motors

This topic contains eight common equations describing DC motors.

1) $\mathrm{Va}=\mathrm{la} \cdot \mathrm{Ra}+K v \cdot \varphi \cdot \Omega$
2) $T=K T \cdot \varphi \cdot l a$
3) $T=\frac{K T \cdot \varphi}{R a} \cdot(V a-K v \cdot \varphi \cdot \Omega)$
4) $P$ in $=V a \cdot l a+V a \cdot I f$
5) $\mathrm{Va} \cdot \mathrm{la}=E g \cdot l a+I a^{2} \cdot R a$
6) $K T=\frac{60 \cdot K v}{2 \cdot \pi}$
7) $E g \cdot l a=T \cdot \Omega$
8) $T=T L+T l o s s$
9) $\Omega=2 \cdot \pi \cdot \frac{n}{60}$
$\left.\begin{array}{lll}\text { Variable } & \begin{array}{l}\text { Description } \\ \text { applied voltage }\end{array} & \text { Units } \\ \text { Va } & \text { armature current } & 1 \_\mathrm{V}\end{array}\right)$

Example 1: A DC motor is drawing 10_A from a 100 V source. The armature resistance is $2.5 \_\Omega$, has a voltage constant of 2.25 and a flux of 0.5 Wb. Find its rotational speed. Use equation 1.

## Given

$$
\begin{aligned}
& \mathrm{Va}=100 \mathrm{~V} \\
& \mathrm{la}=10-\mathrm{A} \\
& \mathrm{Ra}=2.5 \_\Omega \\
& \mathrm{Kv}=2.25 \\
& \phi=0.5 \_\mathrm{Wb}
\end{aligned}
$$

Example 2: Find the generated voltage for this motor. Use equation 5.

## Given

$\mathrm{Va}=100 \mathrm{~V}$
$\mathrm{la}=10 \mathrm{~A}$
$R \mathrm{Ra}=2.5 \_\Omega$

Result
Eg = 75_V

## Induction Motors

These equations describe the performance of induction motors.

1) $E s=\sqrt{2} \cdot \pi \cdot f \cdot N s \cdot K w s \cdot \varphi p$
2) $s=\frac{\Omega s-\Omega r}{\Omega s}$
3) $\Omega r=\frac{p}{2} \cdot \Omega s \cdot s$
4) $\Omega r=s \cdot \Omega$
5) $E r=\frac{s \cdot N r \cdot K w r}{N s \cdot K w s} \cdot E s$
6) $P r=I r^{2} \cdot r r$
7) $P g=\frac{n \cdot l r^{2} \cdot r r \cdot(1-s)}{s}$
8) $P g=T \cdot \Omega r$
9) $P g=T \cdot \Omega s \cdot(1-s)$
10) Rin $=\frac{r r \cdot(1-s)}{s}$

| Variable | Description | Units |
| :---: | :---: | :---: |
| Es | secondary voltage | 1_V |
| f | elec frequency | 1_Hz |
| Ns | stator windings | 1 |
| Kws | stator winding constant | 1 |
| $\phi$ p | flux/pole | 1_Wb |
| s | slip | 1 |
| $\Omega \mathrm{s}$ | stator angular frequency | 1_r/s |
| $\Omega \mathrm{r}$ | rotor angular frequency | $1 \mathrm{r} / \mathrm{s}$ |
| $\Omega$ | angular frequency | 1_r/s |
| Er | rotor voltage | 1-V |
| Nr | rotor windings | 1 |
| Kwr | rotor winding constant | 1 |
| Pr | rotor power | 1_W |
| Ir | rotor current | 1-A |
| rr | rotor resistance | 1 - $\Omega$ |
| Pg | gap power | 1-W |
| n | number of phases | $1-$ |
| Rin | equiv input resistance | 1 _ $\Omega$ |
| p | number of poles | 1 |
| T | torque | 1_N•m |

Example 1: A $60 \_\mathrm{Hz}$ induction motor has 40 secondary windings, $0.64 \_\mathrm{Wb}$ of flux, and a stator constant of $1.82411 \mathrm{E}-2$. Find the secondary voltage.

## Given

$\mathrm{f}=60 \_\mathrm{Hz}$
Ns $=\overline{40}$
Kws $=1.8241 \mathrm{E}-2$
$\phi \mathrm{p}=0.64 \_\mathrm{Wb}$

## Result

Es = 124.4822_V

Example 2: The rotor resistance of the induction motor is $0.26 \Omega \Omega$ and the stator and rotor angular velocities are 126 _r/s and $120 \_r / s$ respectively. What is the slip and input resistance?

## Given

$\Omega \mathrm{s}=126 \mathrm{r} / \mathrm{s}$
$\Omega r=120 \_r / s$ $\mathrm{rr}=0.26 \_$

## Result

$\mathrm{s}=0.04762$
Rin $=5.2 \Omega$

## Synchronous Machines

This class of machines is governed by the explicit relationship between the frequency of the AC circuit and speed of the rotation of the motor.

1) $p=\frac{2 \cdot 60 \cdot f}{n}$
2) $\varphi r=\frac{\mu 0 \cdot N r \cdot A r}{g \cdot p} \cdot I r$
3) $K \varphi=\frac{\sqrt{2} \cdot N r \cdot N s \cdot \mu o \cdot A r \cdot \Omega s}{\pi \cdot g \cdot p}$
4) $E g=K \varphi \cdot I r$

| Variable | Description | Units |
| :--- | :--- | :--- |
| p | number of poles | 1 |
| f | elec frequency | $1 \_\mathrm{Hz}$ |
| n | revolutions per minute | $1-\mathrm{Wb}$ |
| $\phi \mathrm{r}$ | flux | $1-\mathrm{Wb}$ |
| Nr | number of rotor windings | 1 |
| Ar | rotor cross section | $1-\mathrm{m}^{\wedge} 2$ |
| g | gap length | $1-\mathrm{m}$ |
| lr | rotor current | $1-\mathrm{A}$ |
| $\mathrm{K} \phi$ | rotor gen constant | $1 \_\Omega$ |
| Ns | number of stator windings | 1 |
| $\Omega \mathrm{~s}$ | elec radian frequency | $1-r / s$ |
| Eg | voltage | $1 \_V$ |

Example 1: A 4-pole synchronous machine operates at an electrical frequency of 60 Hz . What is its angular velocity in rpm?

$$
\begin{aligned}
& \text { Given } \\
& p=4 \\
& f=60 \_H z
\end{aligned}
$$

Result

$$
n=1800
$$

## Equation Library

Example 2: If the rotor is carrying 150_A current, has 40 windings, has 48 _ $\mathrm{cm}^{2}$ area of cross section and a gap of 0.18 _ cm , find the flux. Use equation 2.

> Given
> $\mathrm{Ir}=150 \_\mathrm{A}$
> $\mathrm{g}=0.18 \_\mathrm{cm}$
> $\mathrm{p}=4$
> $\mathrm{Nr}=40$
> $\mathrm{Ar}=48 \_\mathrm{cm}^{2}$

## Solid State Devices

This category lists solid state device equations describing PN junctions, NMOS and CMOS transistors, where the software allows you to calculate intrinsic device currents or voltages and draw the current-voltage characteristics.

- PN Junctions
- Currents in PN Junctions
- NMOS Transistors
- Currents in NMOS Transistors
- CMOS
- BJT-Ebers \& Moll Model


## PN Junctions

These equations describe PN junctions using step junction approximation.

1) $V b i=\frac{k \cdot T}{q} \cdot L N\left(\frac{N D \cdot N A}{n i^{2}}\right)$
2) $x d=\left[\frac{2 \cdot \varepsilon o \cdot \varepsilon s i}{q} \cdot(V a-V b i) \cdot\left(\frac{1}{N A}+\frac{1}{N D}\right)\right]^{V_{2}}$
3) $C J=\left[\frac{q \cdot \varepsilon o \cdot \varepsilon S i}{2 \cdot\left(\frac{1}{N A}+\frac{1}{N D}\right) \cdot(V a-V b i)}\right]^{V_{2}}$
4) $E \max =\left(\frac{2 \cdot q \cdot\left(\frac{N A \cdot N D}{N A+N D}\right) \cdot(V a-V b i)}{\varepsilon O \cdot \varepsilon S i}\right)^{1 / 2}$
5) $B V=\frac{\varepsilon S i \cdot \varepsilon o \cdot \varepsilon 1^{2}}{2 \cdot q \cdot\left(\frac{N A \cdot N D}{N A+N D}\right)}$

| Variable | Description |
| :--- | :--- |
| ND | donor density |
| NA | acceptor density |
| T | temperature |
| xd | depletion layer width |
| Va | applied voltage |
| Vbi | built-in voltage |
| CJ | junction capacitance per unit area |
| Emax | maximum field in the depletion region |
| BV | breakdown voltage |

## Units

$1-1 / m^{\wedge} 3$
$1-1 / m^{\wedge} 3$
$1-K$
$1-m$
$1-\mathrm{V}$
$1-\mathrm{V}$
$1-\mathrm{F} / \mathrm{m}^{\wedge}{ }^{\wedge} 2$
$1-\mathrm{V} / \mathrm{m}$
$1-\mathrm{V}$


Example 1: A pn junction is fabricated by a gallium doped p region with a density of 1E19_cm ${ }^{-3}$ and an arsenic doped $n$ region with a density of $1 \mathrm{E} 15 \mathrm{~cm}^{-3}$. At room temperature, calculate the built-in voltages and the depletion layer width. Use the first two equations.

```
Given
\(N D=1 E 15 \mathrm{~cm}^{-3}\)
\(\mathrm{NA}=1 \mathrm{E} 19_{-}^{-} \mathrm{cm}^{-3}\)
\(\mathrm{T}=300 \mathrm{~K}\)
\(\mathrm{Va}=0 \_\overline{\mathrm{V}}\)
```

Result
$\mathrm{Vbi}=0.8179$ V
$x d=1.0372 \mathrm{E}-6 \_\mathrm{m}$

Example 2: If a reverse bias of $10 \_\mathrm{V}$ is applied to the diode in Example 1, find the junction capacitance.

```
Given
\(\mathrm{NA}=1 \mathrm{E} 19_{-} \mathrm{cm}^{-3}\)
\(\mathrm{ND}=1 \mathrm{E} 15_{-} \mathrm{cm}^{-3}\)
```


## Result

$\mathrm{xd}=3.4754 \mathrm{E}-6$ _m
$\mathrm{CJ}=3.0318 \mathrm{E}-5-\mathrm{F} / \mathrm{m}^{2}$
$\mathrm{Va}=10 \_\mathrm{V}$
$\mathrm{Vbi}=0 . \overline{8} 179 \_V$

## Currents in PN Junctions

Calculation of currents in PN junctions is based on the minority carrier recombination model developed by William Shockley.

> 1) $J t=J o \cdot\left(e^{\frac{q \cdot v_{a}}{k \cdot T}}-1\right) \quad$ 2) It $=J t \cdot A J$ 3) $J o=q \cdot n i^{2} \cdot\left(\frac{D p}{N D \cdot L p}+\frac{D n}{N A \cdot L n}\right)$

| Variable | Description | Units |
| :---: | :---: | :---: |
| Jt | total junction current density | 1_A/m^2 |
| Jo | saturation current density | 1-A/m^2 |
| Va | applied voltage | 1_V |
| T | temperature | 1_K |
| It | total current | 1_A |
| AJ | effective junction area | 1_m^2 |
| Dp | diffusion length of holes | 1_m ${ }^{\text {²/s }}$ |
| ND | donor density | $1 \_1 / \mathrm{m}^{\wedge} 3$ |
| Lp | holes diffusion coefficient | 1_m |
| Dn | electron diffusion coefficient | 1_m ${ }^{\text {²/ }}$ / |
| NA | acceptor density | 1_1/m^3 |
| Ln | diffusion length of electrons | 1_m |

Example: A pn junction is constructed with 1E18_cm ${ }^{-3}$ acceptors and 1 E 15 _ $\mathrm{cm}^{-3}$ donors. Find the current at 0.25 _V forward bias at $300 \_K$. Calculate the saturation current density, if the diffusion length for holes and electrons are $11.4 \mu$ and $8.65 \mu$ and the diffusion coefficients for electrons and holes are $35 \mathrm{~cm}^{2} / \mathrm{s}$ and $12 \_\mathrm{cm}^{2} / \mathrm{s}$. The junction area is $0.025 \mathrm{~cm}^{2}$. Find the current.

Given
$\mathrm{Va}=0.25 \mathrm{~V}$
$\mathrm{T}=300 \mathrm{~K}$
$\mathrm{AJ}=0.025 \mathrm{~cm}^{2}$
$\mathrm{Dp}=12 \mathrm{~cm}^{-2} / \mathrm{s}$
$N D=1 \bar{E}^{-15} \mathrm{~cm}^{-3}$
$\mathrm{Lp}=11.4 \bar{\mu}$
$\mathrm{Dn}=35_{-} \mathrm{cm}^{2} / \mathrm{s}$
$\mathrm{NA}=1 \mathrm{E} 18 \_\mathrm{cm}^{-3}$
Ln $=8.65 \mu$

## NMOS Transistors

These equations describe the behavior of voltage relations in an N channel MOS device. They assume that the physical geometry of the device is a rectangle and second order effects are ignored.

1) $W e=W-2 \cdot \delta W$
2) $L e=L-2 \cdot \delta L$
3) $C o x=\frac{\varepsilon o x \cdot \varepsilon o}{t o x}$
4) $\gamma=\frac{1}{C o x} \cdot \sqrt{2 \cdot \varepsilon o \cdot \varepsilon S i \cdot q \cdot N A}$
5) $V t=V t o+\gamma \cdot(\sqrt{2 \cdot A B S(\varphi p)+A B S(V B S)}-\sqrt{2 \cdot A B S(\varphi p)})$
6) $\varphi p=\frac{-(k \cdot T)}{q} \cdot L N\left(\frac{N A}{n i}\right)$

| Variable | Description | Units |
| :---: | :---: | :---: |
| W | drawn width of a MOS transistor | 1_m |
| L | drawn gate length | 1_m |
| $\delta \mathrm{W}$ | width encroachment | 1_m |
| $\delta \mathrm{L}$ | gate length encroachment | 1_m |
| We | effective width | 1-m |
| Le | effective length | 1_m |
| tox | gate oxide thickness | 1_Å |
| Cox | gate capacitance | 1-F/m^2 |
| Vt | threshold voltage | 1-V |
| VBS | substrate voltage | 1-V |
| $\gamma$ | body factor | 1_V^1/2 |
| NA | doping density | 1 - $1 / \mathrm{m}^{\wedge} 3$ |
| Vto | threshold at VBS $=0$ | 1-V |
| $\phi$ p | Fermi potential | 1-V |
| T | temperature | 1_K |



Example: An NMOS device is fabricated with a $10 \mu$ width and a $2 \mu$ gate length. The lateral diffusion encroachment is $0.27 \mu$, and the gate oxide is $200 \_\AA ̊$ thick. If substrate doping is $1 \mathrm{E} 15 \_\mathrm{cm}^{-3}$, find the gate capacitance, Fermi potential, and effective gate length and transitor widths. What is the body coefficient?

| Given | Result |
| :--- | :--- |
| $\mathrm{W}=10 \mu$ | $\mathrm{We}=9.46 \mu$ |
| $\mathrm{~L}=2 \_\mu$ | $\mathrm{Le}=1.46 \_$ |
| $\delta \mathrm{W}=0.27 \_\mu$ | $\gamma=0.1064-\mathrm{V}^{1 / 2}$ |
| $\delta \mathrm{~L}=0.27 \mu \mathrm{~V}$ | $\mathrm{p}=-0.2899$ |
| tox $=200 \AA$ | $\mathrm{Cox}=172656.6625 \_\mathrm{pF} / \mathrm{cm}^{2}$ |
| $\mathrm{~N} A=1 \mathrm{E} 15 \_\mathrm{cm}^{-3}$ |  |
| $\mathrm{~T}=300 \_\mathrm{K}$ |  |

## Currents in NMOS Transistors

These equations describe the behavior of a silicon NMOS transistor. They use a two-port network model, include both linear and non-linear regions in the device characteristics, and are based on a gradual-channel approximation. (The electric fields in the direction of current flow are small compared to the electric fields in the direction perpendicular to current flow). The drain current and transconductance are calculated differently, depending on their region. The geometry of the device is rectangular.

1) $I D S=k n \cdot\left((V G S-V t) \cdot V D S-\frac{V D S^{2}}{2}\right) \cdot(1+\lambda \cdot V D S)$
2) $g d s=I D S \cdot \lambda \quad$ 3) $C o x=\frac{\varepsilon O \cdot \varepsilon O X}{\text { tox }}$
3) $V D s a t=V G S-V t$
4) $g m=\left[\operatorname{Cox} \cdot \mu n \cdot\left(\frac{W e}{L e}\right) \cdot(1+\lambda \cdot V D S) \cdot 2 \cdot I D S\right]^{1 / 2}$
5) $k n=\frac{C o x \cdot \mu n \cdot W e}{L e}$

| Variable | Description | Units |
| :---: | :---: | :---: |
| We | effective width | 1_m |
| Le | effective length | 1_m |
| $\mu \mathrm{n}$ | electron mobility | 1_m ${ }^{\wedge} 2 /(\mathrm{V} \cdot \mathrm{s})$ |
| $\varepsilon \boldsymbol{O X}$ | relative dielectric constant oxide | 1 |
| VDS | drain to source voltage | 1_V |
| VGS | gate to source voltage | 1_V |
| Vt | threshold voltage | 1-V |
| gds | output conductance | 1_S |
| gm | transconductance | 1_AN |
| $\lambda$ | conductance parameter | 1_1/N |
| IDS | drain current | 1_A |
| tox | oxide thickness | 1_Å |
| Cox | oxide capacitance | 1_F/m ${ }^{\text {n }} 2$ |
| VDsat | saturation voltage | 1_V |
| kn | process constant | $1 \_A N{ }^{\text {^2 }}$ |

Example: An NMOS transistor has an effective width of $9.46 \mu$ and a channel length of $1.46 \_\mu$. The electron mobility is $500 \_\mathrm{cm}^{2} / \mathrm{v}-\mathrm{s}$. At a gate and drain voltage of $5 \_\mathrm{V}$, and at a threshold voltage of 0.75 , find the output conductance and drain current. The conductance parameter is $0.1 \_\mathrm{V}^{-1}$ and the oxide permittivity is 3.9 , and the gate oxide is $250 \_\AA$ thick.

Given
$\mu \mathrm{n}=500 \_\mathrm{cm}^{2} N$-s
VDS $=5-\bar{V}$
VGS $=5-V$
$\mathrm{Vt}=0.75 \mathrm{~V}$
$\mathrm{We}=9.4 \overline{6} \mu$
$\mathrm{Le}=1.46 \mu$
$\lambda=0.1 \_\mathrm{V}^{-1}$
$\varepsilon 0 x=3.9$
tox $=250 \_\AA$

Result
IDS $=12.1241 \_m A$
gds $=1.2124 \mathrm{E}-3$ _S
VDsat $=4.25 \mathrm{~V}$
$\mathrm{kn}=4.4749 \mathrm{E}-4-\mathrm{A} \mathrm{N}^{2}$
Cox $=138125 . \overline{3} 300 \_\mathrm{pF} / \mathrm{cm}^{2}$
$\mathrm{gm}=4.0344 \mathrm{E}-3 \_\mathrm{A} / \overline{\mathrm{V}}$

## CMOS

These equations describe the circuit behavior of a CMOS inverter connected to a capacitive load.

1) $I D S p=k p \cdot\left(2 \cdot V D S \cdot(V G S-V t p)-V D S^{2}\right)$
2) $I D S n=k n \cdot\left(2 \cdot V D S \cdot(V G S-V t n)-V D S^{2}\right)$
3) $k p=\frac{W p \cdot \varepsilon o \cdot \varepsilon o x \cdot \mu p}{2 \cdot L p \cdot t o x}$
4) $k n=\frac{W n \cdot \varepsilon o \cdot \varepsilon o x \cdot \mu n}{2 \cdot L n \cdot t o x}$
5) $V i n=\frac{V D S-V t p+V t n \cdot \sqrt{k n / k p}}{1+\sqrt{k n} / k p}$

| Variable | Description |
| :--- | :--- |
| IDSp | drain current in p device |
| kp | process constant p-MOS |
| VDS | drain to source voltage |
| VGS | gate to source voltage |
| Vtp | p-channel threshold voltage |
| IDSn | drain current in n device |
| kn | process constant n-MOS |
| Vtn | n-channel threshold voltage |
| Wp | width of P-MOS device |
| $\mu$ p | hole mobility |
| Lp | gate length of n-MOS device |
| tox | gate oxide thickness |
| Wn | width of n-MOS device <br> $\mu n$ |
| electron mobility |  |
| Ln | gate length of p-MOS device |
| Vin | input voltage when IDSN = IDSP |

## Units

| 1_A |
| :---: |
| 1_AN^2 |
| 1 V |
| 1-V |
| 1_V |
| 1_A |
| $1 \_\mathrm{A} \mathrm{N}^{\wedge} 2$ |
| 1_V |
| 1_m |
| $1 \_m{ }^{\wedge} 2 /(V \cdot s)$ |
| 1_m |
| 1_A |
| 1_m |
| 1_m^2/(V•s) |
| 1_m |
| 1_V |



Example: A CMOS inverter is designed with a p-channel threshold of $-0.75 \_\mathrm{V}$ and an n -channel threshold of $0.75 \_\mathrm{V}$. The transistor sizes for p and n are $10 \times 2$ and $4 \times 2$ in microns, respectively. Find the drain currents when the input voltage is $3 \_\mathrm{V}$ and $\mathrm{VDS}=5 \_\mathrm{V}$, and find the trip level.

$$
\begin{aligned}
& \text { Given } \\
& \text { VDS }=5 \_\mathrm{V} \\
& \text { VGS }=3-\mathrm{V} \\
& \mathrm{Vtp}=0.75 \_\mathrm{V} \\
& \mathrm{Vtn}=0.75-\mathrm{V} \\
& \mathrm{Wp}=10-\mu \\
& \mu \mathrm{p}=200^{-} \mathrm{cm}^{2} /(\mathrm{V} \cdot \mathrm{~s}) \\
& \mathrm{Lp}=2 \mu \\
& \text { tox }=200 \_\AA \\
& \mathrm{Wn}=4 \mu \\
& \mu \mathrm{M}=500 \mathrm{~cm}^{2} /(\mathrm{V} \cdot \mathrm{~s}) \\
& \mathrm{Ln}=2 \mu
\end{aligned}
$$

Result
IDSp $=4.3703 \mathrm{E}-4 \_$A IDSn $=4.3703 \mathrm{E}-4 \mathrm{~A}$
$\mathrm{kp}=8.6328 \mathrm{E}-5 \mathrm{~A}^{\mathrm{A}} \mathrm{N}^{2}$
$\mathrm{kn}=8.6328 \mathrm{E}-5-A \mathrm{~N}^{2}$
$\mathrm{Vin}=2.50 \_\mathrm{V}$

## BJT- Ebers and Moll Equations

These equations describe the behavior of the NPN silicon bipolar transistor. They are based on the original large-signal model developed by J.J. Ebers and J.L. Moll.

1) $I E=-I E S \cdot\left(e^{-\left(\frac{q \cdot v B E}{k \cdot T}\right)}-1\right)+\alpha R \cdot I C S \cdot\left(e^{-\left(\frac{q \cdot v B C}{k \cdot T}\right)}-1\right)$
2) $I C=-I C S \cdot\left(e^{-\left(\frac{\mathrm{q} \cdot \mathrm{VBC}}{\mathrm{k} \cdot \mathrm{T}}\right)}-1\right)+\alpha F \cdot \mathrm{IES} \cdot\left(e^{-\left(\frac{\mathrm{q} \cdot \mathrm{VBE}}{\mathrm{k} \cdot \mathrm{T}}\right)}-1\right)$
3) Is $=\alpha F \cdot I E S$
4) Is $=\alpha R \cdot$ ICS
5) $I B+I C+I E=0$
6) ICO $=\operatorname{ICS} \cdot(1-\alpha F \cdot \alpha R)$
7) $I C E O=\frac{I C O}{1-\alpha F}$
8) VCEsat $\left.=\frac{k \cdot T}{q} \cdot L N\left(\frac{1+\frac{I C}{I B} \cdot(1-\alpha R)}{\alpha R \cdot\left(1-\frac{I C}{I B} \cdot\left(\frac{1-\alpha F}{\alpha F}\right)\right.}\right)\right)$

| Variable | Description <br> total emitter current | Units |
| :--- | :--- | :--- |
| IE | emitter-to-base saturation current | $1 \_A$ |
| IES | base-to-emitter voltage | $1-A$ |
| VBE | reverse common-base current gain | $1-V$ |
| $\alpha$ R | collector-to-base saturation current | $1 \_A$ |
| ICS | base-to-collector voltage | $1 \_V$ |
| VBC | total collector current | $1 \_A$ |
| IC | forward common-base current gain | $1-$ |
| $\alpha$ F | transistor saturation current | $1 \_A$ |
| IS | collector current | $1-A$ |
| ICO | CBopen collector current | $1-A$ |
| ICEO | collector-to emitter saturation voltage | $1-V$ |
| VCEsat | temperature | $1-K$ |
| T | total base current | $1 \_A$ |



Example: A bipolar transistor has a base current of 10 mA aqnd a collector current of 11 _mA. If the forward and reverse common emitter gains are 0.95 and 0.05 respectively, find the saturation voltage at 300 _K.
Given
Result
$l B=10 \_\mathrm{mA}$
VCEsat $=9.7482 \mathrm{E}-2 \mathrm{~V}$
$I C=11$ mA
$\alpha \mathrm{F}=0.95$
$\alpha \mathrm{R}=0.05$
$\mathrm{T}=300 \_\mathrm{K}$

## Chapter 3

## Constants Library

## In This Chapter

The Constants Library is a collection of physical constants commonly used in electrical engineering. This chapter covers:
$\square$ Types of Constants
$\square$ Using the Constants Library
$\square$ Using the ECON Function
$\square$ Constants Library Softkeys

## Types of Constants

The Constants Library lists the symbols, descriptions and SI units of four types of constants, shown below:

## Universal Constants

| $R$ | Universal gas constant |
| :--- | :--- |
| NA | Avogadro's number |
| c | Velocity of light |
| $h$ | Plank's constant |
| $k$ | Boltzmann's constant |
| hb | Dirac's constant |

## Physical Constants

| q | Charge of an electron |
| :--- | :--- |
| $\varepsilon_{0}$ | Permittivity in vacuum |
| me | Electron rest mass |
| re | Classical electron radius |
| mp | Proton rest mass |
| $\mathrm{R} \infty$ | Rydberg's constant |


| $\alpha$ | Fine structure constant |
| :--- | :--- |
| ao | Bohr radius |
| $\mu \mathrm{B}$ | Bohr magneton |
| $\lambda$ | Wavelength of 1 eV quantum |
| $\lambda \mathrm{c}$ | Compton's wavelength |
| $\sigma$ | Stefan-Boltzmann's constant |
| c 1 | First radiation constant |
| c 2 | Second radiation constant |
| Vt | Thermal voltage at 300 K |

## Silicon Properties

| N | Atoms/cm ${ }^{\wedge} 3$ |
| :--- | :--- |
| AW | Atomic weight |
| Siden | Density |
| a | Lattice parameter |
| $\varepsilon$ Si | Relative permittivity |
| Nc | Eff density of states in conduction band |
| Nv | Eff density of states in valence band |
| ml | Longitudinal eff mass of electrons |
| mt | Transverse eff mass of electrons |
| mlh | Eff mass of light holes |
| mhh | Eff mass of heavy holes |
| $\phi$ | Electron affinity |
| Eg | Bandgap at 300 K |
| ni | Intrinsic carrier concentration |
| $\alpha \mathrm{th}$ | Linear coefficient of expansion |
| $\mu \mathrm{n}$ | Drift mobility of electrons |
| $\mu \mathrm{h}$ | Drift mobility of holes |
| MP | Melting point |
| BP | Boiling point |
| kth | Thermal conductivity |
| spht | Specific heat |
| $\rho s i$ | Work function |
| $\varepsilon 1$ | Critical field in PN junction |
| $\varepsilon \mathrm{\varepsilon x}$ | Relative permittivity |
| rad | Radians |

## Magnetic Properties

| $\mu \mathrm{o}$ | Permeability of vacuum |
| :--- | :--- |
| $\phi \mathrm{o}$ | Magnetic flux quantum |
| F | Faraday's constant |
| $\mu \mathrm{e}$ | Electron magnetic moment |
| $\mu \mathrm{P}$ | Proton magnetic moment |

## Using the Constants Library

Select CONSTANTS LIBRARY from the main menu screen. The Constants Library menu displays four classes of constants:


Example: Suppose you want to find the density of pure silicon. Use the cursor keys to move the pointer to SILICON PROPERTIES and press ENER to display the following screen:


Move the pointer to SIDEN. Five softkeys are available at this level and are described at the end of this chapter. To view the value for the SIDEN constant, press the 8 ame softkey. This results in the following display:


To place the value of SIDEN on the stack, press ENER or the MSTM:
softkey. The screen flashes a "Value to stack" message, places the value on the stack as a tagged object, then returns to the SILICON PROPERTIES menu. The value(s) you entered on the stack become available for calculation when you exit the Pac. To remove the tag once the value is on the stack


## Using the ECON Function

You can extract the value of any constant without entering the Electrical Engineering Application Pac with the ECON( ) function. In all cases, the constant name must be prefixed with a ' $\$$ ' symbol, accessed by $\quad$ ( 4 For example, suppose you want to retrieve the speed of light:

## User Program Method

Inside a user program, use the commands '\$c' ECON or 'ECON(\$c)' EVAL to call for the speed of light.

## Stack Method

Type '\$c' into level 1 of the stack and press the Eemem library softkey or type the letters ECON and press ENER.

The constant value will have SI units if units are selected (i.e., if flag 60 is clear); otherwise, the value will have no units.

## Constants Library Softkeys

| \%4*) | Displays the value of the constant with units on the screen. Press ENTE to return to the constants list. |
| :---: | :---: |
| \% | Places a copy of the selected constant on the calculator stack. Whether or not the value has units appended is controlled by the units key setting, which can be toggled at the Equation Library screen. |
| \%\%\% | Toggles between large and small display font. |
|  | Moves up one level in the menu structure. |
| \% \% \% \% | Exits to the main menu. |

Notes:

## Chapter 4 <br> Circuit Analysis Tools

## In This Chapter

This set of tools solves some common problems found in electrical engineering. The following sections should be read in order, since some topics common to all sections are discussed first.

## $\square$ AC Circuit Analysis

$\square$ Fourier \& Laplace Transforms/Gain and Phase Plots
$\square$ Ladder Network Analysis
$\square$ Transmission Lines
$\square$ Two-Port Networks

The first five topics or "tools" in this chapter are accessed directly from the main menu. The screen below shows the options available on the main menu; the 'resume solving' option only appears if you've been using the equation library previously.


Softkeys to access gain plot and phase plot functions are available at the opening screen, prior to starting the EE Application Pac program.


## AC Circuit Analysis

Conversions between wye and delta, single phase to three phase, combinations of series and parallel impedance and admittance, and power analysis are provided under this topic.

## AC Circuit Performance (Z)

A simple AC circuit can be modeled as a source voltage Vs and a source impedance Zs which appears in series with the voltage source. This source drives a load impedance $\mathbf{Z L}$.


When these three variables are specified, a variety of circuit performance properties can be calculated. These values can be complex numbers, which complicates the calculations when they're done by hand. Since the HP 48SX handles complex numbers directly, much of the tedium of working these problems is eliminated.

## IMPORTANT!

When entering values for AC voltage, it's essential to decide what input values you will use, and then to be consistent throughout the calculation. In the following examples, the assumption is made that all AC values are entered in volts RMS, DC values are in volts, and impedances are in ohms.

There is no check on the consistency of units in these sections; the HP 48SX does not provide that feature when working with complex numbers.

## Entering Data

Voltages, currents and impedances may be either real or complex values. In electrical circuits, real numbers represent the resistive component of a voltage or current, and complex numbers represent the reactive (inductive or
capacitive) component. Most real-world values contain both real and reactive components.
These complex numbers can be entered two ways, as a real part and imaginary part, or as a scalar vector and phase angle. In electrical engineering, complex numbers are shown in a notation like $5+\mathrm{j} 22$, where 5 is the real part, and 22 is the complex part. The letter ' $j$ ' represents the square root of -1 , commonly known as ' $i$ ' by mathematicians. Engineers use ' $j$ ' because ' i ' is usually used to denote a current. Complex numbers may also be shown as a 'phasor', such as $22.6 \angle 77$, meaning a magnitude of 22.6 at an angle of 77 degrees from the horizontal.
Both rectangular $(5,22)$ and polar $(22.6, \angle 77)$ modes are supported in the HP 48SX calculator. To enter a complex number in rectangular notation, press $\square \square$ and enter the real part, followed by the imaginary part. Separate the real and imaginary parts with either a space or a comma. To enter the number in polar notation, press $\square$ and enter the magnitude, then the $\angle$ symbol, then the phase angle in degrees.
The calculator knows about the square root of -1 , which it calls ' $i$ ', instead of ' j '. When looking at examples in the calculator owner's manual, keep this difference in mind.

## A simple example; the hair dryer

Let's try a simple example to get the feel of it: A hair dryer plugged into a wall socket.


From the main menu, move to AC CIRCUITS and press ENTER. Press ENTER again at AC CKT PERFORMANCE (Z).
Press ENER a third time to set Vs:

| $\{$ HDME EEAPPD \} | PRG |
| :---: | :---: |
| Enter Vs: |  |
| 110 |  |
| GEKIPEETP + FIUEL | STE |

Enter 110 volts for Vs，（standard U．S．wall－plug voltage）．The phase angle of Vs needs to be referenced somewhere，so consider it zero．Since it＇s zero，just enter 110，and then press ENER．It＇s a real number，so you don＇t see an imaginary part．

| \｛ HDME EEAPPD \} | PRG |
| :---: | :---: |
| Enter 2s： |  |
| （．545） |  |
|  | tsite |

Zs is the source impedance，in this case the impedance looking into the wall plug．Probably about $.5 \Omega$ ，with maybe 5 degrees or so of inductive reactance． With the pointer at Zs ，press ENER．To set polar display mode，press $\square$
 that the calculator will now display complex numbers in polar mode．
To enter $.5 \Omega$ at 5 degrees，press $\square$ 回． 5 目 $\angle 5$ ENER．The data can be entered in either rectangular or polar coordinates，independent of the display mode．The data is displayed by the calculator in the requested mode，but it is always stored internally in the rectangular coordinate system．
Now the pointer is set to ZL，the load impedance．It＇s a heating coil and a motor in parallel，so it has both resistive and inductive components．The coil is about $7 \Omega$ and the motor is probably about $1+\mathrm{j} 100 \Omega$ ，if it＇s driving a big fan．In parallel，this works out to（ $6.961,0.4869$ ）．

| $\{$ HIME EEAPPD \} |  |
| :---: | :---: |
| Enter ZL： |  |
| （6．961 ．4869） |  |
|  | TSIE |

Press ENER then 回 6.961 ． 4869 ENER．You＇ll see the value expressed in polar form as $(6.9780, \angle 4.0011)$ ．

|  |
| :---: |
| CMLC［FINT UP |

Press ©eAma to set the calculator to work：

Examining the Results

| VL | Voltage across the load impedance. |
| :--- | :--- |
| IL | Current through the load. |
| VI | Volt-ampere product in the load (apparent power). |
| P | Real power (the part that causes smoke). |
| RP | Reactive power. |
| PF | Power factor, a ratio of real power to apparent power. |
| Pmax | Theoretical maximum power deliverable from this source. |
| ZL* | Load impedance for maximum power transfer from source. |

VL is about 102 volts, since part of the voltage is dropped across the source impedance. IL is almost 15 amps , close to the limit for a single residential circuit. VI, the volt-ampere product, is 1509 volt-amperes . P, the real power, is 1506 watts, a truly impressive hair dryer.

RP, the reactive power, is 105 VARS. PF, the power factor, is .9976 , indicating an almost-entirely resistive load. Not surprising, since it's a heating coil. Pmax is 6073 watts, representing the maximum power available from this wall-outlet source. This would be available into a load of $(.5, \angle-5)$, which is the value given by $\mathrm{ZL}^{*}$, the complex conjugate of Z s. If you enter the value given by $\mathrm{ZL}^{*}$ for Zs and recalculate, you'll find IL to be 110 amps .

## Inline Computation

You can also do in-line computations when entering values. For example, enter the combination of $7 \Omega$ and $1+\mathrm{j} 100 \Omega$ for ZL. Press $\mathrm{MP}_{\mathrm{M}}$ to back out to the data entry screen. Select ZL and press ENER, then press ":Bem or 四 to clear the line.


Now enter the equation $1 /(1 / 7+(1 /(1+\mathrm{j} 100))$ for the two impedances in parallel by typing $117 \% 1$ 回 $1100 \square \square$ ENER . If this seems
confusing, study the HP 48SX manual regarding Reverse Polish Notation. The result should be ( $6.9781, \angle 4.0010$ ).
In general, computations can be performed at any of the data entry screens except for those which request a list to be entered. If an error is made, the calculator will return an error message and request the data again.

## Display Modes

You may want to change the display mode to show the results in a different notation, such as fixed-point, scientific or engineering. To switch to a different notation, press ENTER at any data entry line (except for list entry). At the end of the line of data, add a command, like '2 FIX' (fixed point, 2 digits after decimal) or ' 3 ENG' (engineering notation, 3 digits after decimal).

| [ hime eeappl $\}$ |  |
| :---: | :---: |
| Enter Vs: |  |
| 110.003 ENG |  |
|  | TSTE |

The default is whatever the calculator was set for when you started the EE application pack software.


## AC Circuit Performance ( $\mathbf{Y}$ )

Admittance is the reciprocal of impedance, and it's a convenient unit to use with parallel loads. The admittances of parallel loads simply add together, giving you the equivalent load of all the parallel elements.
As in the preceding example, three variables must be specified. The source current is Is and the source admittance is Ys. These drive a load admittance, YL. The SI unit for admittance is the Siemens, but many textbook still use 'mho' (ohm, spelled backwards).


For example, imagine a current source of 1 _mA with a source admittance of . 0001 _mho (a $10 \_\mathrm{k} \Omega$ resistor in parallel) and three loads; a 2.7 k $\Omega$ resistor, a $.1 \_\mu \overline{\mathrm{F}}$ capacitor and a $1 \_\mathrm{mH}$ inductor. Assume a frequency of $10 \_\mathrm{kHz}$, and the capacitor becomes j6.283E-03_mho, and the inductor is - $\mathrm{j} 15.92 \mathrm{E}-03$ _mho. The resistor admittance is $370.4 \mathrm{E}-06$ mho. Enter the sum of these three values for YL.
Press ENER at AC Ckt Perf. (Y) to start that topic. Press ENER at Is and type .001 (remember, it's in amps) then press ENER. Move the pointer to Ys and enter .0001. Move down to YL and enter the following:

| $370.4 \mathrm{E}-06$ | $(0,-15.92 \mathrm{E}-03)$ | $(0,6.283 \mathrm{E}-03)$ | ++ ENER |
| :--- | :--- | :--- | :--- |
| (resistor) | (inductor) | (capacitor) | (add together) |

The result should be (9.6441E-3, -87.7990 ) in polar coordinates or ( $3.704 \mathrm{E}-4,-9.637 \mathrm{E}-3$ ) expressed in rectangular terms.

|  |
| :---: |
| [GTLC [ FONT [PP |

Press \$क्सs. As before, the results for the combined load are displayed:


VL is the load voltage, about 100 mV .
IL is the load current, about 1 _mA.

VI is the apparent power in the load, about .1_mW.
P , the real power is about $4 \_\mu \mathrm{W}$, and reactive power is about $100 \_\mu \mathrm{W}$.
PF , the power factor is about .04 , so the vast majority of the 'power' is reactive, through the inductor and capacitor. Their admittances are much greater than that of the $2.7 \_\mathrm{K} \Omega$ resistor, so it makes sense.
Pmax is the maximum power from this source; about 2.5 mW.
$\mathrm{YL}^{*}$ is the load admittance that would produce maximum power transfer from the source, given its source admittance Ys. Power transfer is maximized when the real parts of Ys and YL are equal, and their imaginary parts are opposite in sign; that is, $\mathrm{YL}^{*}$ is the complex conjugate of Ys.

## Z's (Impedances) in Series

This calculation determines the voltage drop across each impedance in a series string. It takes two arguments; a voltage V and a list of impedances. Let's plug in four impedances and see what we get.
A stereo amplifier has an output voltage of $15 \_\mathrm{V}$, for example, and it's driven with a 2_kHz signal for test purposes. On the output is 100 feet of two-conductor wire, an $8 \_\Omega$ "tweeter" speaker and a $10 \mu \mathrm{~F}$ capacitor to block the low frequencies.

100 ft wire $\times .005 \_\Omega / \mathrm{ft}=.5 \_\Omega$
tweeter $=2+2 \cdot \pi \cdot 2000 \cdot 600 \mu \mathrm{H}=(2+\mathrm{j} 7.54) \_$
capacitor $=.01+1 /\left(2 \cdot \pi \cdot 2000 \cdot 10 \_\mu \mathrm{F}\right)=(.01-\mathrm{j} 7.96) \_\Omega$
$100 \_\mathrm{ft}$ wire $\times .005 \_\Omega / \mathrm{ft}=.5 \_\Omega$
First enter $\mathrm{V}, 15$. Next, enter the four impedances inside the brackets. $\{.5(2,7.54)(.01,-7.96) .5\}$

|  |
| :---: |
| CMLC ${ }^{\text {a }}$ FINT |

 to V4 correspond to the four impedances in the order they were entered:


From the results in the screen above，it＇s apparent that the wires leading to the speaker cause a substantial voltage drop（V1 and V4）．In a real stereo system，low－resistance wires would improve performance．The voltage drop across the wires represents wasted power and lower output level．
The speaker（V2）and the capacitor（V3）are splitting the rest of the voltage drop about equally．At lower frequencies，the capacitor bears most of the load，and at higher frequencies the speaker takes over．In a real sound system，multiple sizes of speakers are used，along with a＇crossover＇network which routes low and high frequencies to the appropriate－sized speaker．

## Y＇s（Admittances）in Parallel

Series calculations are easiest to handle with impedances；they＇re just added up．Similarly，parallel admittances add up too，so they work out well for parallel kinds of problems．This calculation figures the current through each admittance in a parallel set．


Imagine a 10 mA current source driving three parallel load resistances；100， 270 and $910 \Omega$ ．What＇s the current through each load？Take the reciprocal of each impedance to get the admittance．
$1 / 100=.01 \quad 1 / 270=.0037 \quad 1 / 910=.0011$
First，enter the current Is（ $10 \mathrm{E}-3$ ）．Next，enter the admittances．Since this is a list，you can＇t do in－line calculations，so you need to already know admittance． If you had calculated the values outside of the Electrical Engineering Application Pac，you could leave them on the stack in a list．Then，enter this
 and $⿴ 囗 ⿰ 丿 ㇄$ of this example，just type in the numbers：$\{.01 .0037 .0011\}$

Admittance in Parall... $\rightarrow 15: 10.0 \mathrm{E}-3 \mathrm{~B}$ (

CGILC FINTIUP


|  |  |
| :---: | :---: |
|  |  |

It's no surprise that the $100 \_\Omega\left(.01 \_\right.$mho $)$admittance gets most of the current.

## Z's (Impedances) in Parallel

This works like the last section, but the loads are expressed in impedance terms (ohm) instead of admittance. This saves you from having to convert to admittance if the original data is for impedance. Try the same example as in the previous section. The results should come out the same.


## Phase Conversions, $\mathbf{3} \phi-1 \phi$ and $1 \phi-3 \phi$

Power systems often use three-phase power for the generation and transmission of electricity, and industrial plants commonly use it in large motors and machines. Many analyses are simplified if only one phase is considered, but this requires that load conditions be re-computed for a single-phase-equivalent load.

## $3 \phi$ to $1 \phi$ Conversion

The $3 \phi-1 \phi$ conversion requires two arguments; line-to-line voltage, and phase current.


As an example, enter a value of 240 volts line-to-line for $V \phi 3$, and $(6,2)$ amps for $I \phi 3$. This implies a capacitive load, since current is leading voltage (the reactive component is positive, and the voltage is referenced to zero).
Press the © ©


## Examining the Results

VARS Total volt-ampere product in the load.
PF Power factor, where 1 is purely resistive and 0 is purely reactive.

V1 Equivalent line voltage that this load would see if connected to a single-phase source. This is the same as the line-neutral voltage for a Y-connected load or line-line voltage for a $\Delta$-connected load.

I1 Equivalent load current that this load would carry if connected to a single phase source. This is the same as the line current for a Y-connected load, or the line current for a $\Delta$-connected load divided by $\sqrt{3}$.

## 1 $\phi$-3 $\mathbf{\phi}$ Conversion

The $1 \phi-3 \phi$ conversion requires two arguments; load voltage and load current. Use the hair dryer example again. What would the conditions be for three hair dryers connected as a balanced load in both Y and $\Delta$ ?

## Circuit Analysis Tools



Let's plug in 110 volts for $V \phi 1$, and $(14.6,-1)$ for $I \phi 1$. Press ©esaw. The results are shown in the screen below:


## Examining the Results

VARS Volt-amperes reactive, is 4829. That's three hair dryers' power, since these are configured as a balanced three-phase load.

PF Power factor is nearly 1 . Given the relatively small reactive component to IL, that makes sense.

V3 Line-to-line voltage, if these loads were Y-connected. The load current would remain the same, (14.6,-1). The voltage is higher because in Y -connection the voltage is impressed across multiple loads simultaneously.

I3 Phase current, if these loads were $\Delta$-connected. Line-to-line voltage would still be 110 volts. The current is higher because in $\Delta$-connection the current splits between two loads.

## Impedance conversion, $\mathbf{Y}-\Delta$ and $\Delta-\mathbf{Y}$

Circuit analysis sometimes requires that a Y-connected set of impedances be converted to their equivalent in a $\Delta$-connection, or vice versa. These calculations take the Y or $\Delta$-connected impedances and transform them to the other form.


The transformation between Y and $\Delta$ is such that from the outside terminals A, B and C it is not possible to tell which way the loads are connected. Since they are equivalent, circuits can often be simplified by substituting one for the other.


In a Y connection, Z 1 is $15 \Omega, \mathrm{Z} 2$ is $\mathrm{j} 30 \Omega$ and Z 3 is $25-\mathrm{j} 20 \Omega$. The equivalent $\Delta$ parameters are:

$$
\begin{aligned}
Z A & =65.0+\mathrm{j} 60.0 \Omega \\
Z B & =30.0-\mathrm{j} 32.5 \Omega \\
Z C & =6.22+\mathrm{j} 41.0 \Omega
\end{aligned}
$$



Try these values in a $\Delta-\mathrm{Y}$ conversion to get $\mathrm{Z} 1, \mathrm{Z} 2$ and Z 3 again.

## Fourier and Laplace Transforms

The section on Fourier and Laplace transforms has tabular data on common transforms. Pole/zero entry can be used to derive a transfer function, and gain/phase plots can be created based on the derived function. A finite

Fourier transform is provided to create a transfer function from data points, and the inverse function can be used to recreate a set of data from a known transfer function.

## Laplace Transform Pairs

This section lists transform pairs in s-plane and time domain forms.
Use $\square$ UREE see the rest of the line if it extends off the end of the screen. These transforms are used to reduce complex differential equations in the time-domain into simple algebraic expressions in the frequency domain. Although there are many Laplace transforms, these are the ones most commonly used for circuit analysis.


## Inverse Transfer Functions

This section lists selected transfer function pairs, with the s-domain function on the left, and the equivalent time-domain expression on the right. These transforms are used when returning from the frequency domain to the time domain after analyzing the response of a circuit.


## Pole-Zero Analysis

In this section a transfer function can be derived from the poles and zeros of an s-parameter function. A constant multiplier is entered ( 1 is typical), along with a list of poles and a list of zeros, mapped in the s-plane. This transfer function can be used as input to the phase and gain plot functions.
As an example, consider the equation below
$F(s)=\frac{10(s+5)(s+3-j 4)(s+3+j 4)}{s(s+10)(s+6-j 8)(s+6+j 8)}$

The poles of this function are $0,-10,-6+\mathrm{j} 8$ and $-6-\mathrm{j} 8$. The zeros are $-5,-3+\mathrm{j} 4$ and $-3-\mathrm{j} 4$. To get a transfer function, first enter a constant ( 1 is fine), and then a list of zeros and a list of poles. See the screen below:


Press ©A\&E to compute the transfer function.

```
New Xfer Function
->XFER FUN: '(.234375,-.257B125)`(S...
    HSTE:FONT UP
```

The function is automatically stored in the HOME EEAPPD directory, ready for use by the GPLOT and PHPLOT functions.
To plot phase and gain, exit the application pack by pressing $\mathbf{1} \mathrm{P}$ 迸
UR Im. Press the ©Rem menu key to start the gain plot program.


The first screen asks whether the existing picture should be cleared before plotting a new function. Ordinarily the answer is YES, unless you want to overlay multiple plots to look at how a function changes.


Next, select a horizontal range appropriate to the function you want to plot. It may be necessary to experiment a bit to get the right value. For this example,

## Circuit Analysis Tools

use . 14 as the horizontal range, representing a frequency range from $10^{-1}$ to $10^{4} \mathrm{~Hz}\left(3 \_\mathrm{Hz}\right.$ to $10 \_\mathrm{kHz}$ ). The X-axis represents frequency, and the result is a Bode plot with frequency plotted logarithmically, and gain plotted linerarly.


Vertical range is selected next, and it can be auto-scaled to the horizontal range if desired. The range will be selected such that it fills the screen. Just press EENER to select auto-ranging.

| ( hime eenppo | PRG |
| :---: | :---: |
| Enter vertical range for EĂTER for AUTOS': |  |
|  |  |
|  |  |

The plot program will autoscale the function, then begin plotting. It may take a few minutes, depending on the function to be plotted.


Labels for the axes, and the function will be printed on the display. To remove these, and clear up the display, press the ex med and softkeys. For more information on the plotting softkeys, read Chapter 18 of the HP 48SX Owner's Manual.


The phase plot program uses the same sequence of operations, and is started by pressing the EBP E © menu key at the opening screen.

## Fourier Transform Pairs

Like the earlier Laplace section, selected transform pairs are listed.


## Fourier Coefficients

A summary of coefficients are listed for common waveforms.

```
*Fourier Coefficients
->RECT PULSE - AN E*NXTOCTXSINCC...
SMM TRIONGIGN O
SYM TRIANGULAR - AN 䱚D'TXCSl...
SHM TRAPEZ
HALF SINEWAUE - - BN OXHxCTU-T...
HALF SINEWAVE - AN AXTDSTXCSIN...
    - BNO
    #STF: FONT UP
```


## Finite Fourier Transform

For a single cycle of an arbitrary waveform, a set of data points may be taken at discrete intervals. A Fourier series may be obtained by analysis of these data points, representing the frequency components present in this single cycle of the signal. Frequency components up to $1 / 2$ the sampling frequency may be obtained.
The data is entered as a list; the assumption is made that the data samples were taken over one full cycle of the waveform, and that the sampling rate remained constant over the sampling interval.
To use the FFT, enter a list of data points sampled over a single cycle of a waveform as shown below. These are just the amplitudes; the time interval is assumed to be constant between samples.

## Circuit Analysis Tools

```
Finite Fourier Trans..
->DATA:{21 -2 3-1-11}
EHLE FONT IP
```

After entering the list of sample data, press exime to compute the coefficients of the Fourier series.


To see all of the coefficients, press 国 HIST.


If the list is extremely long, it won't all fit on one screen. If that's the case, use the \$BEK menu key to place the coefficient list on the stack, then exit the application pack and examine the list using the stack editing functions. Go ahead and press S.s. St to save the list on the stack, for use in the next example.

## Inverse FFT

This transform takes the Fourier series coefficients as input and returns a list containing an evenly spaced set of data points over one cycle of a repetitive signal.

```
FFT Coefficients \(\rightarrow\) Dat. \(\rightarrow\) DATA: 1 ( \(2.0,-1 . \mathrm{BE}-12\) ) ( \(1.0,-4.1 \mathrm{E}-\ldots\)
-STK FINT UP
```

Enter the coefficients you just calculated by retrieving them from the stack. Press ENTER to enter coefficients, then the \#\# STIN menu key to get to the stack. Press the \#\# ma key to copy the level 1 stack contents back to the data entry line. Press $0 \mathbb{N}$ to leave the stack. You'll need to edit out an extra set of brackets, and the ':Coefficients:' label, but it's better than typing in all the numbers by hand.
Now that the numbers are in, press stum to compute values for the data points. Neglecting round-off errors, you should get some data points that look the same as the ones you typed in originally, except for round-off errors and the fact that all data is now converted to complex numbers.


## Ladder Network Analysis

Ladder network analysis interactively constructs a network from the load-impedance end. Load conditions are computed when the ladder section is complete. Illustrated below are the various kinds of elements that can be used to form a ladder network.



This routine provides a method of reducing multi-element ladder networks to a single equivalent impedance. It constructs a matrix as each element is added to the network. When a calculation is requested, the matrix is evaluated and the transfer characteristics are computed. The Zin which results is substituted for the original ZL , and another network can then be added using the old network as the new load impedance. From this point, the transfer characteristics for the next ladder network are computed using the previous network as a load, and the original load impedance ZL no longer appears. The assumption is made that the operating frequency is specified first and not changed for the remainder of the analysis.
A ladder network can be composed of many different kinds of devices, placed in series or parallel with the load. These may include passive components such as resistors, capacitors and inductors, or active devices like gyrators and current sources. The following table shows the different ladder elements and the data which must be entered for each.

| Series or Parallel Resistor | Enter resistor value in $\Omega$. |
| :--- | :--- |
| Series or Parallel Capacitor | Enter capacitor value in F. |
| Series or Parallel Inductor | Enter inductor value in H. |
| Series or Parallel RL | Enter resistor and inductor values. |
| Series or Parallel LC Tank | Enter the inductor and capacitor values. |
| Series or Parallel RC | Enter resistor and capacitor values. |
| Transformer | Enter turns ratio $\mathrm{n},(10: 1=10)$. |
| Gyrator | Specify the gyration resistance, $\alpha$. |
| VCIS, Voltage-Controlled <br> Current Source | Specify gm, the transconductance, and $r \mathrm{~b}$, <br> the input resistance. |


| ICIS, Current-Controlled <br> Current Source | Specify $\beta$ (or $\left.h_{\mathrm{fe}}\right)$, the current gain, and $r_{\mathrm{b}}$, <br> the input resistance. |
| :--- | :--- |
| Transmission Line, Open <br> Stub and Shorted Stub | Specify the stub length $\Theta$ in degrees and <br> the characteristic impedance $\mathrm{Z}_{0}$. |

To evaluate a ladder network, begin at the output by entering the initial load impedance and the operating frequency. Press mak to load these initial conditions and bring up a list of ladder elements. Next, add a single element by selecting it from the list and entering its value. Do not try to enter more than one element; the first one encountered will be used. Press Mex to combine the new element into the network.
For each subsequent item, select it, enter its value and press U-X E . Repeat the process for each element, working from the load down to the input of the ladder. When finished, press © \& to compute the input impedance, current and voltage gains, transconductance, and power transferred. To return to the previous level to restart the ladder analysis, press बTIN.

## Transmission Lines

Transmission line calculations find propagation constants, phase velocity, VSWR, and other factors given the physical characteristics of the line. 'Smith chart' calculations replace the traditional graphic methods of determining line impedances with the more-accurate method of direct computation. The equations provide input, output, short-and open circuit impedances and VSWR without the need for paper charts.
Transmission lines are key elements in every electronic system. They may be used for power transmission, strung between towers and separated by many feet, or they might be twisted pairs carrying telephone conversations, packed by the hundreds into a cable measuring an inch in diameter.
The uses of transmission lines are so varied that it is difficult to cover every specific case, but a generalized form often serves to approximate the real-life situation. This model assumes two wires, separated by some insulating medium and carrying a signal at some fixed frequency for a fixed distance. The properties that must be known are the inductance, capacitance and leakage conductance between the two wires, and the series resistance per unit length.

## Transmission Line Parameters

For a unit length, the following parameters are entered:

| $\mathbf{r}$ | resistance |
| :--- | :--- |
| $\mathbf{l}$ | inductance between conductor pair |
| $\mathbf{g}$ | inter-conductor conductance (leakage) |
| $\mathbf{c}$ | capacitance between conductor pair |

The general conditions for the line are then entered:

| f | operating frequency |
| :--- | :--- |
| $d$ | length of line (in terms of unit length) |
| ZL | terminating impedance |

For two conductors in a flat ribbon cable, for example, the values are:

| r | $.067 \_\Omega / \mathrm{ft}$ |
| :--- | :--- |
| l | $2 \mathrm{E}-6 \_\mathrm{H} / \mathrm{ft}$ |
| g | $1 \mathrm{E}-9 \mathrm{mho} / \mathrm{ft}$ |
| c | $18 \mathrm{E}-12 \_\mathrm{F} / \mathrm{ft}$ |
| f | $1 \mathrm{E} 6-\mathrm{Hz}$ |
| d | $150-\mathrm{ft}$ |
| ZL | $100 \_\Omega$ |

When data entry is complete, you should see a screen something like this:


Press


## Examining the Results

- $\alpha$ is the attenuation constant, in nepers per unit length. The signal is reduced in strength as it travels along the line. It's about 100E-6_ft.
$\square \phi$ is the phase constant in radians per unit length. It represents the amount of phase shift which takes place as the signal propagates down the line. It's about 38E-3_r/ft.
- $\mathrm{V}_{\mathrm{ph}}$ is the phase velocity, in this case feet per second. The speed of light is about 982.1E6_ft/sec, and in this case $\mathrm{V}_{\mathrm{ph}}$ is 167E6_ft/s. This phase velocity represents the speed at which a signal propagates along this transmission line. It is possible to get values greater than the speed of light, if the values you choose for L and C are physically impossible. The limits for L and C (in the real world) are the permeability and permittivity of free space.
- $\phi \mathrm{Zo}$ is the phase angle of the characteristic impedance in radians.
- Mag Zo is the magnitude of the line's characteristic impedance.
- VSWR is the voltage standing wave ratio. It's a measure of how well the terminating impedance on the line matches the characteristic impedance. If the two are identical, the VSWR is 0 . Try repeating the calculation with a terminating resistance of $333 \_\Omega$, and see how the VSWR changes.
For non-zero values, energy in the signal is reflected back along the line to the source. In an application such as a broadcast station or amateur radio transmitter, minimizing this figure is critical to achieving maximum signal strength and preventing damage to the transmitter.


## Smith Chart Impedance Calculations

The Smith chart is a polar representation of normalized resistance and reactance curves. The traditional method of calculating open- and short-circuit conditions for a transmission line has been through graphical construction on a Smith chart, with the results read from the chart and interpolated. The equations provided in this section use the same data that would be entered on the chart in graphic form, but they solve for the results directly.


Using the same data as in the previous section, try the problem again. It computes the input, output, open-circuit and short-circuit impedances for a transmission line, as well as VSWR, the voltage standing wave ratio.

| ```Impedance calculatio.. ->21N: (35.星0.<-53.05E) 20: (9732.300,<-152.5E-3)```  ```WSWR: (5ja.5E-3.Ci7g.9E0)``` |
| :---: |
| -STEFINNT UP |

## Two-Port Networks

The two-port network section does performance calculations for a two- port with known z , h or y parameters, given the characteristics of the source and load. Conversions are provided between $\mathrm{z}, \mathrm{h}, \mathrm{a}$ and y parameters.
Combinations of two-port networks can be evaluated in series, parallel and hybrid topologies. The combined networks can be reduced to an equivalent set of $z$-parameters which can then be combined with additional networks to form complete systems.


In this section a variety of tools are provided for evaluating two-port networks. The two-port concept is widely used to simplify electronic circuits or to model a subcircuit too complex to evaluate directly. The parameters can be measured on the bench, and the results plugged in to predict performance in an electronic system.

The two-port network is used as a model for many different kinds of systems. Transistors are modeled as two-ports; in fact, h -parameters such as $\mathrm{h}_{\mathrm{fe}}$ are commonly listed in data sheets for transistors. Operational amplifiers can also be modeled in this way, as well as very complex systems whose internal design may not even be known. As long as the required parameters can be
measured from the outside, the internal configuration of the two-port is irrelevant.

Two-port networks do have to follow some basic rules in order for an analysis to be valid.

- Energy storage (like a battery) is not permitted within the circuit.
- Independent sources of voltage or current are not permitted within the two-port, although dependent sources are allowed.
- Current into a port must equal current out of the port.
- All connections between the two ports must be made internally; an external connection between port 1 and port 2 is not permitted.
- Currents are positive when flowing IN to a port, and negative when flowing OUT. Negative results for impedances are a sure sign that you forgot which way a current was going.
Given these restrictions, and the four two-port parameters, any number of these blocks may be connected together and analyzed as a complete system.


## Measuring Two-Port Parameters

Listed below are the formulas for $\mathrm{z}, \mathrm{y}, \mathrm{h}$ and a parameters.
$z_{11}=\left.\frac{V_{1}}{I_{1}}\right|_{\mathrm{I}_{2}=0} z_{12}=\left.\frac{V_{1}}{I_{2}}\right|_{\mathrm{I}_{1}=0} z_{21}=\left.\frac{V_{2}}{I_{1}}\right|_{\mathrm{I}_{2}=0} z_{22}=\left.\frac{V_{2}}{I_{2}}\right|_{\mathrm{I}_{1}=0}$
$y_{11}=\left.\frac{I_{1}}{V_{1}}\right|_{\mathrm{V}_{2}=0} y_{12}=\left.\frac{I_{1}}{V_{2}}\right|_{\mathrm{V}_{1}=0} y_{21}=\left.\frac{I_{2}}{V_{1}}\right|_{\mathrm{V}_{2}=0} y_{22}=\left.\frac{I_{2}}{V_{2}}\right|_{\mathrm{V}_{1}=0}$
$h_{11}=\left.\frac{V_{1}}{I_{1}}\right|_{\mathrm{V}_{2}=0} h_{12}=\left.\frac{V_{1}}{V_{2}}\right|_{\mathrm{I}_{2}=0} h_{21}=\left.\frac{I_{2}}{I_{1}}\right|_{\mathrm{V}_{2}=0} h_{22}=\left.\frac{I_{2}}{V_{2}}\right|_{\mathrm{I}_{1}=0}$
$a_{11}=\left.\frac{V_{1}}{V_{2}}\right|_{\mathrm{I}_{2}=0} a_{12}=-\left.\frac{V_{1}}{I_{2}}\right|_{\mathrm{V}_{2}=0} a_{21}=\left.\frac{I_{1}}{V_{2}}\right|_{\mathrm{I}_{2}=0} a_{22}=-\left.\frac{I_{1}}{I_{2}}\right|_{\mathrm{V}_{2}=0}$
As an example, $\mathrm{z}_{11}$ is the voltage across port 1 , divided by the current into port 1 (port 1 input impedance), given that current into port 2 is zero; that is, port 2 is open-circuit. It's clear that these parameters can be measured with a meter, for any random collection of circuitry, if it follows the rules listed above.

## Conversions Between Parameters

A variety of conversions are provided to make it easy to get $\mathrm{z}, \mathrm{y}, \mathrm{a}$ or h parameters if any of the types are known. Select a conversion from the type you have to the type you want, and enter the values.
For example, let's calculate the [z] parameters for a simple two-port network; a wall-plug power supply, like the sort used to power calculators or modems. Such a supply might have characteristics something like this:
Input, $110 \_$VAC rms, $30 \_\mathrm{mA}$ maximum (3.3_W). If the output is open-circuit, input current is about $1 \_\mathrm{mA}$. When the output is shorted, input current rises to 30 _mA. In normal operation, it's somewhere in between these extremes.

Output, 6_VAC rms nominal, 7.5_V open circuit, 400_mA short-circuit current. The output voltage will drop to about 6_V under load, with the output voltage varying significantly as the load current changes. No regulation or other control is assumed.

| $a_{11}=\left.\frac{V_{1}}{V_{2}}\right\|_{I_{2}=0}=\frac{110}{7.5}=14.67$ | $a_{12}=-\left.\frac{V_{1}}{I_{2}}\right\|_{V_{2}=0}=-\frac{110}{-.400}=275.0$ |
| :--- | :--- |
| $a_{21}=\left.\frac{l_{1}}{V_{2}}\right\|_{I_{2}=0}=\frac{.001}{7.5}=133.3 E-6$ | $a_{22}=-\left.\frac{l_{1}}{l_{2}}\right\|_{V_{2}=0}=-\frac{.030}{-.400}=.075$ |

Select CONVERT $\mathrm{A} \rightarrow \mathrm{Z}$ and enter the values given above. The calculations can be done on the input line, or you can just type in the values.
Press \&सte and the converted parameters will be displayed.


The z-parameters should be:
z11 110,000 (input impedance).
z12 7975 (transfer impedance).
z21 7500 (reverse transfer impedance).
z22 562.5 (ouput impedance).

If you got a negative result for any of the impedances, go back a moment and think about the sign convention for current (particularly I2).
We'll use these results in a moment to examine how this power supply would work under load. Use the cursor keys and the \$STK menu key to place each z -parameter value on the stack.

## Circuit Performance

The circuit performance topic provides three ways to enter the two-port parameters; as impedance, Z ; as admittance, Y ; or as the hybrid h-parameters.
After entering the appropriate parameters, the source voltage Vg , source impedance Zs and load impedance ZL are entered. Let's use the previous results to see how the power supply works with a 20 ohm resistor attached as a load.

Select CIRCUIT PERFORMANCE (Z SPEC) and retrieve the four z-parameters from the stack. To do that, press ENTER at each z-parameter, then the © menu key. Use the cursor keys to point to the right value (it's
 keys to skip over the numeric value and delete the tag.
Enter 110 for Vg , the 'generator' voltage.
Enter zero for Zs , the source resistance (it's a wall outlet, and we'll assume its resistance is negligible).
Finally, enter 20 for Zl , the load resistance on the output side of the two-port.


Press © A \& to see the results.


## Examining the Results

- Zin is the input impedance, which is influenced by the load. It's a little over 7_k $\Omega$. This is the impedance which the wall outlet looks into.
- I2 is the current flowing in the load, about $200 \_\mathrm{mA}$; negative, since it's flowing out of the two-port and into the load.
- Vt is the Thevenin voltage seen at the output of the network into an open circuit. It's exactly 7.5 _V, as expected from the initial assumptions made about this power supply.
- Zt is the Thevenin impedance seen at the output of the network into a short circuit. From the load's point of view, this power supply looks like a 7.5 _V source with an 18.75 _ $\Omega$ resistor in series.
- I2/I1 is the current gain. It's a measure of the current attenuation or amplification which takes place through the network. As expected, it's negative, since current flows IN (+) the input and OUT (-) the output, and it's just under 13, so this network amplifies current (no surprise, it's a power supply!).
- V2/V1 is the voltage gain, analogous to current gain, measured across the terminals of the network. This gain figure neglects the effect of source impedance on the total gain across the network.
- $\mathrm{V} 2 / \mathrm{Vg}$ is the voltage gain, measured from the voltage source to the load impedance. Multiply this value by the generator voltage Vg to get the actual voltage across the load resistance. The voltage gain figures are small, since this supply steps down a high voltage to a lower one. If a significant source resistance Zs had been included, the overall gain $\mathrm{V} 2 / \mathrm{Vg}$ would be lower, due to losses in the source resistance.
If it's more convenient to enter $h$ - or $y$-parameters, select the appropriate topic from the menu and enter the data in the same way.


## Connections of Two-port Networks

Two-port networks can be connected together in a variety of topologies. The ability to combine two networks into a single one lets you stack small sections together to create a complex system. The sketch below illustrates the connections.


Select the kind of connection needed, and enter the Z-parameters for each of the networks. If you don't have the Z-parameters, use the conversions to solve for them first. Although the textbook methods specify different parameters, depending on the connection type, these calculations uniformly use Z-parameters and convert internally to do the calculations.
Once the eight parameters are entered, press enue to get the equivalent parameters for the connected networks. These results can be plugged into the next combination to solve a large network.

## Circuit Analysis Tools

Notes:

## Appendix A Warranty and Service

## Pocket Professional Support

You can get answers to your questions about using your Pocket Professional card from Sparcom. If you don't find the information in this manual or the HP 48SX Owner's Manual, contact us in writing, at 897 N.W. Grant, Corvallis, OR 97330, U.S.A., or by calling us at 503-757-8416.

## Limited One-Year Warranty

## What Is Covered

The Pocket Professional is warranted by Sparcom Corporation against defects in material and workmanship for one year from the date of original purchase. If you sell your card or give it as a gift, the warranty is automatically transferred to the new owner and remains in effect for the original one-year period. During the warranty period, we will repair or replace (at no charge) a product that proves to be defective, provided you return the product and proof of purchase, shipping prepaid, to Sparcom.

## What Is Not Covered

This warranty does not apply if the product has been damaged by accident or misuse or as the result of service or modification by other than Sparcom.

No other warranty is given. The repair or replacement of a product is your exclusive remedy. ANY OTHER IMPLIED WARRANTY OF MERCHANTABILITY OR FITNESS IS LIMITED TO THE ONE-YEAR DURATION OF THIS WRITTEN WARRANTY. IN NO EVENT SHALL SPARCOM CORPORATION BE LIABLE FOR CONSEQUENTIAL DAMAGES.

Products are sold on the basis of specifications applicable at the time of manufacture. Sparcom shall have no obligation to modify or update products, once sold.

## If the Card Requires Service

Sparcom will repair a card, or replace it with the same model or one of equal or better functionally, whether it is under warranty or not. There is a service charge for service after the warranty period. Cards are usually serviced and reshipped within five working days.
Send the card to Sparcom Corporation, 897 N.W. Grant, Corvallis, OR 97330, U.S.A.

## Service Charge

Contact Sparcom for the standard out-of-warranty repair charges. This charge is subject to the customers local sales or value-added tax wherever applicable.

Cards damaged by accident or misuse are not covered by the fixed charges. These charges are individually determined based on time and material.

## Shipping Instructions

If your card requires service, ship it to Sparcom.

- Include your return address and a description of the problem.
- Include proof of purchase date if the warranty has not expired.
- Include a purchase order, along with a check, or credit card number and expiration date (VISA or MasterCard) to cover the standard repair charge.
- Ship your card postage prepaid in adequate protective packaging to prevent damage. Shipping damage is not covered by the warranty, so we recommend that you insure the shipment.


## Environmental Limits

The reliability of the Pocket Professional depends upon the following temperature and humidity limits:

- Operating temperature: 0 to $45^{\circ} \mathrm{C}\left(32\right.$ to $\left.113^{\circ} \mathrm{F}\right)$.
- Storage temperature: -20 to $60^{\circ} \mathrm{C}\left(-4\right.$ to $\left.140^{\circ} \mathrm{F}\right)$.
- Operating and storage humidity: $90 \%$ relative humidity at $40^{\circ} \mathrm{C}(104$ ${ }^{\circ} \mathrm{F}$ ) maximum.

Warranty and Service

## NOTES

## Appendix B

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## Appendix C <br> Questions and Answers

## Questions Commonly Asked

Q．I can＇t find the EEARP ． verify that the card and the calculator are functioning properly？
A．There are several possibilities：
a．Check to make sure that the card is properly seated in the calculator port．
b．Turn the calculator off and on．
c．The calculator checks the application card when it turns on．If an ＂Invalid Card Data＂or a＂Port Not Available＂message is displayed， the card may require service．

Q．What do three dots（．．．）mean at the end of a display line？
A．The three dots indicate that the object is too long to show on one line． To view the complete object，use the cursor keys to move the arrow to the object and press $⿴ 囗 ⿰ 丿 ㇄$ original display of the item．

Q．I＇m using the Equation Library to solve a problem．After selecting the equations and entering values for the variables，the calculator displays ＂Too many unknowns．＂What＇s wrong？
A．Not enough variables were specified to completely solve the problem． You will have to specify more values and solve again．

Q．I＇m using the Equation Library to solve a problem．After selecting the equations，I＇m ready to enter values for my variables．I find that some of the variables have values already displayed．What＇s wrong？
A．The variables with values displayed indicate that these variable names have been used in solving another equation．To start with a clean slate of values，you can use ©̈＝An：to reset the values of all variables to 0 ．
Q. While using the Equation Library, I turned units off and all the numbers changed. What's wrong?
A. In no-units mode, the Equation Library assumes that all values are SI in order for the equations to solve correctly. Therefore, when units are turned off, all values are first converted to SI units, then the unit tags are eliminated.
Q. While using the Equation Library to solve an equation set, intermediate answers are given. Why?
A. The Sparcom's equation solver engine has the ability to solve a set of equations in a systematic fashion. The result of computation from each equation is reported, to keep you informed of the solver's progress.
Q. The calculator displays "Bad Guess(es)" while running the Equation Library. What's wrong?
A. The HP 48SX root finder encountered variable values or units that prevented a solution. You may need to start the root finding process by providing a "guess" value. See Chapter 1 for details.
Q. While solving for an angle, I got an answer that was too large: For example, 8752 degrees instead of the expected answer of 112 degrees.
A. The calculated result may be offset by integer multiples of 360 degrees. By entering a "guess" value, or by solving in no-units mode, you should be able to avoid this problem.
Q. I solved a problem some time ago, and I'm trying to recall those calculated values for a problem I'm working on now. The values from the past calculation have changed. What's wrong?
A. Most likely, the same variable name was used in solving another equation, so you will not be able to recall the old values.
Q. While searching a list of information, I used the alpha key, but the search function didn't work. Why?
A. Since the search function is case-sensitive, you most likely entered the letters in the wrong case.

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Notes:

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