

Pocket Professional[™] OWNER'S MANUAL



The Pocket Professional[™]

Electrical Engineering Application Pac

Owner's Manual

SPARCOM[®]

Edition 2 August, 1991

Manual Reorder No. 11051-A Software Reorder No. 10051-1A

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Sparcom Corporation 897 N.W. Grant Avenue Corvallis, OR 97330 U.S.A.

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Chapter 1 Getting Started

In This Chapter

Sparcom's Pocket Professional- software is the first of its kind, developed to provide speed, efficiency and portability to students and professionals in the technical fields. When you slide the Pocket Professional- Electrical Engineering Application Pac into your HP 48SX, your calculator is instantly transformed into an electronic "textbook," ready to efficiently solve your electrical engineering problems. The Pac is organized into seven sections: Equation Library, AC Circuits, Fourier/Laplace Transforms, Ladder Network Analysis, Transmission Lines, Two-port Networks, and Constants Library... all available in an efficient, menu-driven format.

This chapter covers:

- □ Installing and Removing the Card
- Using the Main Menu
- Using the Equation Library
- □ What You Should Know About the Solver
- □ Summary of Functions
- □ Summary of Softkeys

Installing and Removing the Card

The HP 48SX has two ports for installing plug-in cards. You can install your Electrical Engineering Application Pac card in either port. Be sure to **turn off the HP 48SX** while installing or removing the card. Otherwise, user memory may be erased.

To Install the Application Card

- 1. Turn off the HP 48SX. Do not press M until you have completed the installation procedure.
- 2. Remove the port cover. Press against the grip lines and push forward. Lift the cover to expose the two plug-in ports, as shown below:



- 3. Select either empty port for the Pocket Professional card, and position the card just outside the slot. Point the triangular arrow on the card toward the calculator port opening, as shown above.
- 4. Slide the card firmly into the slot. After you first feel resistance, push the card about 1/4 inch further, until it is fully seated.



5. Replace the port cover.

Memory Requirements

The EE Application Pac requires some RAM memory in order to work. This memory is used for temporary storage, and for saving variables such as equations to be plotted later. Errors may be encountered if the available memory is less than about 4000 bytes. For more information, see Chapter 5 of the *HP 48SX Owner's Manual*.

To Remove an Application Card

- 1. Turn the HP 48SX off. Do not press 🕅 until you have completed the removal procedure.
- 2. Remove the port cover. Press against the card's grip lines and push forward. Lift the cover to expose the two plug-in ports, as shown below:



- 3. Press against the card's grip lines and slide the card out of the port, as shown above.
- 4. Replace the port cover.

Accessing the Electrical Engineering Application Pac

After you turn on your HP 48SX by pressing \bigcirc , there are three ways to start the application.

First Method: Press **E LEAPP** to display all libraries available to the HP 48SX. Find and press **EAPP** to enter the Electrical Engineering Application Pac library directory. The screen displays new menu keys or "softkeys" along the bottom, as shown:

ł	HOME	EEAPPD }
14	:	
13	:	
ĺŹ	:	
Ī	:	
â	ENPP E	CON DEREC DERUB GPLO PHPLO

Press the **EEAPP** softkey again to start the application.

The **ECON** softkey accesses the Constants Library function, described in Chapter 3. **DEREC** and **DERUB** are functions required by the software, but are not available to the user. **GPLO** and **PHPLO** are two programs available to the user to plot the gain and phase of a transfer function, and are explained in Chapter 4.

Additional softkeys are accessed by pressing the **NT** key. One of these keys is the **ABOUT** softkey. Pressing this key displays a screeen containing the revision number of the Electrical Engineering Application Pac. (Press **TTN** to exit the revision screen).

Second Method: Type in EEAPP (using alpha entry mode) and press ENTER.

Third Method: Add the command "EEAPP" to the CST (custom) menu (for more information, refer to Chapter 15 of the *HP-48SX Owner's Manual*, "Customizing the Calculator"). After the command has been added, press [ST], then press **EAPP** to start the software.

Using the Main Menu

After you start the application, the main menu screen appears:



The main menu lists the seven major areas of application in a menu-driven format. Menu-driven means that the information is selected by moving the pointer to an item in the menu and pressing **ENTER**.

Applications in the Main Menu

Each application in the main menu is briefly described below and is discussed in detail in the next three chapters of this manual.

Equation Library	Allows you to solve, plot and analyze over 300 equations commonly used by electrical engineers
Constants Library	Lists over 20 universal and physical contants,
	plus 22 silicon properties and 5 magnetic prop- erties
AC Circuits	Solves problems in 9 topics including imped-
	ance, admittance, power factor, and star-to-
	delta circuit transformation
Fourier/Laplace Trans-	Lists tables of transforms; includes a pole zero-
forms	analysis section, and FFT and inverse FFT
	computation
Ladder Network Analy-	Computes performance parameters for a
sis	loaded ladder network
Transmission Lines	Allows you to compute propagation character-
	istics, impedance, and VSWR for a transmis-
	sion line
Two-port Networks	Computes circuit performance parameters for
	a given source and load impedance, converts
	between z, y, a and h parameters; and com-
	bines two-ports into equivalent networks

The "softkeys" located along the bottom of each screen give you options that relate to the information displayed on any given screen. The following softkeys appear along the bottom of the main menu. A summary of common softkeys used throughout the Pac is given at the end of this chapter.

- FONT Toggle between the large and small font for easy viewing of results
- QUIT Exits the Electrical Engineering Application Pac

Moving Around the Screen

Use the **A** and **v** keys to move the pointer up and down in the menu list. Pressing **v** moves the pointer to the bottom of the screen, or pages down (one screen at a time) if the pointer is already at the bottom of the screen. Pressing **v** moves the pointer to the top of the screen, or pages up. Pressing $\square \square$ moves the pointer to the bottom of the list and $\square \square$ moves the pointer to the begining of the menu.

Viewing Items Too Wide for the Display

If the text of a menu item is too wide to fit within the display, an ellipsis (...) appears at the end of the line. Press I to display the rest of the text. Press I or ENTER to return to the original display of the item.

Changing the Font Size

The default font for the Electrical Engineering Application Pac displays information in small, uppercase letters only. Pressing **FONT** displays the information in a larger, medium-sized font, which is case-sensitive. The font size stays medium (shown below) until you press **FONT** again:



Using the Search Mode

When menu lists are long, it is faster to locate an item using the search mode. To initiate a search, press \square to display the following screen:

{ HOME EEAPPD }	PRG
Search for:	
eskipiskipalenet inela lins i	

The calculator is now in *alpha* entry mode, as indicated by the alpha (α) annunciator at the very top of the screen. Alpha entry mode overrides the function of the standard keyboard. This means that each key that has a white capital letter printed to its lower right loses its original function and types that letter onto the command line when pressed. (See Chapter 2 of the *HP* 48SX Owner's Manual for a complete description of how the alpha mode operates). Type the first letter or letters of the name you want to search for, to create a search string, and press ENTER. The search function is

case-sensitive. To enter a lower case letter in the alpha entry mode, preceed the letter with the **G** key.

Pressing Im returns you to the main menu.

Editing Text Entries

The search mode softkeys, shown on the screen above, are command line editing keys. They are built into the HP 48SX and allow you to edit the search string. Their functions are summarized below:

-SKIP	Moves the cursor to the beginning of the current word.
SKIP→	Moves the cursor to the beginning of the next word.
*DEL	Deletes all the characters in the current word to the left of the cursor.
DEL-+	Deletes all the characters from the cursor's current position to the first character of the next word.
INS	Toggles between insert and typeover modes.

Using the Equation Library

The Equation Library contains over 300 equations commonly used by today's electrical engineering professionals and students. The Equation Library enables you to:

- Select the equation category and topic from the main menu.
- List all the equations in a topic.
- Solve a specific equation or set of equations.
- View a description of the variables.
- View a figure that illustrates the problem, when available.
- Plot the equation.

The next few pages show you how to solve a single equation. Solving multiple equations systematically is discussed later in the chapter. For this example,

let's suppose you want to calculate the resistance of a wire 1.569_cm long and 0.00245_cm² in area with a resistivity of $1.5_{\mu}\Omega \cdot \text{ cm}$.

Accessing Equations

The first step in solving this problem is to locate the necessary equation in the Equation Library. At the main menu, move the pointer to EQUATION LIBRARY and press ENTER. This displays the list of 11 main categories:



Move the pointer to CIRCUIT ELEMENTS, and press ENTER to display the list of topics in this category:



Selecting and Displaying Equations

Move the pointer to the topic RESISTANCE and press [MER], or [EQNS] to display the equation set for resistance:



This screen lists all the equations in the current topic. In this case, there are four. You may choose to solve all the equations systematically or solve any one equation. Solving multiple equations will be discussed later in this chapter.

For this example, the resistance of the wire is given by the first equation in the set:

$$R = \frac{\rho \cdot len}{A}$$

where ρ is the resistivity, len is the length, A is the area of cross section, and R is the resistance. Any equation may be selected by moving the pointer to the desired equation and pressing **SELECT**. If no equation set is selected, then all equations will be solved systematically. When an equation is selected, a triangular tag is placed in front of the equation:



If you want to view the equation in its full "textbook" form, place the pointer at the equation and press **ENTER**. This displays the equation on the screen:



Press ENTER or ATM to return to the list of equations.

When displaying a lengthy equation from the Equation Library, pressing or scrolls the screen to the left or to the right revealing the entire equation. Pressing > moves the display window to the end of the equation, and pressing > moves the display window to the beginning of the equation.

Viewing Variable Definitions

You can view a list that defines all the variables in the selected equation, or set of equations, by pressing the **WARS** softkey at the equations screen. The screen below displays the definitions screen for the first equation of the RESISTANCE topic:



To continue solving the problem, you need to invoke the solver function.

Using the Solver Function

The Sparcom "solver" is a software function that simplifies the job of setting up equations to be calculated by the HP 48SX. The solver function is discussed in more detail later in this chapter, under "What You Need to Know About the Solver."

Enter the solver function of the Pac by pressing **SOLVE**. At the solver screen, the units key becomes available. To work with units for this example, press the **UNITS** toggle key until it reads **UNITE**. The variables for the selected equation(s) now appear on the screen, with units, waiting for you to enter values:



To enter the resistivity, move the pointer to ρ and press ENTER. This displays the following screen:

{ HOM	IE EE	APPD }	PRG
Set	۶,	resistiv	ity:
•			
9%M	-2%	(# <u>8</u> #	

Enter the resistivity value at the prompt:

{ HOME EEAPPD }	PRG
Set p, resistivity:	
1.5•	
_9.1M _9.1C _#9.1	

After the entering a value, there are two ways to assign units to your entry. The easiest way is by selecting one of the unit softkeys provided on the menu line, or typing in your own choice of units.

If you choose **not** to add units, just press **ENTER** at the prompt, and the software will assign SI units. In some cases, more units are available than the six softkeys displayed in the first screen. In these cases, press **NUT** to display the additional units. For a complete description of units supported by the HP 48SX and their respective symbols, see the HP 48SX Owner's Manual. For this example, press **NUT** to add units of $\mu\Omega$ · cm to your entry:

{ HOME EEAPPD }	PRG
Set P, resistivity:	
1 5 u0×cm	
1.J_XX*CII 288M 2880 2888	

Now press ENTER to store this value into the variable ρ . This returns you to the solver screen with $1.5 \ \mu\Omega$ · cm stored into the variable, ρ :



The triangular tag indicates that ρ is a known variable. Repeat this procedure for the other known variables, A and len. This results in the following screen:



With three of the four variables known in this equation, you may now solve the equation for the resistance by pressing **CALC**. After a few moments, the calculator returns to this screen with the calculated value of R:

→XR: ' ÞA: ' ÞLEN Þ <i>P</i> : '	R(.00094 .0025_ : '1.56 '1.5_#3	2515 14_0' (M^2' 19_CM' 23CM'	tand	e	
CALC	EQNS	VARS	UNITE	÷stk	UP

The * by R indicates that this value was calculated and was not initially specified.

Converting Data to Different Units.

Suppose you want to convert the resistance (R) from ohms to kilo ohms. First press the key to reveal the next page of softkeys available for this display:



Move the pointer to the variable R and press **CONV**. This lists all of the possible units for R:



Now move the pointer to $k\Omega$ and press ENTER:

Resistance →XR: ',0000009414_KQ' ▶A: '.0025_CM'2' ▶LEN: '1.563_CM' ▶¢: '1.5_⊬QXCM'
(KNOW)WANT CLEAR PURGE CONV UP

This converts the resistance in Ω to $k\Omega$. If you want to use the data for further calculations, move the pointer to the data item and press and then press **STR** to place it on the calculator stack.

Options After Solving the Equation

Pressing A exits the Electrical Engineering Application Pac and places you in the calculator operating environment. Pressing **CLEAR** resets all entries in the current topic to zero. Pressing **PURCE** deletes the global copies of each variable in the currently selected set of equations that reside in the EEAPPD directory.

To return to the main menu screen press **UP** multiple times. At the main menu, a new RESUME SOLVING... entry will have been added to the list, as shown:



Selecting the RESUME SOLVING function returns you directly to the equation set you were working with, with all previous entries still intact.

Managing Units

When solving an equation, **WINITS** (a toggle key) controls whether the calculations are performed in your choice of units, or in Systeme Internationale d'Unites (SI) units. When the **UNITS** softkey appears, it means that all entries are converted to SI units and the unit designations are removed. **UNITS** indicates that the software is managing units, and that all values will contain the unit designations that you specify.

Using designated units usually increases the processing time.

Solving Multiple Equations

For many problems, the result of one calculation acts as the input to another. The Electrical Engineering Application Pac is capable of solving multiple equations, systematically.

Selecting the Equation Set

Suppose you want to calculate the performance characteristics of an ideal transformer. From the Equation Library menu screen, move the pointer to TRANSFORMERS and press ENTER. This category contains only one topic, IDEAL TRANSFORMERS.

The equations for this topic are displayed on the screen when you move the pointer to IDEAL TRANSFORMERS and press **ENTER**:



These are the five equations in their written form:

1) $\frac{V_1}{V_2} = \frac{n_1}{n_2}$ 2) $l_1 \cdot n_1 = l_2 \cdot n_2$

3)
$$Rin = \frac{R^2}{a^2}$$
 4) $a = \frac{n^2}{n^1}$

5) V2=I2 · R2

To view the variables for this equation set, press **VARS**. All the variables for the IDEAL TRANFORMERS topic are listed in the following table:

Variable	Description	Default Units
V1	primary voltage	1 V
V2	secondary voltage	1 [¯] V
n1	number of turns in primary	1
n2	number of turns in secondary	1
11	current in primary	1 A
12	current in secondary	1 [¯] A
R2	secondary load resistance	1_Ω
Rin	resistance at primary	1 <u>Ω</u>
а	turns ratio	1

Solving the Equation Set

Press **SELECT** to select the desired equation to be solved. In the following, the top four equations have been selected as indicated by the triangular tags to the left of the equations.

Ideal →▶¥1/¥2=N1/N ▶11¥N1=12¥N2 ▶RIN=R2/A^2 ▶A=N2/N1 ¥2=12¥R2	transf Z	orme	r
MAIN SELECT V	ARS PLOT	SOLVE	UP

Press **SOLVE** to enter the solver function for these four equations. Enter all the information pertaining to the problem, using the procedure described previously. Press **CALC** to start the solver. The solver then steps through each equation in the list, solving those equations that contain sufficient data to calculate an unknown variable. When all known variables are found, or all remaining equations have more than one unknown variable, the solver stops. It then lists the variables it can't find, and returns to the solver screen. The given variables and calculated results for all four selected equations are shown below:

Given	Result
$V2 = 10_V$	V1 = 5.7143 V

 $\begin{array}{ll} n1 &= 100 & & l2 &= 7.1429\text{E-}2_\text{A} \\ n2 &= 175 & & \text{Rin} &= 0 \\ l1 &= 125_\text{mA} & & a &= 1.75 \\ & & & \text{R2} &= 0 \ \Omega \end{array}$

With the information given, the solver finds all the variables except Rin and R2. The calculator beeps and indicates that all the variables cannot be calculated. Then, all the known and calculated variables are shown on the solver screen. Notice that Rin and R2 are not marked by an asterisk *:



Tagging Variables

If you want to solve for only one variable in the list, you can tag is as "wanted." Move the pointer to the variable you want to tag, press with to display the additional softkeys for this screen, and press want . This places a "?" tag in front of the variable you want to solve for:



If you tag Rin and press **CALC**, the solver stops when it finds a value for Rin, rather than solving for the entire set. It is possible to tag more than one variable in the list as wanted.

Plotting One Equation

Any equation in the Equation Library that is of the form y = f(a,b,c...) can be easily plotted using the Electrical Engineering Application Pac. To plot an equation, the dependent variable on the left (y) and the desired independent variable (a or b or c...) on the right side must be unknown (no triangular tag). However, all other variables must be known.

Finding and Selecting the Equation

As an example, plot the variation of capacitance of a pn junction as a function of applied voltage. The equations that describe the capacitance of pn junctions are filed in the SOLID STATE DEVICES category, under the topic PN JUNCTIONS. The equation screen for this topic is shown below:



Move the pointer the third equation in the list and press **SELECT**. Press ENTER to view the written out form of the equation, or **VARS** to view the subset of variables for this equation. The equation and a table of its variables are shown below:

$$CJ = \left(\frac{q \cdot \varepsilon o \cdot \varepsilon Si}{2\left(\frac{1}{NA} + \frac{1}{ND}\right) \cdot \left(Va - Vbi\right)}\right)^{\frac{1}{2}}$$

Variable	Description	Customary Units
ND	donor density	1 1/cm ³
NA	acceptor density	1 ⁻ 1/cm ³
Va	appliced voltage	1 ⁻ V
Vbi	built-in voltage	1 ⁻ V
CJ	junction capacitance per unit area	1_pF/cm ²
60	*permittivity of free space (no user entry)	1
εSi	*relative permittivity of Si (no user entry)	1
q	*charge on the electron (no user entry)	1_C

*These variables are not visible on the screen and are automatically extracted by the software from the Constants Library. No user entry is needed.

Tagging and Entering the Variables

To plot the capacitance curve (CJ versus Va), NA, ND, and Vbi must be tagged as known variables. Move the pointer to the third equation and press **SELECT**. Then press **SOLVE** to specify values for the following known variables:

 $ND = 1E15_{cm}^{-3}$ $NA = 1E18_{cm}^{-3}$ $Vbi = 0.7583_{V}$

With these three variables entered, return to the equations screen by pressing **EGNS**. Move the pointer to the third equation and press **PLOT**. Since this equation is of the proper form, and all but Va and CJ have been specified on the right hand side, it may be plotted.

Entering the X and Y Coordinates

The first prompt asks whether you want to erase the previous plot and reset the axes **YES**, or whether you want the new plot drawn over any existing graphics already on the screen **NC**. To continue with this example, at the prompt enter **YES** to clear all previous plots from the screen.

Now enter the minimum and maximum values for the x coordinate for the graph. Type the coordinates for the plot on the same line, separated by a space (use the relation key). For this example plot, select the no units option (**UNITS**), then enter 1 10 for Va; (the assumed units are _V). This results in the following screen:

{ HOME E	EAPPD }	PRG
Enter for T <min></min>	horizontal (K): <max></max>	range
1 10+	P→ { DEL DEL→ IN	S • † STK

The plot function now prompts for the limits of the y-axis (in this case, CJ, the capacitance in pF/m^2 units). You can either enter the lower and upper limits for y, or allow the system to auto range when **ENTER** is pressed. For this example, press **ENTER** to auto range a plot of CJ versus Va over the range of 1 to 10_V, shown below:



Plotting Speed

If units are on (the **UNITE** key is displayed at the solver screen) a plot can take up to 10 minutes to display. If you turn the units off (i.e., toggle the units key to remove the box) the plot function performs in approximately one tenth of the time.

However, as described earlier in this chapter under "Managing Units," when you turn off units, all values are converted to SI units. Therefore, when you enter the x-axis coordinates, you need to enter them as low limit and upper limit. The plot will also be displayed in the default SI units.

Softkeys for the Plot Function

The softkeys shown in the above plot are plot function keys in the HP 48SX. For example, pressing **COORD** displays the (x,y) coordinates of any point on the screen indicated by the cursor. For a description of the behavior of the plot function softkeys, see Chapter 18 of the HP 48SX Owner's Manual.

Note that the **SLOPE** and **F** keys, inside the HP 48SX **FCN** submenu, are supported by the Electrical Engineering Application Pac only when units are off. You can remove the softkeys from the plot to expose more of the graph by pressing **NUT NUT** and **KEYS**. Press **ATN** to interrupt the plotting of an equation or to return to the equation screen.

Making Multiple Plots of an Equation

In some cases, you may want to graph an equation on the same axes several times. To do this, simply answer **NO** to the "Clear plot first?" prompt after you have pressed **PLOT**.

For example, suppose you're interested in plotting a new capacitance curve for a higher doping (e.g., $ND = 1E16_1/cm^3$). Return to the solver screen by pressing **SOLVE** and enter the new value for ND. Then go to the equations screen, move the pointer to the capacitance equation, and press the **PLCT** softkey. At the prompt, press **NO**. The new graph will plot over the previous one, as shown:



There is no limit to the number of graphs that may be plotted on a given axis. However, the HP 48SX plot/graphics function keys support only the most recent plot.

What You Should Know About the Solver

As you have seen in the examples in this chapter, the Sparcom solver allows you to easily specify the values and units of your equation or set of equations before sending the data to the HP 48SX numerical root-finder. For the selected equations(s), the solver screen lists all the variables, shows whether they are known (triangular tag), unknown (no tag), wanted ("?" tag), or just calculated (*), and whether units are on or off.

Once you set these parameters, pressing **CALC** activates the HP 48SX root-finder to calculate the solution(s). The root-finder requires an initial value on which to base its search. You can provide a "guess" for the calculator to use, or the solver will provide a "guess" value of 1. The root-finder then generates pairs of intermediate values and interpolates between them to find the solution. The time required to find the root depends on how close the initial guess is to the actual solution.

Speeding Up Computing Time

You can speed up computing time by providing the calculator a "guess" value close to the expected solution. At the variables screen, enter your guess value into the "unknown" variable. The variable will then be tagged as "known" (triangle). Press the **KNOW** softkey to toggle the variable back to "unknown" (no tag). Now press **CALC**

"Bad Guess" Message

If the calculator displays the message, "Bad Guess(es)," it indicates an error has been made in setting up the problem. Go back through the set up process and check for errors in specifying data.

For more information, refer to Chapter 17 of the HP 48SX Owner's Manual .

Loading Values from the Stack

There are two methods of entering a value into the Sparcom solver directly from the calculator stack:

First Method: Make sure the value you want is on the stack. Press EEAPP, then choose an equation set to solve, or select RESUME SOLVING to return to the equation set you're last working with. At the variables screen, move the pointer to the variable that will incorporate the value currently on the stack and press [MTE]. A prompt message asks you to enter the value. Press [] [] to reveal the command line editing keys. Press the [] [] STIK softkey to invoke a limited version of the HP 48SX Interactive Stack. Move the pointer to the appropriate stack level and press ECHO then [] MTE]. This takes you back to the "Enter value" prompt message. Press [] again to store the echoed value into the current variable and return to the solver screen.

Second Method: Alternatively, store the desired value into a global variable in the EEAPPD directory under the same name as the equation variable. When the solver is entered, it will automatically recall the value and load it into the selected equation variable.

Sparcom's "EEAPPD" Directory

When you plug in the Electrical Engineering Application Pac for the first time, the software creates its own directory, EEAPPD, in the HOME directory of the HP 48SX. ALL operations performed by the software take place in the EEAPPD directory. It is, therefore, the only place where global variables are created or purged by the solver. If you purge this directory by mistake, it will be recreated in its entirety, but all the values that you previously stored will be lost.

The variable created in the EEAPPD directory and its functions is described below:

EEpar The parameter EEpar is utilized to provide a direct path from the main menu to the solver level. EEpar is created (or rewritten) whenever the equation, solver, or variable levels of the the Equation Library is exited. The three possible exit routes that trigger an EEpar update are: 1) Pressing IT to quit the software and exit to the calculator stack, 2) Pressing UP to return to the topic level, or 3) pressing to return to the main menu level.

Summary of Functions

The following figure diagrams the basic flow and function of each level of the equation library and solver. On the following page, the softkeys available at each level are explained in more detail.



Summary of Softkeys

- **CALC** Stores all variable values and iterates through the set of selected equations in an attempt to find values for all wanted variables. After completion of the solver process, the user is returned to the solver level, where newly found variables are marked with "*".
- **CLEAR** Resets the values of the current variable set to zero.
- **EGNS** Enters the equation level of the current topic.
- **FIGURE** Displays a figure for the currently selected topic or displays "No figure".
- **FONT** Toggles between small and medium fonts.
- **KNOW** Toggles the currently selected variable between known and unknown, adding or removing the triangular tag.
- MAIN Returns to the main menu.
- **PLOT** Plots the selected equation, prompting the user for x-axis and y-axis values. This feature works only for equations of the form y = f(a,b,) where y and one variable on the right are unknown.
- **PURGE** Purges the global copies (in the EEAPPD directory) of the current variable set displayed in the solver level.
- **QUIT** Exits the Electrical Engineering Application Pac.
- **SELECT** Marks or unmarks the currently selected equation with the triangular tag. Only variables in the marked equations will appear in the solver and variable levels (with the exception of constants). If no equations are selected, all will be used.
- **SOLVE** Enters the solver level of the currrent topic.
- STK Copies selected entry to calculator stack.
- **UNITH** Toggle key. Indicates that units are on.

- **UNITS** Toggle key. Inducates units are off. When off, all variables are assumed to be SI if entered with no units or are converted to SI units, if entered in other units.
- **UP** Moves up one level in the menu structure.
- VARS Displays the variable screen for the current topic.
- **VIEW** Displays the full text entry for a variable description or value if the description is too wide to fit on the screen.
- Toggles the currently selected variable between wanted and not wanted, adding or removing the symbol "?". If no variables are marked "wanted," all variables are assumed to be wanted.
- ENTER Prompts for the value of the currently selected variable. If the selected variable already contains a value, that value is copied to the command line for editing. Pressing And clears the command line, or returns you to the variables screen if the command line is already empty.

Used to exit the application.

Notes
Chapter 2 Equation Library

In This Chapter

The Equation Library contains over 300 equations organized into 11 main categories. Each category contains several topics. Each topic includes an equation set, a complete list of variables, sometimes a figure illustrating the equation set, and a set of units for all variables. This chapter describes these topics and provides one or more examples using the equation set. The computed results for all examples have been rounded off to the fourth decimal place.

- Circuit Elements
- Simple DC Circuits
- **RL and RC Circuits**
- **RLC** Circuits
- Electrical Resonance
- **OpAmp Circuits**

□ Simple AC Circuits

Transformers

- **Transmission Lines**
- Motors and Generators
- □ Solid State Devices

Circuit Elements

The following topics calculate values of electrical circuit elements from first principles.

- Resistance
- Cylinder/Coaxial Cable
- Spherical Shell, R/C
- Solenoid/ Toroid/ Loops
- Parallel Plate Capacitor

Resistance

The four equations in this set compute the resistance or conductance of a rectangular bar, and show the reciprocal relationship betweeen resistivity and conductivity, and calculate the effect of temperature on resistance.

1)
$$R = \frac{\rho \cdot len}{A}$$
 2) $G = \frac{\sigma \cdot A}{len}$ 3) $\rho = \frac{1}{\sigma}$

4) $R = Ro \cdot (1 + \alpha \cdot (T - To))$

Variable	Description	Units
R	resistance of a bar (at temperature T)	1Ω
G	conductance of a bar	1 S
Α	uniform area of cross section	1_cm^2
len	length of the bar	1 cm
ρ	resistivity	1_Ω · m
σ	conductivity	1_S/m
Ro	resistance at temperature To	1_Ω
α	temperature coefficient of resistance	1_1/K
Т	temperature	1_℃
То	reference temperature	1_°C



Example 1: A rectangular bar 1.41_cm long and 0.00425_cm² in area has a conductivity of 10.5_S/cm. What is its resistance, resistivity in $\Omega \cdot$ cm and conductance? Use equations 1, 2 and 3.

Given	Result
$A = 0.00425 \text{ cm}^2$	$R = 31.5966_{\Omega}$
len = 1.41_cm	$G = 3.1649E-2_S$
$\sigma = 10.5 \text{_}\text{S/cm}$	$ ho$ = 9.5238E-2_ Ω · cm

Example 2: A filament resistor measures 58.5_{Ω} at 100_{C} and 50_{Ω} at a reference temperature. Its temperature coefficient of resistance is 0.0025 1/°C. What is the reference temperature in °F? Use equation 4.

Result

 $To = 89.6 \ ^{\circ}F$

Given $R = 58.5_{\Omega}$ $Ro = 50_{\Omega}$ $\alpha = 0.0025_{1}^{\circ}C$ $T = 100_{C}^{\circ}C$

Cylinder/Coaxial Cable

The equations in this topic calculate the radial resistance of a thin cylindrical tube and the inductance or capacitance of a coaxial cable.

1)
$$R = \frac{r2 - r1}{2 \cdot \pi \cdot r2 \cdot len} \cdot \rho$$

2) $L = \frac{\mu o \cdot \mu r \cdot len}{2\pi} \cdot LN\left(\frac{r2}{r1}\right)$
3) $C = \frac{2 \cdot \pi \cdot \varepsilon o \cdot \varepsilon r \cdot len}{LN\left(\frac{r2}{r1}\right)}$

Variable	Description	Units
R	radial resistance of a cylinder	1_Ω
r1	inner conductor radius	1_cm
r2	outer conductor radius	1_cm
len	length of cable	1_cm
ρ	resistivity	1_Ω · cm
Ĺ	inductance	1_mH
μr	relative permeability	1
С	capacitance	1_μF
Er	relative permittivity	1



Example 1: A thin 20 _ft long, cylindrical tube has an inner radius of 0.15_cm and an outer radius of 0.75_cm. The cylindlindrical tube is made of material with a resistivity of $0.75_{\Omega} \cdot \text{cm}$. Find the radial resistance using the first equation.

Given r1 = 0.15_cm r2 = 0.75_cm len = 20_ft $\rho = 0.75$ _ $\Omega \cdot$ cm **Result** $R = 1.5665E-4 \Omega$

Example 2: A 20_ft cable with an inner conductor radius of 0.15_cm and an outer conductor radius of 0.75_cm is filled with either a magnetic material with a relative permeability of 1.25 or a dielectric material with a relative permittivity of 4.9. Find the total inductance and capacitance for these cables in mH and μ F respectively. Use equations 2 and 3.

GivenResultr1 = 0.15_cmL = 2.4528E-3_mHr2 = 0.75_cmC = 1.0325E-3_ μ Flen = 20_ft ϵ r = 4.9 μ r = 1.25

Spherical Shell, R/C

These formulas cover the calculation of the radial resistance and capacitance of a thin spherical shell.

$r_2 - r_1$	$c = \frac{4 \cdot \pi \cdot \varepsilon o \cdot \varepsilon r}{1 \cdot \pi \cdot \varepsilon o \cdot \varepsilon r}$
$n = \frac{1}{4 \cdot \pi \cdot r1 \cdot r2} \cdot p$	C =
	$\frac{1}{r_1}$

Variable	Description	Units
R	radial resistance	1_Ω
r1	inner spherical radius	1_m
r2	outer spherical radius	1_m
ρ	resistivity	1_Ω·m
Ċ	capacitance	1_F
Er	relative permittivity	1

Example: A spherical shell has a resistance of 0.0125_Ω . The inner radius of the shell is 0.85_{cm} and the outer radius is 0.985_{cm} . Find the resistivity of the material of the shell. If the shell is replaced by a dielectric material with a permittivity of 11.7, what is the capacitance of the shell?

Given	Result
$R = 0.0125_{\Omega}$	$\rho = 0.9742 \Omega \cdot \text{cm}$
r1 = 0.85 cm	$C = 8.0736E-5_{\mu}F$
$r2 = 0.985$ _cm	
$\varepsilon r = 11.7$	

Solenoid/Toroid/Loops

The first equation calculates the inductance of a solenoid of length, len, and a cross-sectional area A, with a core whose relative permeability is μr . For completeness, equations which compute the inductance of toroid and the self-inductance of a wire loop are also included in this subtopic.

1)
$$Ls = \mu o \cdot \mu r \cdot n^2 \cdot len \cdot A$$

2)
$$Lt = \frac{\mu o \cdot \mu r \cdot Nt^2 \cdot h}{2 \cdot \pi} \cdot LN\left(\frac{ro}{ri}\right)$$

3)
$$LI = \mu o \cdot a \cdot \left(LN \left(\frac{8 \cdot ro}{a} \right) - 1.75 \right)$$

Variable	Description	Units
Ls	solenoid inductance	1_H
Lt	toroid inductance	1_H
LI	loop inductance	1_H
n	number of turns per unit length	1_1/m
len	length of solenoid	1_m
Α	area of cross section	1_m^2
μr	relative permeability	1
Nt	number of turns	1
ri	inside toroid radius	1_m
ro	outside toroid radius/mean loop radius	1_m
h	thickness of the toroid	1_m
а	wire radius	1_m



Example 1: A 25_cm long solenoid has a coil of 15 turns/cm. The core of the solenoid has an area of cross-section of 3.25_{in}^2 and is filled with a magnetic material with a relative permeability of 1000. Find the inductance of the solenoid, using equation 1.

Given	Result
len = 25_cm	L = 1482.1194 mH

 $n = 15_1/cm$ A = 3.25_in² $\mu r = 1000$

Example 2: A 150 twin toroid has an inner radius of 1.00_in and an outer radius of 1.25_in. The toroid has a relative permeability of 650 and a thickness of 0.15 in. Using equation 2, find the inductance of the toroid.

Given $\mu r = 650$ Nt = 150 ri = 1_in ro = 1.25_in h = 5.7_cm

Result Lt = 37.2036_mH

Parallel Plate Capacitor

This formula computes the capacitance between two parallel plates separated by a small spacing, d (ignoring fringing field effects).

$$C = \frac{\varepsilon \, o \cdot \varepsilon \, r \cdot A}{d}$$

Variable	Description	Units
С	capacitance	1 F
Er	relative permittivity	1
Α	area of cross section	1 m^2
d	plate separation	1_m
	Ŷ	



Example: A parallel plate capacitor is built using a dielectric with a relative permittivity of 3.9 and a plate separation of 1.56E-6_cm. The plate area is 2.8 cm². What is the capacitance in μ F?

 Given
 Result

 $\varepsilon r = 3.9$ $C = 0.6198 \mu F$
 $A = 2.8 cm^2$ d = 1.56E-6 cm

Simple DC Circuits

This category covers circuit fundamentals, including Ohm's law, combining two circuit elements of the same type in series or parallel, energy stored in reactive elements, circuit performance parameters, and the Wheatstone's bridge. These topics focus on basic circuit principles of equivalence, energy storage, and power delivered to a load.

- Ohm's Law and Power
- Combination of 2 R's, 2 C's or 2 L's
- Energy Stored in L or C
- DC Circuit Properties
- Wheatstone's Bridge

Ohm's Law and Power

The relationship between current, voltage, resistance, and power is based on Ohm's law. The equations in this set show the interrelationship between these four variables.

1) $V = I \cdot R$ 2) $P = V \cdot I$

3)
$$P = l^2 \cdot R$$
 4) $P = \frac{V^2}{R}$

Variable	Description	Units
V	voltage	1_V
I	current	1_A
R	resistance	1_Ω
Р	power dissipated	1_W

Example: A 5_V battery with no internal resistance has a load of 1250_{Ω} . Calculate the current in the load and the power dissipated in the load.

Given	Result
$V = 5_V$	I = 0.004 A
$\mathbf{R} = 1250_{\Omega}$	P = 0.02 W

Combination of 2 R's,2 C's or 2 L's

This equation set covers the effects of combining two resistors, two inductors or two capacitors in either series or parallel.

1) $Rs = R1 + R2$	2) $\frac{1}{Rp} = \frac{1}{R1} + \frac{1}{R2}$
3) $Ls = L1 + L2$	4) $\frac{1}{Lp} = \frac{1}{L1} + \frac{1}{L2}$
1 1 1	

CS $C1$ $C2$	5)	$\frac{1}{Cs} =$	$\frac{1}{C1}$ +	$\frac{1}{C2}$	6)	Ср	=	C1	+	C2
--------------	----	------------------	------------------	----------------	----	----	---	----	---	----

Variable	Description	Units
Rs	equivalence of 2 resistors in series	1Ω
Rp	equivalence of 2 resistors in parallel	1_Ω
R1	resistance 1	1_Ω
R2	resistance 2	1_Ω
Ls	equivalence of 2 inductors in series	1_H
Lp	equivalence of 2 inductors in parallel	1_H
L1	inductor 1	1_H
L2	inductor 2	1_H
Cs	equivalence of 2 capacitors in series	1_F
Ср	equivalence of 2 capacitors in parallel	1_F
C1	capacitor 1	1_F
C2	capacitor 2	1_F



Example: Calculate the effect of two 1250_ Ω and 28050_ Ω resistors, two 275_ μ H and 1.225_mH inductors, and two 0.65_ μ F and 0.52_ μ F capacitors connected in series and parallel.

Given Resistor	Result
$R1 = 1250_{\Omega}$	$Rs = 4100_\Omega$
$R2 = 2850 \Omega$	$Rp = 868.9024_{\Omega}$
$L1 = 275 \mu H$	Ls = 1.5 mH
L2 = 1.225 mH	Lp = 0.2246 mH.
$C1 = 0.68 \mu F$	$Cs = 0.2947 \mu F$
$C2 = 0.52 \mu F$	$Cp = 1.2 \mu F$

Energy Stored in L or C

An inductor carrying current stores magnetic potential energy in the magnetic field surrounding the conductor. Equations 2 and 3 describe the relationship between charge, energy stored in the electric field, and voltage across the capacitor.

1) $E = \frac{L}{2}$	$\frac{ \cdot ^2}{2} \qquad 2) \ Q = C \cdot V$	$3) E = \frac{C \cdot V^2}{2}$
Variable	Description	Units
E	stored energy	1 J
L	inductance	1_H
I	current	1_A
Q	charge on C	1 C
С	capacitance	1_F
V	capacitor voltage	1_V

Example 1: A 4.2 mH inductor carries a current of 1.89 A. What is the energy stored in the inductor? Use equation 1.

Given	Result
L = 4.2 mH	E = 0.0075 J
I = 1.89 A	_

Example 2: A 6.8 μ F capacitor is charged to a level of 2 V. Using equations 2 and 3, find the charge on the capacitor and the energy stored.

Given	Result
$C = 6.8 \mu F$	$Q = 0.0000136_C$
V = 2V	E = 0.0000136 J

DC Circuit Properties

These equations describe two valuable parameters in circuit analysis that complement each other: Thevenin's voltage source and Norton's current source. The equations compute load current, load voltage, power dissipated in the load, and maximum power available to the load from the source.

1) $Vs = Is \cdot Rs$ 2) $IL = \frac{Vs}{Rs + RI}$ 3) $VL = IL \cdot R/$ 6) $PLmax = \frac{Vs^2}{A + Pc}$ 4) $PL = VL \cdot IL$ 5) $PL = IL^2 \cdot RI$

Variable	Description	Units
Vs	source voltage	1_V
ls	short circuit current	1_A
Rs	source resistance	1_Ω
VL	load voltage	1_V
IL	load current	1_A
RI	load resistance	1 <u>Ω</u>
PL	power in load	1_W
PLmax	maximum power available in RI	1_W



Example: A 10_V battery with a 50_ Ω internal resistance is supplying power to a load of 125_ Ω . Find the circuit performance parameters for this circuit.

Given	Result
Vs = 10V	Is = 0.2 A
$Rs = 50 \Omega$	IL = 5.7143E-2_A
$RI = 125_{\Omega}$	VL = 7.1429 V
_	PL = 0.4082 W
	PLmax = 0.5 W

Wheatstone's Bridge

These equations describe the relationship between current and voltage in branches of a Wheatstone's bridge circuit. The equations for Ra, Rb and Rc describe intermediate equivalent values to handle equations for current in the galvanometer circuit. They do not represent any physical resistors.

1)
$$\frac{R1}{R2} = \frac{R3}{R4}$$

2)
$$Ra = \frac{R1 \cdot Rg}{R1 + R3 + Rg}$$

3)
$$Rb = \frac{R3 \cdot Rg}{R1 + R3 + Rg}$$

4)
$$Rc = \frac{R1 \cdot R3}{R1 + R3 + Rg}$$

5)
$$Va = \frac{Vs \cdot (Ra \cdot R4 - Rb \cdot R2)}{Vs \cdot (Ra \cdot R4 - Rb \cdot R2)}$$

5)
$$Vg = \frac{VS (Ra + Rb + R2)}{Rc \cdot (Ra + Rb + R2 + R4) + (Rb + R4) \cdot (Ra + R2)}$$

6)
$$lg = \frac{Vg}{Rg}$$

Variable	Description	Units
Vg	Thevenin voltage	1 V
lg	galvanic l	1_A
Ř1	resistance of arm 1 of the bridge	1_Ω
R2	resistance of arm 2 of the bridge	1_Ω
R3	resistance of arm 3 of the bridge	1_Ω
R4	resistance of arm 4 of the bridge	1_Ω
Rg	resistance in galvanic kg	1_Ω
Ra	equivalent resistance	1_Ω
Rb	equivalent resistance	1_Ω
Rc	equivalent resistance	1_Ω
Vs	source voltage	1_V



Example 1: Four resistors, 1200_{Ω} , 2500_{Ω} , 2000_{Ω} and 4000_{Ω} form the four branches of a Wheatstone's bridge. The bridge is driven by a 10_{V} source. The galvanometer in the bridge link has a series resistance of 10000_{Ω} . Find the galvanometer current and the bridge voltage.

Given	Result
$R1 = 1200 \Omega$	$Vg = -7.4184E-2_V$
$R2 = 2500 \Omega$	lg = -7.4184E-6_A
$R3 = 2000 \Omega$	$Ra = 909.0909_{\Omega}$
$R4 = 4000 \Omega$	$Rb = 1515.1515_{\Omega}$
Vs = 10V	$Rc = 181.8182_{\Omega}$
$Rg = 10000_{\Omega}$	
Note: Ra, Rb and Rc have been calculated	, but have no physical significance.

Example 2: In the Wheatstone's bridge in Example 1, replace R1 by 1250_{Ω} to make a balanced bridge. Find the new galvanometer current and bridge voltage.

Given	Result
$R1 = 1250 _ \Omega$	$Vg = 4.8401E-12_V$

Equation Library

 $\begin{array}{ll} R2 &= 2500_\Omega & & Ig &= 4.84018E\text{-}16_A \\ R3 &= 2000_\Omega & & Ra &= 943.3962_\Omega \\ R4 &= 4000_\Omega & & Rb &= 1509.4340_\Omega \\ Vs &= 10_V & & Rc &= 188.6792_\Omega \\ Rg &= 10000_\Omega \\ Note: \ \mbox{Ra, Rb and Rc have been calculated, but have no physical significance.} \end{array}$

The result for Vg given in the example above represents a calculation within the accuracy of the HP 48SX and should, for practical purposes, be interpreted as 0_V .

RL and RC Circuits

This category covers the response of RC and RL circuits to a step function and converting from series to parallel equivalents.

- RL Circuit Response
- RC Circuit Response
- **RL** Series $\leftarrow \rightarrow$ Parallel Conversion
- **RC** Series $\leftarrow \rightarrow$ Parallel Conversion

RL Circuit Response

These equations describe inductor current and voltage in response to a step function input stimulus. The first two equations characterize an RL series circuit, while the last pair describe a parallel RL circuit.

1)
$$vl = (Vs - Io \cdot R) \cdot e^{-R \cdot t/L}$$

2) $il = \frac{Vs}{R} + (Io - \frac{Vs}{R}) \cdot e^{-R \cdot t/L}$

3)
$$vI = R \cdot (ls - lo) \cdot e^{-R \cdot t/L}$$

Variable	Description	Units
vl	inductor voltage	1_V
L	inductance	1_H

il inductor current 1 A Vs source voltage 1 V resistance R 1Ω current at t = 0lo 1 A t time 1 s ls source current 1 A γi

Example 1: A 120_ Ω resistor and a 0.18_mH inductor are connected in series and are subjected to a 5_V step at t=0. The inductor carries no current before the voltage stimulus. Calculate the current through the inductor and the voltage across it after a time lapse of 1.75_ μ s. Use equations 1 and 2 to solve this problem.

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Given L = 0.18 mH Vs = 5 V $R = 120 \Omega$ lo = 0 A $t = 1.75 \mu s$ **Result** vI = 1.5570_V iI = 2.8692E-2_A

Example 2: A 1500_ Ω resistor and a 0.15_mH inductor are connected in parallel at t = 0 to a current source delivering 176_mA. The inductor carries no current at t = 0. Find the current in the inductor and the voltage across it, 0.75_ μ s after the current stimulus has been applied. Use the last two equations in this set to solve this problem.

RC Circuit Response

These four equations describe the current and voltage response in a series RC and parallel RC circuit to an input voltage step.

1)
$$ic = \frac{Vs - Vo}{R} \cdot e^{-t/(R \cdot C)}$$

2) $vc = Vs + (Vo - Vs) \cdot e^{-t/(R \cdot C)}$
3) $vc = Is \cdot R + (Vo - Is \cdot R) \cdot e^{-t/(R \cdot C)}$
4) $ic = \frac{Is \cdot R - Vo}{R} \cdot e^{-t/(R \cdot C)}$

Variable	Description	Units
vc	capacitor voltage	1 V
С	capacitance	1_F
ic	capacitor I	1 [¯] A
Vs	source voltage	1 [¯] V
R	resistance	1 Ω
Vo	capacitor voltage at $t = 0$	1 V
t	time	1 ⁻ s
ls	source current	1_A



Example 1: A 5_Ω resistor and a 0.18_μ F capacitor are connected in series and are stimulated by a 5_V step function. The capacitor has an initial voltage of -0.5_V . Find the capacitor current and voltage across the capacitor 0.75_μ s after the input stimulus has been applied. Use equations 1 and 2.

 Given
 Result

 $C = 0.18 \mu F$ vc = 2.6097 V

 Vs = 5 V ic = 0.4781 A

 $R = 5 \Omega$ Vo = -0.5 V

 Vo = -0.5 V $t = 0.75 \mu s$

Example 2: A parallel RC circuit, using a $10_k\Omega$ resistor and a 0.33_μ F capacitor with an initial voltage of 0.25_V , is stimulated by a 5.8_mA step

current source. Find the capacitor current after a time lapse of 0.075_ms. What is the voltage across the capacitor?

 Given
 Result

 $C = 0.33 \ \mu F$ $vc = 1.5477 \ V$
 $R = 10 \ k\Omega$ $ic = 5.6452E \ -3 \ A$
 $Vo = 0.25 \ V$ $t = 0.075 \ ms$
 $Is = 5.8 \ mA$ Result

RL Series ←→ Parallel Conversion

These five equations convert a series RL circuit to its parallel equivalent, and vice versa. Equations 1, 2 and 3 help convert a series RL circuit to a parallel equivalent circuit. Equations 3, 4 and 5 convert a parallel RL circuit to its series equivalent.

1)
$$R2 = \frac{R1^2 + \omega^2 \cdot L1^2}{R1}$$

2) $L2 = \frac{R1^2 + \omega^2 \cdot L1^2}{\omega^2 \cdot L1}$
3) $\omega = 2 \cdot \pi \cdot f$
4) $L1 = \frac{R2^2 \cdot L2}{R2^2 + \omega^2 \cdot L2^2}$

5)
$$R1 = \frac{\omega^2 \cdot L2^2 \cdot R2}{R2^2 + \omega^2 \cdot L2^2}$$

Variable	Description	Units
R1	series resistance	1_Ω
L1	series inductance	1_H
R2	parallel resistance	1_Ω
L2	parallel inductance	1_H
ω	radian frequency	1_r/s
f	frequency	1_Hz

$$\begin{array}{c} & & \\ & & \\ & & \\ & & \\ & & \\ \end{array} \end{array} \Rightarrow \begin{array}{c} & & \\ R2 \\ & \\ L1 \\ \end{array}$$

Example 1: An inductor has a series resistance of 0.1_{Ω} and an inductance of 0.015_{mH} . At 1.25_{MHz} . Calculate its parallel equivalent using equations 1-3.

Given	Result
$R1 = 0.1_\Omega$	$R2 = 3515.725_{\Omega}$
L1 = 0.015 mH	$L2 = 1.5000E-5_H$
f = 1.25_MHz	ω = 7853981.634_r/s

Example 2: A 1000_ Ω resistor and an inductor of 0.015_mH are connected in parallel. At 1_MHz, what is its series equivalent? Use equations 3, 4, and 5.

Given	Result
$R2 = 1000_{\Omega}$	$R1 = 0.2249_{\Omega}$
L2 = 0.015 mH	$L1 = 1.4997\overline{E}-5_H$
f = 1.0 MHz	$\omega = 6283185.3072$ r/s

RC Series ←→ Parallel Conversion

Equations 1, 2 and 3 convert a series RC circuit to its parallel equivalent. Equations 3, 4 and 5 convert an RL parallel circuit to its series equivalent.

1) $R2 = \frac{1 + \omega^2 \cdot R1^2 \cdot C2^2}{\omega^2 \cdot R1 \cdot C1^2}$	2) C2 = $\frac{C1}{1 + \omega^2 \cdot R1^2 \cdot C1^2}$
3) $\omega = 2 \cdot \pi \cdot f$	4) $R1 = \frac{R2}{1 + \omega^2 \cdot C2^2 \cdot R2^2}$
5) $C1 = \frac{1 + \omega^2 \cdot R2^2 \cdot C2^2}{2}$	

$$\omega^2 \cdot R2^2 \cdot C2$$

Variable	Description	Units
R1	series resistance	1Ω
C1	series capacitance	1_F
R2	parallel resistance	1 [¯] Ω
C2	parallel capacitance	1_F
ω	radian frequency	1_r/s
f	frequency	1_Hz



Example 1: A 10_ Ω resistor and a 0.015_ μ F capacitor are connected in series. At 1.0_MHz, find its parallel equivalent using equations 1-3.

Given $R1 = 10_{\Omega}$ $C1 = 0.015_{\mu}F$ f = 1.0 MHz

Result R2 = 454.0092_{Ω} C2 = $1.4670E-8_{F}$ $\omega = 6283185.3072_{r/s}$

Example 2: A 10_k Ω resistor and a 0.005_ μ F capacitor are connected in parallel. At 1.00_MHz, what is its series equivalent? Use equations 3-5.

Given	Result
$R2 = 10_k\Omega$	R1 = 3.9984_{Ω}
$C2 = 0.005 \mu F$	$C1 = 0.0050 \mu F$
$f = 1.0$ _MHz	$\omega = 6283185.3072 r/s$

RLC Circuits

This category includes descriptions of steady-state and transient behavior of RLC circuits.

- Impedance Series for RLC Circuit
- Admittance Parallel for RLC Circuit
- Overdamped RLC Circuit
- Critically Damped RLC Circuit
- Underdamped RLC Circuit

Impedance Series for RLC Circuit

The equations below calculate the magnitude and phase of impedance for a series RLC circuit.

1) $\omega = 2 \cdot \pi \cdot f$ 2) $XI = \omega \cdot L$

- 3) $Xc = \frac{1}{\omega \cdot C}$
- 5) Zi = XI Xc

7)
$$\varphi = ASIN\left(\frac{Zi}{Z}\right)$$

4) 21 - 1	4) Zr	=	R
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6)
$$Z = \sqrt{Zr^2 + Zi^2}$$

Variable	Description	Units
ω	radian frequency	1 r/s
f	frequency	1 Hz
R	series resistance	1_Ω
L	series inductance	1 ⁻ H
С	series capacitance	1 ⁻ F
Zr	real part of impedance	1_Ω
Zi	imaginary part of impedance	1_Ω
Z	total impedance	1_Ω
ϕ	phase angle of impedance	1 ⁻ °
XI	inductive reactance	1_Ω
Хс	capacitive reactance	1_Ω



Example: A series RLC circuit consists of a 10_ Ω resistor, a 0.25_ μ H inductor and a 0.0033_ μ F capacitor. What is its impedance and phase angle at 1_MHz?

Given	Result
$f = 1_MHz$	$Zr = 10_\Omega$
$R = 10_{\Omega}$	$Zi = -46.6580 \Omega$
$L = 0.25 \mu H$	$\phi = -77.9031^{\circ}$
$C = 0.0033 \mu F$	$Z = 47.7176 \Omega$
	$XI = 1.5708 \Omega$
	$Xc = 48.2288_{\Omega}$
	$\omega = 6283185.30718$ _r/s
	$Xc = 48.2288_{\Omega}$ $\omega = 6283185.30718_r/$

Note: XI, Xc, and ω are listed for reference. Solutions for ϕ often result in numbers that may seem strange at first; the extraction of angles show results that may be offset in integer multiples of 2π (or 360°).

Admittance Parallel for RLC Circuit

These equations calculate the impedance and admittance of a parallel RLC circuit.

1) $\omega = 2 \cdot \pi \cdot f$ 2) $XI = \omega \cdot L$ 3) $Xc = \frac{1}{(\omega \cdot C)}$ 4) $Yr = \frac{1}{R}$ 5) $Yi = \frac{1}{Xc} - \frac{1}{XI}$ 6) $Y = \sqrt{Yr^2 + Yi^2}$ 7) $\varphi y = ATAN\left(\frac{Yi}{Yr}\right)$ 8) $Z = \frac{1}{Y}$

9)
$$\varphi z = -\varphi y$$

Variable	Description	Units
ω	radian frequency	1 r/s
f	frequency	1 ⁻ Hz
R	parallel resistance	1_Ω
L	parallel inductance	1_H
С	parallel capacitance	1 F
XI	inductive reactance	1_Ω
Хс	capacitive reactance	1_Ω
Y	total admittance	1 [¯] S
Yr	real part of admittance	1 ⁻ S
Yi	imaginary part of admittance	1 ⁻ S
Z	total impedance	1 Ω
φy	admittance phase angle	1 ⁻ °
φz	impedance phase angle	1_°
		-



Example: A parallel RLC circuit has a 10 K Ω resistor, a 0.25_mH inductor and a 0.033_ μ F capacitor in parallel. Find its admittance and impedance at 1_MHz.

Given	Result
f = 1_MHz	$\omega = 6283185.3071 r/s$
$R = 10 k\Omega$	$XI = 157.0746_{\Omega}$
L = 0.025 mH	$Xc = 4.8288_{\Omega}$
$C = 0.033 \mu F$	$Y = 0.20098$ _S
_	$Yr = 0.0001_S$
	Yi = 0.20098 S
	$Z = 4.9756_{\Omega}$
	ϕ y = 89.9715_°
	$\phi z = -89.9715^{\circ}$

Overdamped RLC Circuit

These equations describe the response of an RLC circuit to a step function DC input voltage stimulus.

1) $\alpha p = \frac{1}{2}$	$\frac{1}{\cdot R \cdot C} 2) \alpha s = \frac{R}{2 \cdot L}$	3) $\alpha = \alpha p$
4) $\alpha = \alpha s$	5) $\omega o = \frac{1}{\sqrt{L \cdot C}}$	6) $\omega o = 2 \cdot \pi \cdot f o$
7) s1 = -	$\alpha + \sqrt{\alpha^2 - \omega o^2}$ 8) s2	$= -\alpha - \sqrt{\alpha^2 - \omega o^2}$
9) A 1 = -	$\frac{\left(Vo \cdot s2 + \frac{1}{C} \cdot \left(\frac{Vo}{R} + Io\right)\right)}{(s2 - s1)}$	
10) A 2 =	$\frac{\left(Vo \cdot s1 + \frac{1}{C} \cdot \left(\frac{Vo}{R} + Io\right)\right)}{(s2 - s1)}$	
11) $v = A_1$	$1 \cdot e^{s1 \cdot t} + A2 \cdot e^{s2 \cdot t}$	
Variable R L	Description resistance inductance	Units 1_Ω 1_Η

L	Inductance	1_11
С	capacitance	1_F
αs	Napiere's frequency for a series circuit	1_1/s
α p	Napiere's frequency for a parallel circuit	1_1/s
ωο	natural radian frequency	1 r/s
fo	natural frequency	1 Hz

Equation Library

s1	first natural root	1 1/s
s2	second natural root	1 ⁻ 1/s
A1	constant 1	1_V
A2	constant 2	1_V
Vo	DC stimulus voltage	1_V
lo	current in inductor at $t=0$	1_A
v	time dependent voltage	1_V
t	time	1_s
α	Napiere's constant	1_1/s

Example 1: A parallel RLC circuit has a 100_{Ω} resistance, a 40_{mH} inductance and a 0.25_{μ} F capacitance. Find its natural frequencies. Use equations 1, 3, 5, 6, 7, and 8.

Given	Result
$R = 100_{\Omega}$	$\alpha p = 20000_1/s$
L = 40 -mH	$\omega o = 10000 r/s$
$C = 0.25 \mu F$	$fo = 1591.5494$ _Hz
	s1 = -2679.4919_1/s
	s2 = -37320.5081_1/s

Critically Damped RLC Circuit

These equations describe the response to a DC input step function for a critically damped RLC circuit.

1) $\alpha p = \frac{1}{2 \cdot R \cdot C}$ 2) $\alpha s = \frac{R}{2 \cdot L}$ 3) $\alpha = \alpha p$ 4) $\alpha = \alpha s$ 5) $\omega o = \frac{1}{\sqrt{L \cdot C}}$ 6) $\omega o = 2 \cdot \pi \cdot fo$ 7) $D1 = \alpha \cdot Vo - \frac{1}{C} \cdot \left(\frac{Vo}{R} + Io\right)$ 8) D2 = Vo

9) $v = D1 \cdot t \cdot e^{-\alpha \cdot t} + D2 \cdot e^{-\alpha \cdot t}$

Variable	Description	Units
R	resistance	1Ω
L	inductance	1 ⁻ H
С	capacitance	1_F
α	Napiere's constant	1_1/s

ωο	natural radian frequency	1_r/s
fo	natural frequency	1 Hz
D1	constant 1	1 [¯] V/S
D2	constant 2	1_V
v	time dependent voltage	1 V
t	time	1_s
αp	Napiere's constant, parallel	1_1/s
αs	Napiere's constant, series	1_1/s
lo	inductor current at time $t=0$	1 [¯] A
Vo	source voltage	1_V

Example 1: A series RLC circuit has a 200 ' Ω resistance, a 40_mH inductance and a 0.25 μ F capacitor. Is the circuit critically damped?

Given	Result
$R = 200_{\Omega}$	$\omega o = 10000 r/s$
L = 40 mH	fo = 1591.5494_Hz
$C = 0.25 \mu F$	$\alpha s = 10000_{1/s}$
	α = 10000_1/s

Underdamped RLC Circuit

These equations describe the transient response of an RLC circuit to an input DC stimulus when the circuit is underdamped.

1)
$$\alpha p = \frac{1}{2 \cdot R \cdot C}$$
 2) $\alpha s = \frac{R}{2 \cdot L}$ 3) $\alpha = \alpha p$
4) $\alpha = \alpha s$ 5) $\omega o = \frac{1}{\sqrt{L \cdot C}}$ 6) $\omega o = 2 \cdot \pi \cdot fo$
7) $\omega d = \sqrt{\omega o^2 - \alpha^2}$
8) $v = B1 \cdot \left(e^{-\alpha \cdot t} \cdot COS(\omega d \cdot t)\right) + B2 \cdot \left(e^{-\alpha \cdot t} \cdot SIN(\omega d \cdot t)\right)$
9) $B1 = Vo$ 10) $B2 = -\left(\frac{\alpha}{\omega d}\right) \cdot (Vo + 2 \cdot lo \cdot R)$
Variable Description Units

R	resistance	1_Ω
L	inductance	1_H

С	capacitance	1_F
αs	Napiere's constant for a series circuit	1_1/s
αp	Napiere's constant for a parallel circuit	1_1/s
α	Napiere's constant	1_1/s
ωο	natural radian frequency	1_r/s
ωd	damped frequency	1_r/s
fo	natural frequency	1_Hz
B1	constant 1	1_V
B2	constant 2	1_V
Vo	DC voltage stimulus	1_V
lo	current in inductance at $t=0$	1_A
v	time dependent voltage	1_V
t	time	1_s

Example 1: A parallel RLC circuit has a 400_ Ω resistor, a 40_mH inductor and a 0.25_ μ F capacitor. Find the natural frequency and the damped frequency for this circuit.

Given	Result
$R = 400 \Omega$	$\alpha p = 5000_1/s$
L = 40 mH	$\alpha = 5000_1/s$
$C = 0.25 \mu F$	$\omega o = 10000 r/s$
	$\omega d = 8660.2540 r/s$
	fo = 1591.5494_Hz

Electrical Resonance

This category includes bandwidth and quality factor calculations for series or parallel resonance circuits.

- RLC Resonance
- Q of a Series RLC
- Q of a Parallel RLC

RLC Resonance

This equation set characterizes properties of an RLC circuit at resonance.

1)
$$\omega o = \frac{1}{\sqrt{L \cdot C}}$$
 2) $\omega o = 2 \cdot \pi \cdot fo$ 3) $Zs = R$

4) $Zp = \omega o \cdot C \cdot R^2$

Variable	Description	Units
R	resistance	1Ω
L	inductance	1 ⁻ H
С	capacitance	1 ⁻ F
ωο	natural radian frequency	1_r/s
fo	frequency	1 [–] Hz
Zs	impedance of a series RLC	1 [¯] Ω
Zp	impedance of a parallel RLC	1_Ω



Example: A tank circuit used for an IF transformer in a super heterodyne receiver has a capacitance of $0.1392 \,\mu$ F, an inductance of $0.8782 \,\mu$ H, and has a resistance of $100 \,\text{k}\Omega$. Find the frequency of resonance and the impedance at resonance.

Given	Result
$R = 100 k\Omega$	$\omega o = 2860116.1216 r/s$
$L = 0.8782 \mu H$	fo = 455201.6186 Hz
$C = 0.1392 \mu F$	$Zs = 100000 \Omega$
_	$Zp = 3981.2816 M\Omega$

Q of a Series RLC

This equation set describes the series resonant circuit in terms of the quality factor.

1)
$$\omega o = \frac{1}{\sqrt{L \cdot C}}$$
 2) $Q = \frac{\omega o \cdot L}{R}$ 3) $Q = \frac{1}{\omega o \cdot R \cdot C}$
4) $Q = \frac{1}{R} \cdot \sqrt{L/C}$
5) $\omega 1 = \omega o \cdot \left(\frac{-1}{2 \cdot Q} + \sqrt{1 + \frac{1}{(2 \cdot Q)^2}}\right)$

6)
$$\omega 2 = \omega o \cdot \left(\frac{1}{2 \cdot Q} + \sqrt{1 + \frac{1}{(2 \cdot Q)^2}}\right)$$

7) $\beta = \omega 2$	$2 - \omega 1$ 8) $\omega o = \sqrt{\omega} 1 \cdot \omega 2$ 9)	$\omega o = 2 \cdot \pi \cdot f o$
Variable	Description	Units
ωο	natural radian frequency	1_r/s
fo	natural frequency	1_Hz
Q	quality factor	1
ω1	lower 3dB cutoff radian frequency	1_r/s
ω2	upper 3dB cutoff radian frequency	1_r/s
β	3dB bandwidth radian frequency	1_r/s
Ĺ	inductance	1_H
С	capacitance	1_F
R	resistance	1_Ω
		-

Example : Suppose you have a series RLC circuit with an inductance of 0.8782_{μ} H, a capacitance of 0.1392_{μ} F and a resistance of 0.3_{Ω} . Find its resonance frequency, 3dB bandwidth, and lower and upper cutoff radian frequencies.

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Given	Result
$R = 0.3_\Omega$	$\omega o = 2.8601 Mr/s$
$L = 0.8782 \mu H$	$fo = 455.2016$ _kHz
$C = 0.1392 \mu F$	Q = 8.3725
	ω 1 = 2.6944_Mr/s
	$\omega 2 = 3.0360$ _Mr/s
	eta = 0.3416_Mr/s

Q of a Parallel RLC Circuit

This equation set describes a parallel resonant circuit in terms of quality factor.

1)
$$\omega o = \frac{1}{\sqrt{L \cdot C}}$$
 2) $Q = \omega o \cdot R \cdot C$ 3) $Q = R \cdot \sqrt{C/L}$

4)
$$\omega 1 = \omega o \cdot \left(\frac{-1}{2 \cdot Q} + \sqrt{1 + \frac{1}{(2 \cdot Q)^2}}\right)$$

5) $\omega 2 = \omega o \cdot \left(\frac{1}{2 \cdot Q} + \sqrt{1 + \frac{1}{(2 \cdot Q)^2}}\right)$

6) $\beta = \omega 2 - \omega 1$ 7) $\omega o = \sqrt{\omega 1 \cdot \omega 2}$ 8) $\omega o = 2 \cdot \pi \cdot fo$

Variable	Description	Units
ωο	natural radian frequency	1 r/s
fo	natural frequency	1 Hz
Q	quality factor	1
ω1	lower 3dB cutoff radian frequency	1 r/s
ω2	upper 3dB cutoff radian frequency	1 r/s
β	3dB bandwidth radian frequency	1 r/s
Ĺ	inductance	1 H
С	capacitance	1 F
R	resistance	1_Ω

Example: A parallel RLC circuit is constructed with a $100_k\Omega$ resistor, a 40_mH inductor and a 0.15_μ F capacitor. Find the electrical characteristics of this tank circuit.

R	=	100	kΩ
L	=	40_r	nH
С	=	0.15	μF

Result

 $\omega o = 12909.9445_r/s$ fo = 2054.6815_Hz Q = 193.6492 $\omega 1 = 12876.6542_r/s$ $\omega 2 = 12943.3209_r/s$ $\beta = 66.6667_r/s$

OpAmp Circuits

This category consists of OpAmp circuits that focus on five specific configurations: A basic inverting amplifier, a non-inverting amplifier, a current amplifier, a current to voltage converter, and a voltage to current converter. You can use these OpAmp equations in designing OpAmp building blocks.

- Inverting OpAmp
- Non-Inverting OpAmp
- Current OpAmp
- Current to Voltage Converter
- Voltage to Current Converter

Inverting OpAmp

The equations below represent design equations for an inverting OpAmp. The equations cover ideal and actual OpAmp cases. The impact of non-ideal opamp parameters on Avc is evident from the equations below.

1)
$$Avc = \frac{\frac{-Rf}{R1}}{1 + \frac{1}{\beta \cdot Av}}$$

2) $\beta = \frac{R1}{R1 + Rf}$
3) $Avc = \frac{-\left(\frac{Rf}{R1}\right)}{1 + \frac{Rf + Ro}{\beta \cdot Av \cdot Rf}}$
4) $Rfopt = \sqrt{\frac{Rid \cdot Ro}{2 \cdot \beta}}$
5) $Rin = R1 \cdot \left(1 + \frac{Rf}{Avo \cdot R1}\right)$
6) $Rout = \frac{Ro}{1 + \beta \cdot Av}$
7) $fcp = \frac{fop \cdot Avo \cdot R1}{Rf}$
8) $tr = \frac{0.35 \cdot Rf}{fop \cdot Avo \cdot R1}$
9) $Rp = \frac{R1 \cdot Rf}{R1 + Rf}$
Variable Description Units
Av open loop voltage gain 1

Avo Avc	open loop DC voltage gain closed loop voltage gain	1
AVCO R	foodback ratio	1
ρ fcp	3dB bandwidth	ı 1 Hz
fop	first pole of OpAmp	1_Hz
RÍ	input resistor	1_Ω
Rf	feedback resistor	1_Ω
Rfopt	optimum Rf for minimum gain error	1_Ω
Rid	differential input resistance	1_Ω
Rin	load resistance of circuit	1_Ω
Ro	output resistance of OpAmp	1_Ω
Rout	output resistance	1_Ω
tr	rise time 10-90%	1 s
Rp	optimum resistance	1_Ω



Example 1: For an OpAmp with an input resistance of $10_k\Omega$, a feedback resistance of $50_k\Omega$ and an openloop gain of 100,000, find the closed loop voltage gain and feedback ratio. Use equations 1 and 2.

Given	Result
Av = 100000	Avc = -4.9997
$R1 = 10 k\Omega$	$\beta = 0.1667$
$Rf = 50 k\Omega$	

Example 2: Continuing the example above, if you include a 150_{Ω} output resistance, you get the following results, using equations 2 and 3.

Given	Result
Av = 100000	Avc = -4.9997
$R1 = 10_k\Omega$	$\beta = 0.1667$
$Rf = 50 k\Omega$	
$Ro = 150 \Omega$	

Non-Inverting OpAmp

This equation set provides the key design equations for a non-inverting amplifier. As in the inverting OpAmp case, ideal and practical cases are included.

1)
$$Avc = \frac{1 + \frac{Rf}{R1}}{1 + \frac{1}{\beta \cdot Av}}$$

2) $\beta = \frac{R1}{R1 + Rf}$
3) $Avc = \frac{1 + \frac{Rf}{R1}}{1 + \frac{1}{\beta \cdot Av} + \frac{2 \cdot Rf}{Av \cdot Rid}}$
4) $Avc = \frac{1 + \frac{Rf}{R1}}{1 + \frac{R1 + Rf + Ro}{Av \cdot Rf}}$
5) $Rfopt = \sqrt{\frac{Rid \cdot Ro \cdot Rf}{2 \cdot R1}}$
6) $Rin = \frac{\beta \cdot Av \cdot Rid^2 \cdot Rf}{(Rf + Ro) \cdot (Rid + 2 \cdot \beta \cdot Rf)}$
7) $Rout = \frac{Ro \cdot (Rf + Ro) \cdot (Rid + 2 \cdot \beta \cdot Rf)}{\beta \cdot Av \cdot Rf \cdot Rid}$
8) $fcp = \frac{fop \cdot Avo \cdot R1}{Rf + R1}$
9) $tr = \frac{0.35 \cdot (R1 + Rf)}{fop \cdot Avo \cdot R1}$
10) $Rp = \frac{R1 \cdot Rf}{R1 + Rf} - Rs$
Variable Description Linits
Avo open loop voltage gain 1

closed loop voltage gain	1
feedback ratio	1
3 dB bandwidth	1 Hz
first pole of OpAmp	1 Hz
input resistor	1 Ω
feedback resistor	1 Ω
optimum Rf for minimum gain error	1 Ω
differential input resistance	1 Ω
input resistance of circuit	1 Ω
output resistance of OpAmp	1 Ω
output resistance	1 Ω
rise time 10-90%	1 s
optimum value of Rp	1 Ω
source resistance	1_Ω
	closed loop voltage gain feedback ratio 3 dB bandwidth first pole of OpAmp input resistor feedback resistor optimum Rf for minimum gain error differential input resistance input resistance of circuit output resistance of OpAmp output resistance rise time 10-90% optimum value of Rp source resistance



Example 1: An ideal non-inverting OpAmp has an open loop voltage gain of 1000, a 15 k Ω feedback resistor and a 1 k Ω input resistor. Calculate the feedback ratio and the closed loop voltage gain. Use equations 1 and 2.

Given Av = 1000 $R1 = 1_k\Omega$ $Rf = 15_k\Omega$ **Result** Avc = 15.7480 β = 0.0625

Result

Avc = 15.7094

Example 2: The above amplifier has a differential input resistance of 12 k Ω . Calculate the revised value of closed loop, gain, using equation 3.

Given Rid = $12_k\Omega$ R1 = $1_k\Omega$ Av = 1000 β = 0.0625 Rf = $15_k\Omega$

Current Amplifier

This equation set describes the behavior of a current amplifier.

1) Aic =
$$\frac{1 + \frac{Rr}{Rs}}{1 + \frac{1}{\beta \cdot Av}}$$
 2) $\beta = \frac{Rs}{Rs + R1}$

3) Aic =
$$\frac{(Rs + Rf) \cdot Av}{Rl + Ro + Rs \cdot (1 + Av)}$$

4) Aic =
$$\frac{\text{Rid} \cdot ((\text{Rf} + \text{Rs}) \cdot \text{Av} + \text{Rs})}{(\text{Rid} + \text{Rf}) \cdot (\text{Rs} + \text{RI} + \text{Ro}) + \text{Rs} \cdot (\text{RI} + \text{Ro}) + (\text{Rs} \cdot \text{Rid} \cdot \text{Av})}$$

5)
$$Rin = Rf \cdot (1 + Av)$$

6)
$$\operatorname{Rin} = \frac{\operatorname{Rid} \cdot (\operatorname{Rf} \cdot (\operatorname{Rs} + \operatorname{Rl} + \operatorname{Ro}) + \operatorname{Rs} \cdot (\operatorname{Rl} + \operatorname{Ro}))}{(\operatorname{Rid} + \operatorname{Rf}) \cdot (\operatorname{Rs} + \operatorname{Rl} + \operatorname{Ro}) + \operatorname{Rs} \cdot (\operatorname{Rl} + \operatorname{Ro}) + \operatorname{Rs} \cdot \operatorname{Rid} \cdot \operatorname{Av}}$$

7) Rout =
$$Rs \cdot (1 + Av)$$

 $\mathsf{Rout} = \frac{(\mathsf{Rf} + \mathsf{Rs}) \cdot (\mathsf{Ro} + \mathsf{Rs}) \cdot ((\mathsf{Rid} + \mathsf{Rf}) \cdot (\mathsf{Rs} + \mathsf{Ro}) + \mathsf{Rs} \cdot \mathsf{Ro} + \mathsf{Rs} \cdot \mathsf{Rid} \cdot \mathsf{Av})}{(\mathsf{Rid} + \mathsf{Rf} + \mathsf{Rs}) \cdot (\mathsf{Rf} \cdot \mathsf{Rs} + \mathsf{Rf} \cdot \mathsf{Ro} + \mathsf{Rs} \cdot \mathsf{Ro})}$

8) (below)

Variable	Description	Units
Aic	closed loop current gain	1
Av	closed loop voltage gain	1
β	feedback ratio	1
R1	resistance	1Ω
Rf	feedback resistance	1_Ω
Rid	differential input resistance	1_Ω
Rin	input resistance	1_Ω
RI	load resistance	1_Ω
Ro	output resistance of OpAmp	1_Ω
Rout	output resistance	1_Ω
Rs	resistance	1 [¯] Ω



Example 1: A current amplifier has a 25_k Ω feedback resistor, a load resistance of 1500_ Ω and a source resistance of 50_ Ω . If the open loop gain is 50, find the feedback ratio and current gain. Use equations 1 and 2.

Given	Result
Av = 1000	Aic = 485.9360
$RI = 1500_{\Omega}$	eta = 3.2258E-2
$Rf = 25 k\Omega$	-
$Rs = 50_{\Omega}$	

Example 2: The amplifier described above has an output resistance of 10 Ω . Find the closed loop current gain, in this case. Use equation 3.

Given	Result
$Ro = 10_\Omega$	Aic = 485.8417

Current to Voltage Converter

These equations model a current-voltage converter that provides an output voltage proportional to input current. The circuit is characterized by zero input resistance and zero output resistance for an ideal circuit.

1) Arc =
$$\frac{-(Rf \cdot Rid \cdot Av)}{Rf + Rid \cdot (1 + Av)}$$

2)
$$Rin = \frac{Rf}{1 + Av}$$

3)
$$Rin = \frac{Rid \cdot (Ro + Rf)}{Ro + Rf + Rid \cdot (1 + Av)}$$

4) Rout =
$$\frac{Ro \cdot (Rf + Rid)}{Ro + Rf + Rid \cdot (1 + Av)}$$

Variable	Description	Units
Arc	closed loop transresistance	1_Ω
Av	open loop voltage gain	1
Rf	feedback resistor	1_Ω
Rin	input resistance	1_Ω
Ro	output resistance of OpAmp	1_Ω
Rout	output resistance	1_Ω

vo output voltage Rid input differential resistance





Example 1: A current to voltage converter is being designed using a $100_k\Omega$ feedback resistor, a $12_k\Omega$ differential input resistor, with an output resistance of 250_Ω and an open-loop voltage gain of 10000. Find the input and output resistances and transfer resistance for an ideal converter. Use equations 1, 2 and 4.

Given	Result
Av = 10000	Arc = -99906.7537Ω
$Rf = 100 k\Omega$	$Rin = 9.9990_{\Omega}$
$Rid = 12 k\Omega$	Rout = 0.2331Ω
$Ro = 250_{\Omega}$	_

Example 2: Using the same example, calculate the output and input resistance for the non-ideal converter. Use equation 3 instead of equation 2.

Given	Result
Av = 10000	$Rin = 10.0156 \ \Omega$
$Rf = 100 k\Omega$	Rout = $0.2331^{\circ}\Omega$
$Rid = 12 k\Omega$	-
$Ro = 250^{\circ} \Omega$	

Voltage to Current Converter

This topic describes design equations for a voltage to current amplifier. This circuit is characterized by an output current proportional to input voltage.

1)
$$Agc = \frac{\frac{1}{Rs}}{1 + \frac{1}{\beta \cdot Av}}$$
 2) $\beta = \frac{Rs}{Rs + Rl + Ro}$

3) Agc = $\frac{\text{Rid} \cdot \text{Av} - \text{Rg}}{(\text{Ro} + \text{R1}) \cdot (\text{Rs} + \text{Rid} + \text{Rg}) + \text{Rs} \cdot (\text{Rid} + \text{Rg}) + \text{Rs} \cdot \text{Rid} \cdot \text{Av}}$

4)
$$Rin = Rid \cdot (1 + Av \cdot \beta)$$

5) $Rin = Rid + Rg + \frac{Rs \cdot (Ro + Rl + Rid \cdot Av)}{Rs + Rl + Ro}$

6) Rout = Ro +
$$\frac{Rs \cdot (Rg + Rid \cdot (1 + Av))}{Rs + Rid + Rs}$$

Variable	Description	Units
Agc	transconductance	1_S
Rs	general resistance	1_Ω
β	factor	1
Av	open loop gain	1
RI	current sensing R	1_Ω
Ro	output resistance of OpAmp	1_Ω
Rid	differential input resistance	1_Ω
Rin	input resistance	1_Ω
Rg	resistor	1_Ω
Rout	output resistance	1_Ω



Example: A voltage to current converter needs to be designed with a general resistance of 100_{Ω} , an open loop voltage gain of 1000, a generator resistance of 50_{Ω} a differential input resistance of 500_{Ω} , a load of 320_{Ω} and an OpAmp output resistance of 725_{Ω} . Find the transconductance, transfer factor, and input and output resistance for an ideal converter.

Given	Result
Av = 1000	$Agc = 9.8868E-3_S$
$Rs = 100_{\Omega}$	β = 8.7336E-2
$Rg = 50_{\Omega}$	$Rin = 44168.1223_{\Omega}$
$Rid = 500 \Omega$	Rout = 72232.1429_{Ω}
$Ro = 725 \Omega$	_
$RI = 320 \Omega$	

(φy)

Simple AC Circuits

The simple AC circuit equations cover rules for combining two AC impedance elements. Series and parallel combinations, power dissipation in a load, and power factor calculations are included.

- Impedance (Z) to Admittance (Y) Conversion
- Admittance (Y) to Impedance (Z) Conversion
- Two Impedances in Series
- Two Impedances in Parallel
- AC Circuit Calculations (Current in Load)

Impedance (Z) to Admittance (Y) Conversion

1) $Z = \sqrt{Zr^2 + Zi^2}$	2) $\varphi z = ATAN \left(\frac{Zi}{Zr}\right)$
3) $\varphi y = -\varphi z$	4) $Y = \frac{1}{Z}$

5) $Yr = Y \cdot COS(\varphi y)$	6)	Yi	=	Y	۰S	'IN
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Variable	Description	Units
Z	impedance	1Ω
Zr	real part of impedance	1 Ω
Zi	imaginary part of impedance	1 [¯] Ω
φz	phase angle of impedance	∖ 1 ¯°
Ý	admittance	1 ⁻ S
Yr	real part of admittance	1 ⁻ S
Yi	imaginary part of admittance	1 ⁻ S
<i>Ф</i> у	phase angle of admittance	1_°

Example: Convert an impedance with a real part and imaginary part of 100_{Ω} and 125_{Ω} to an admittance. Find the phase angle.

Given	Result
Zi = 125 Ω	$Z = 160.0781 \Omega$
$Zr = 100^{\circ}\Omega$	Y = 6.2470E-3 S
-	Yr = 3.9024E-3
	Yi = -4.8780E-3_S
	$\phi z = 51.3402^{\circ}$
	$\dot{\phi}y = -51.3402^{\circ}$

Note: ϕz and ϕy are reduced to -180° + 180° range only.

Admittance to Impedance Conversion

Conversion of admittance to impedance is covered in this set of equations.

6) $Zi = Z \cdot SIN(\varphi z)$

- 1) $Y = \sqrt{Yr^2 + Yi^2}$ 2) $\varphi y = ATAN\left(\frac{Yi}{Yr}\right)$
- 3) $\varphi z = -\varphi y$ 4) $Z = \frac{1}{\gamma}$

5) $Zr = Z \cdot COS(\varphi z)$

Variable	Description	Units
Z	impedance	1Ω
Zr	real part of impedance	1_Ω
Zi	imaginary part of impedance	1_Ω
φz	phase angle of impedance	1_°
Ý	admittance	1_S
Yr	real part of admittance	1_S
Yi	imaginary part of admittance	1_S
φy	phase angle of admittance	1_°



Example: Using the values for real and imaginary parts of admittance from the previous example, calculate the impedance.

Given	Result
$Yr = 3.9024E-3_S$	$Y = 6.2469E-3_S$
$Yi = -4.8780E-3$ _S	ϕ y = -51.3402_°
$$Z = 160.0781_{\Omega}$$

 $Zr = 100_{\Omega}$
 $Zi = 125_{\Omega}$
 $\phi z = 51.3402^{\circ}$

Two Impedances in Series

The equations in this topic cover combining two impedances algebraically.

1) $Zr = Z1$	r + Z2r	2)	Zi = .	Z1i + 2	Z2i
3) Zs = $$	$Zr^2 + Zi^2$	4)	φ s =	ATAN	$\left(\frac{Zi}{Zr}\right)$
5) Z1 = √	$Z1r^2 + Z1i^2$	6)	φ1 =	ATAN	$\left(\frac{Z1i}{Z1r}\right)$
7) Z2 = √	$Z2r^2 + Z2i^2$	8)	φ2 =	ATAN	$\left(\frac{Z2i}{Z2r}\right)$
Variable	Description				Units
Z1	impedance of ac element	1			1Ω
Z1r	real part of Z1				1_Ω
Z1i	imaginary part of Z1				1_Ω
ϕ 1	phase angle of Z1				1 ⁻ °
Ž2	impedance of ac element	2			1_Ω
Z2r	real part of Z2				1_Ω
Z2i	imaginary part of Z2				1_Ω
φ2	phase angle of Z2				1_°
Żs	equivalent impedance				1 Ω
Zr	real part of Zs				1_Ω
Zi	imaginary part of Zs				1_Ω
φs	phase angle of Zs				1°



Example: An impedance consisting of 100_{Ω} resistor and 125_{Ω} inductive reactance is connected in series with an impedance with 125_{Ω} resistance

and 180_ Ω capacitive reactance. Find the resulting impedance and phase angle.

Given	Result
$Z1r = 100_{\Omega}$	$Z1 = 160.0781_{\Omega}$
Z1i = 125_Ω	ϕ 1 = 51.3402_°
$Z2r = 125_{\Omega}$	$Z_2 = 219.1461_{\Omega}$
$Z2i = -180_{\Omega}$	ϕ 2 = -55.2222_°
	$Zs = 231.6247_{\Omega}$
	$Zr = 225_\Omega$
	Zi = -55_Ω
	$\phi s = -13.7363^{\circ}$

Two Impedances in Parallel

This equation set finds the result of two impedances connected in parallel.

1) Z1 =	$\sqrt{Z1r^2+Z1i^2}$	2) <i>φ</i> 1 =	$= ATAN\left(\frac{Z1i}{Z1r}\right)$
3) Z2 =	$\sqrt{Z2r^2+Z2i^2}$	4) φ2 =	$= ATAN\left(\frac{Z2i}{Z2r}\right)$
5) Zp =	$\left(\frac{(Z1r \cdot Z2r - Z1i \cdot Z2i)^2 + (Z1r + Z2r)^2}{(Z1r + Z2r)^2}\right)$	+ (Z1r · Z2 + (Z1i + Z2	$\left(\frac{i+Z2r\cdot Z1i}{2i}\right)^{2}$
6) <i>\varphip</i> =	$ATAN \left(\frac{Z1r \cdot Z2i + Z2r \cdot Z}{Z1r \cdot Z2r - Z1i \cdot Z} \right)$	2 <u>1i</u>) – ATA 22i)	$N\left(\frac{Z1i+Z2i}{Z1r+Z2r}\right)$
7) Zr = .	Ζ $p \cdot COS (\varphi p)$	8) <i>Zi</i> =	Zp \cdot SIN (φp)
7) $Zr = A$ Variable	Zp · COS (φp) Description	8) Zi =	Zp · SIN (φp) Units
7) Zr = . Variable Z1	Zp · COS (φp) Description impedance of ac elemer	8) <i>Zi</i> =	$Zp \cdot SIN(\varphi p)$ Units
7) $Zr = .$ Variable Z1 Z1r	Zp · COS (φp) Description impedance of ac elemer real part of Z1	8) <i>Zi</i> =	$Zp \cdot SIN (\varphi p)$ Units 1_{Ω} 1 Ω
7) <i>Zr</i> = . Variable Z1 Z1r Z1i	Zp · COS (φp) Description impedance of ac elemer real part of Z1 imaginary part of Z1	8) <i>Zi</i> =	$Zp \cdot SIN (\varphi p)$ $Units$ 1_{Ω} 1_{Ω} 1_{Ω}
7) $Zr = .$ Variable Z1 Z1r Z1i ϕ 1	Zp · COS (φp) Description impedance of ac elemer real part of Z1 imaginary part of Z1 phase angle of Z1	8) <i>Zi</i> =	$Zp \cdot SIN (\varphi p)$ $Units$ 1_{Ω} 1_{Ω} 1_{Ω} 1_{Ω} 1_{Ω} 1°
7) $Zr = .$ Variable Z1 Z1r Z1r Z1i ϕ 1 Z2	$Zp \cdot COS (\varphi p)$ Description impedance of ac element real part of Z1 imaginary part of Z1 phase angle of Z1 impedance of ac element	8) <i>Zi</i> = nt 1	$Zp \cdot SIN (\varphi p)$ $Units$ 1_{Ω} 1_{Ω} 1_{Ω} 1_{Ω} 1_{Ω} 1_{Ω}
7) $Zr = .$ Variable Z1 Z1r Z1r Z1i ϕ 1 Z2 Z2r	$Zp \cdot COS (\varphi p)$ Description impedance of ac elemer real part of Z1 imaginary part of Z1 phase angle of Z1 impedance of ac elemer real part of Z2	8) <i>Zi</i> = nt 1 nt 2	$Zp \cdot SIN (\varphi p)$ $Units$ 1_{Ω} 1_{Ω} 1_{Ω} 1_{Ω} 1_{Ω} 1_{Ω} 1_{Ω} 1_{Ω} 1_{Ω}
7) $Zr = .$ Variable Z1 Z1r Z1i ϕ 1 Z2 Z2r Z2i Z2i	$Zp \cdot COS (\varphi p)$ Description impedance of ac elemer real part of Z1 imaginary part of Z1 phase angle of Z1 impedance of ac elemer real part of Z2 imaginary part of Z2	8) <i>Zi</i> = nt 1 nt 2	$Zp \cdot SIN (\varphi p)$ $Units$ 1_{Ω} 1_{Ω} 1_{Ω} 1_{Ω} 1_{Ω} 1_{Ω} 1_{Ω} 1_{Ω} 1_{Ω}
7) $Zr = .$ Variable Z1 Z1r Z1r Z1i ϕ 1 Z2 Z2r Z2i ϕ 2	$Zp \cdot COS (\varphi p)$ Description impedance of ac elemer real part of Z1 imaginary part of Z1 phase angle of Z1 impedance of ac elemer real part of Z2 imaginary part of Z2 phase angle of Z2	8) <i>Zi</i> = nt 1 nt 2	$Zp \cdot SIN (\varphi p)$ $Units$ 1_{Ω} 1_{Ω} 1_{Ω} 1_{\circ} 1_{Ω} 1_{Ω} 1_{Ω} 1_{Ω} 1_{Ω} 1_{Ω} 1_{Ω} 1_{0} 1_{0} 1_{0} 1_{0} 1_{0} 1_{0} 1_{0} 1_{0} 1_{0} 1_{0} 1_{0} 1_{0} 1_{0}
7) $Zr = .$ Variable Z1 Z1r Z1r Z1i ϕ 1 Z2 Z2r Z2i ϕ 2 Zp	$Zp \cdot COS (\varphi p)$ Description impedance of ac elemer real part of Z1 imaginary part of Z1 phase angle of Z1 impedance of ac elemer real part of Z2 imaginary part of Z2 phase angle of Z2 equivalent impedance	8) <i>Zi</i> = nt 1 nt 2	$Zp \cdot SIN (\varphi p)$ $Units$ 1_{Ω} 1_{Ω} 1_{Ω} 1_{\circ} 1_{Ω} 1_{Ω} 1_{Ω} 1_{Ω} 1_{Ω} 1_{Ω} 1_{Ω}

Ziimaginary part of Zp ϕ pphase angle of Zp

1_Ω 1_°



Example: Two impedances (212, 185) and (475, -874) are connected in parallel. Find the combined impedances.

Given	Result
$Z1r = 212_{\Omega}$	$Z1 = 281.3699_{\Omega}$
Z1i = 185_Ω	ϕ 1 = 41.1093_
$Z2r = 475 \Omega$	$Z2 = 994.7366_{\Omega}$
Z2i = -874_Ω	ϕ 2 = -61.4769 [°]
	$Zp = 287.6615_{\Omega}$
	$\phi p = 24.7157_{\circ}$
	$Zr = 261.3099_{\Omega}$
	$Zi = 120.2759_{\Omega}$

Current in Load

These equations calculate the current in a load ZL from a voltage source with internal impedance Zg.

Zi = Zgi + ZLi
$\varphi s = ATAN\left(\frac{Zi}{Zr}\right)$
$\varphi g = ATAN \left(\frac{Zgi}{Zgr}\right)$
$\varphi L = ATAN\left(\frac{ZLi}{ZLr}\right)$
)) $\varphi i = -\varphi s$
2) $\varphi V = \varphi L + \varphi i$

14) $VI = VL \cdot IL$

15) $pf = COS(\varphi V + \varphi i)$

Variable	Description	Units
Zg	impedance of ac voltage source	1_Ω
Zgr	real part of Zg	1_Ω
Zgi	imaginary part of Zg	1_Ω
φg	phase angle of Zg	1_°
ZL	load impedance	1_Ω
ZLr	real part of ZL	1_Ω
ZLi	imaginary part of ZL	1_Ω
ϕ L	phase angle of ZL	1_°
Zs	combined series impedance	1Ω
Zr	real part of Zs	1_Ω
Zi	imaginary part of Zs	1_Ω
φs	phase angle of Zs	1_°
IL	current in load	1_A
ϕ i	phase angle of current	1°
VL	voltage across the load	1_V
ϕ V	phase angle of load voltage	1°
PL	power in the load	1_W
VI	volt-amps	1_W
pf	power factor	1
Vg	source voltage	1_V



Example: A 100_V voltage source, with an impedance of (10, 25), drives a load of (30, -40). Calculate voltage across the load, power in the load, and power factor.

Given	
Vg = 100V	
$Zgr = 10_\Omega$	
Zgi = 25_Ω	
$ZLr = 30_{\Omega}$	
$ZLi = -40_\Omega$	

Result IL = 2.3408_A VL = 117.0411_V PL = 267.9677_W pf = 0.9781

Transformers

The equations in this category describe ideal transformers.

1) $\frac{V1}{V2} =$	$\frac{n1}{n2}$ 2) $11 \cdot n1 = 12 \cdot n2$	
3) <i>Rin</i> =	$\frac{R2}{a^2}$ 4) $a = \frac{n2}{n1}$	5) $V2 = I2 \cdot R2$
Variable	Description	Units
V1	primary voltage	1 V
V2	secondary voltage	1 [¯] V
n1	number of turns in primary	1
n2	number of turns in secondary	1
I1	current in primary	1 A
I2	current in secondary	1 [¯] A
R2	secondary load resistance	1 [¯] Ω
Rin	resistance at primary from R2	1_Ω
а	turns ratio	1



Example: An ideal transformer has 20 primary turns and 40 turns in the secondary winding. The input voltage is 5_V , the load secondary resistance is 15_Ω , and the primary current is 0.75_A. Find the secondary current and voltage.

Given	Result
$V1 = 5_V$	V2 = 10V
n1 = 20	I2 = 0.375 A
n2 = 40	$Rin = 3.75 \Omega$
I1 = 0.75 A	a = 2.0
$R2 = 15_{\Omega}$	

Transmission Lines

The transmission line category includes skin effect and ideal transmission line calculations.

- Skin Effect
- Ideal Transmission Line

Skin Effect

The resistance of a conductor carrying a current is distributed uniformly over the cross sectional area at low frequencies. However, at higher frequencies, the self inductance forces the current to crowd toward the surface. Skin depth δ represents effective depth of penetration of the RF signal.

1)
$$\delta = \frac{1}{\left(\frac{\pi \cdot f \cdot \mu o}{\rho}\right)^{\nu_2}}$$

2)
$$Rf = \frac{Rdc}{1 - \left(1 - \frac{\delta}{r}\right)^2}$$

Variable	Description	Units
δ	skin depth	1 m
f	frequency	1_Hz
ρ	resistivity	1_Ω·m
Rf	resistance at frequency f	1_Ω
Rdc	resistance at dc	1_Ω
r	radius of wire	1_m

Example 1: A conductor with a radius of 0.1_cm carries a 50_MHz signal in a material with a resistivity of $0.0000025_{\Omega} \cdot \text{cm}$. Find the skin depth of this material in μ .

Given	Result
f = 50 MHz	$\delta = 11.2539 \mu$
$\rho = 0.0000025 \ \Omega \cdot cm$	_

Ideal Transmission Line

Assumes that the transmission lines are ideal, allowing you to calculate various parameters, such as characteristic impedance, VSWR.

- 1) $Zo = \sqrt{l/c}$ 2) $\beta = \omega \cdot \sqrt{l \cdot c}$ 3) $VSWR = \frac{1-\rho}{1+\rho}$ 4) $Zinqrt = \frac{Zo^2}{Zl}$
- 5) $\rho = \frac{ZI Zo}{ZI + Zo}$ 6) $\omega = 2 \cdot \pi \cdot f$

Variable	Description	Units
Zo	characteristic impedance	1Ω
ZI	load impedance	1_Ω
I	inductance/unit length	1_H/m
С	capacitance/unit length	1_F/m
Zinqrt	input impedance at quarter wave length	1_Ω
β	phase constant	1_r/m
ω	radian frequency	1_r/s
VSWR	Voltage Standing Wave Radio	1
ρ	reflection coefficient	1
f	frequency	1_Hz

Example: An ideal transmission line has a series inductance of 1E-8_H/m, a shunt capacitance of 7.0359E-14_F/m, and a load impedance of 1000_{Ω} . Calculate the transmission line parameters at 100 kHz.

Given	Result
$I = 1E-8_H/m$	$Zo = 376.999_{\Omega}$
c = 7.0359E-14F/m	eta = 1.6666E-5_r/m
$ZI = 1000_{\Omega}$	$Zinqrt = 142.1282_{\Omega}$
f = 100 kHz	ho = 0.4524
	VSWR = 0.37699
	ω = 628318.5307_r/s

Motors and Generators

This category covers basic properties of motors and generators.

Transmission Lines

The transmission line category includes skin effect and ideal transmission line calculations.

- Skin Effect
- Ideal Transmission Line

Skin Effect

The resistance of a conductor carrying a current is distributed uniformly over the cross sectional area at low frequencies. However, at higher frequencies, the self inductance forces the current to crowd toward the surface. Skin depth δ represents effective depth of penetration of the RF signal.

1)
$$\delta = \frac{1}{\left(\frac{\pi \cdot f \cdot \mu o}{\rho}\right)^{V_2}}$$

2)
$$Rf = \frac{Rdc}{1 - \left(1 - \frac{\delta}{r}\right)^2}$$

Variable	Description	Units
δ	skin depth	1 m
f	frequency	1_Hz
ρ	resistivity	1_Ω·m
Rf	resistance at frequency f	1 Ω
Rdc	resistance at dc	1_Ω
r	radius of wire	1_m

Example 1: A conductor with a radius of 0.1_cm carries a 50_MHz signal in a material with a resistivity of $0.0000025_{\Omega} \cdot \text{cm}$. Find the skin depth of this material in μ .

Given	Result
$f = 50_MHz$	$\delta = 11.2539 \mu$
$ ho$ = 0.0000025_ Ω · cm	_

Ideal Transmission Line

Assumes that the transmission lines are ideal, allowing you to calculate various parameters, such as characteristic impedance, VSWR.

1) $Zo = \sqrt{I/c}$ 2) $\beta = \omega \cdot \sqrt{I \cdot c}$ 3) $VSWR = \frac{1-\rho}{1+\rho}$ 4) $Zinqrt = \frac{Zo^2}{ZI}$ 5) $\rho = \frac{ZI-Zo}{ZI+Zo}$ 6) $\omega = 2 \cdot \pi \cdot f$

Variable	Description	Units
Zo	characteristic impedance	1Ω
ZI	load impedance	1 Ω
I	inductance/unit length	1_H/m
С	capacitance/unit length	1_F/m
Zinqrt	input impedance at quarter wave length	1_Ω
β	phase constant	1_r/m
ω	radian frequency	1 r/s
VSWR	Voltage Standing Wave Radio	1
ρ	reflection coefficient	1
f	frequency	1_Hz

Example: An ideal transmission line has a series inductance of 1E-8_H/m, a shunt capacitance of 7.0359E-14_F/m, and a load impedance of 1000_{Ω} . Calculate the transmission line parameters at 100 kHz.

Given	Result
$I = 1E-8_H/m$	$Zo = 376.999 \ \Omega$
c = 7.0359E-14F/m	$\beta = 1.6666 \text{E-5} \text{r/m}$
$ZI = 1000_{\Omega}$	$Zinqrt = 142.1282_{\Omega}$
$f = 100_kHz$	$\rho = 0.4524$
	VSWR = 0.37699
	$\omega = 628318.5307 r/s$

Motors and Generators

This category covers basic properties of motors and generators.

- DC Generators
- DC Motors
- Induction Motors
- Synchonous Machines

DC Generators

These equations govern voltage generation in a DC generator and its relationships to the mechanical energy input.

1)
$$Eg = Kv \cdot \varphi \cdot n$$

2) $Eg = Va \cdot \frac{IL}{Ia} + It^2 \cdot \frac{Rf}{Ia} + Ia \cdot Ra$
3) $T = KT \cdot \varphi \cdot Ia$
4) $KT = \frac{p \cdot Zc}{2 \cdot \pi \cdot ap}$
5) $KT = \frac{60 \cdot Kv}{2 \cdot \pi}$
6) $Eg = \frac{p \cdot Zc}{60 \cdot ap} \cdot \varphi \cdot n$
7) $Kv = \frac{p \cdot Zc}{60 \cdot ap}$
8) $Pm = Pr + T \cdot \Omega$
9) $\Omega = \frac{n}{60} \cdot 2\pi$

Variable	Description	Units
Eg	generated voltage	1_V
Kv	voltage constant	1
ϕ	magnetic flux	1_Wb
Ω	mech angular velocity	1_r/s
Va	terminal voltage	1_V
Ra	armature resistance	1_Ω
IL	load current	1_A
lf	field current	1_A
la	armature current	1_A
Rf	field resistance	1_Ω
Т	torque	1_N∙m
кт	torque constant	1
р	number of poles	1

Zc	number of armature wires	1
ар	number of parallel paths	1
n	rotational speed (rpm)	1
Pm	mech power	1_W
Pr	mech power loss	1_W

Example 1: A DC generator has four poles rotating with an angular velocity of 150_rpm. If the flux at each pole is 0.5_Wb, calculate the generated voltage if the voltage constant is 2.25. Use equation 1.

 Given
 Result

 Kv = 2.25
 Eg = 168.75_V

 ϕ = 0.5_Wb
 ω = 150 rpm

Example 2: For the generator in Example 1, if there are 148 armature wires with four parallel paths, calculate the torque constant and torque with an armature current of 10 A. Use equations 1, 3 and 4.

Given	Result
p = 4	KT = 23.5549
Zc = 148	$T = 117.7747 N \cdot m$
ap = 4	
la = 10_A	

DC Motors

This topic contains eight common equations describing DC motors.

1) $Va = Ia \cdot Ra + Kv \cdot \varphi \cdot \Omega$ 2) $T = KT \cdot \varphi \cdot Ia$ 3) $T = \frac{KT \cdot \varphi}{Ra} \cdot (Va - Kv \cdot \varphi \cdot \Omega)$ 4) $Pin = Va \cdot Ia + Va \cdot If$ 5) $Va \cdot Ia = Eg \cdot Ia + Ia^2 \cdot Ra$ 6) $KT = \frac{60 \cdot Kv}{2 \cdot \pi}$ 7) $Eg \cdot Ia = T \cdot \Omega$

8)
$$T = TL + Tloss$$
 9) $\Omega = 2 \cdot \pi \cdot \frac{n}{60}$

Variable	Description	Units
Va	applied voltage	1_V
la	armature current	1_A
Ra	armature resistance	1_Ω
Kv	voltage constant	1
ϕ	flux	1_Wb
Ω	angular velocity	1_r/s
Т	torque	1_N∙m
KT	torque constant	1
Pin	power input	1_W
lf	field current	1_A
Eg	generated voltage	1_V
TL	load torque	1_N∙m
Tloss	torque loss	1_N∙m
n	speed in rpm	1

Example 1: A DC motor is drawing 10_A from a 100_V source. The armature resistance is 2.5_{Ω} , has a voltage constant of 2.25 and a flux of 0.5 Wb. Find its rotational speed. Use equation 1.

Given $Va = 100_V$ $Ia = 10_A$ $Ra = 2.5_\Omega$ Kv = 2.25 $\phi = 0.5_Wb$ $\begin{array}{l} \textbf{Result} \\ \Omega \ = \ 66.6667 \text{_r/s} \end{array}$

Example 2: Find the generated voltage for this motor. Use equation 5.

Given Va = 100_V la = 10_A Ra = 2.5_Ω **Result** Eg = 75_V

Induction Motors

These equations describe the performance of induction motors.

1)
$$Es = \sqrt{2} \cdot \pi \cdot f \cdot Ns \cdot Kws \cdot \varphi p$$

2) $s = \frac{\Omega s - \Omega r}{\Omega s}$ 3) $\Omega r = \frac{p}{2} \cdot \Omega s \cdot s$

4)
$$\Omega r = s \cdot \Omega$$

5) $Er = \frac{s \cdot Nr \cdot Kwr}{Ns \cdot Kws} \cdot Es$
7) $Pg = \frac{n \cdot Ir^2 \cdot rr \cdot (1 - s)}{s}$

$$6) Pr = Ir^2 \cdot rr$$

3

9) $Pg = T \cdot \Omega s \cdot (1-s)$

8)
$$Pg = T \cdot \Omega r$$

10) $Rin = \frac{rr \cdot (1-s)}{s}$

Variable	Description	Units
Es	secondary voltage	1 V
f	elec frequency	1 [–] Hz
Ns	stator windings	1
Kws	stator winding constant	1
φp	flux/pole	1 Wb
S	slip	1
Ωs	stator angular frequency	1 r/s
Ωr	rotor angular frequency	1 [¯] r/s
Ω	angular frequency	1 [¯] r/s
Er	rotor voltage	1_V
Nr	rotor windings	1
Kwr	rotor winding constant	. 1
Pr	rotor power	1_W
lr	rotor current	1_A
rr	rotor resistance	1_Ω
Pg	gap power	1_W
n	number of phases	1
Rin	equiv input resistance	1_Ω
р	number of poles	1
Т	torque	1_N · m

Example 1: A 60_Hz induction motor has 40 secondary windings, 0.64 Wb of flux, and a stator constant of 1.82411E-2. Find the secondary voltage.

Given Result f = 60 HzEs = 124.4822 VNs = 40Kws = 1.8241E-2 $\phi p = 0.64 \text{ Wb}$

Example 2: The rotor resistance of the induction motor is 0.26_{Ω} and the stator and rotor angular velocities are $126_{r/s}$ and $120_{r/s}$ respectively. What is the slip and input resistance?

Given	Result
$\Omega s = 126 r/s$	s = 0.04762
$\Omega r = 120 r/s$	Rin = 5.2_Ω
$rr = 0.26 \Omega$	

Synchronous Machines

This class of machines is governed by the explicit relationship between the frequency of the AC circuit and speed of the rotation of the motor.

1) $p = \frac{2}{3}$	$\frac{2 \cdot 60 \cdot f}{n}$	2) $\varphi r = \frac{\mu o \cdot Nr \cdot Ar}{g \cdot p} \cdot Ir$
3) <i>K</i> φ =	$\frac{\sqrt{2} \cdot Nr \cdot Ns \cdot \mu o \cdot Ar \cdot G}{\pi \cdot g \cdot p}$	$4) Eg = K\varphi \cdot Ir$
Variable	Description	Units
n	number of poles	1

van a bio	Decomption	011110
р	number of poles	1
f	elec frequency	1_Hz
n	revolutions per minute	1
φr	flux	1_Wb
Ňr	number of rotor windings	1
Ar	rotor cross section	1_m^2
g	gap length	1_m
lr	rotor current	1_A
Kφ	rotor gen constant	1_Ω
Ns	number of stator windings	1
Ωs	elec radian frequency	1_r/s
Eg	voltage	1_V

Example 1: A 4-pole synchronous machine operates at an electrical frequency of 60 Hz. What is its angular velocity in rpm?

Given	Result
p = 4	n = 1800
f = 60 Hz	

Example 2: If the rotor is carrying 150_A current, has 40 windings, has 48_cm² area of cross section and a gap of 0.18_cm, find the flux. Use equation 2.

Given $lr = 150_A$ $g = 0.18_cm$ p = 4 Nr = 40 $Ar = 48_cm^2$ **Result** ϕ r = 5.0265E⁻³_Wb

Solid State Devices

This category lists solid state device equations describing PN junctions, NMOS and CMOS transistors, where the software allows you to calculate intrinsic device currents or voltages and draw the current-voltage characteristics.

- PN Junctions
- Currents in PN Junctions
- NMOS Transistors
- Currents in NMOS Transistors
- CMOS
- BJT-Ebers & Moll Model

PN Junctions

These equations describe PN junctions using step junction approximation.

1)
$$Vbi = \frac{k \cdot T}{q} \cdot LN\left(\frac{ND \cdot NA}{ni^2}\right)$$

2) $xd = \left[\frac{2 \cdot \varepsilon o \cdot \varepsilon si}{q} \cdot (Va - Vbi) \cdot \left(\frac{1}{NA} + \frac{1}{ND}\right)\right]^{V_2}$
3) $CJ = \left[\frac{q \cdot \varepsilon o \cdot \varepsilon Si}{2 \cdot \left(\frac{1}{NA} + \frac{1}{ND}\right) \cdot (Va - Vbi)}\right]^{V_2}$

4) Emax =
$$\left(\frac{2 \cdot q \cdot \left(\frac{NA \cdot ND}{NA + ND}\right) \cdot (Va - Vbi)}{\varepsilon o \cdot \varepsilon Si}\right)^{V_2}$$

5)
$$BV = \frac{\varepsilon Si \cdot \varepsilon o \cdot \varepsilon 1^2}{2 \cdot q \cdot \left(\frac{NA \cdot ND}{NA + ND}\right)}$$

Variable	Description	Units
ND	donor density	1 1/m^3
NA	acceptor density	1_1/m^3
т	temperature	1_K
xd	depletion layer width	1_m
Va	applied voltage	1_V
Vbi	built-in voltage	1_V
CJ	junction capacitance per unit area	1_F/m ^ 2
Emax	maximum field in the depletion region	1_V/m
BV	breakdown voltage	1_V



Example 1: A pn junction is fabricated by a gallium doped p region with a density of 1E19_cm⁻³ and an arsenic doped n region with a density of 1E15_cm⁻³. At room temperature, calculate the built-in voltages and the depletion layer width. Use the first two equations.

$\begin{array}{l} \textbf{Given} \\ \textbf{ND} = 1 \texttt{E15}_\texttt{cm}^{-3} \\ \textbf{NA} = 1 \texttt{E19}_\texttt{cm}^{-3} \\ \textbf{T} = 300_\texttt{K} \\ \textbf{Va} = 0_\texttt{V} \end{array}$

Result Vbi = 0.8179_V xd = 1.0372E-6_m

Example 2: If a reverse bias of 10_V is applied to the diode in Example 1, find the junction capacitance.

Given		
NA =	1E19_	cm ⁻³
ND =	1E15	_cm ⁻³

Result xd = $3.4754E-6_m$ CJ = 3.0318E-5 F/m² Va = 10_V Vbi = 0.8179_V

Currents in PN Junctions

Calculation of currents in PN junctions is based on the minority carrier recombination model developed by William Shockley.

Variable	Description	
3) <i>Jo</i> = 6	$q \cdot ni^2 \cdot \left(\frac{Dp}{ND \cdot Lp} + \frac{1}{2}\right)$	Dn NA · Ln
1) $Jt = J$	$o \cdot \left(e^{\frac{\mathbf{q} \cdot \mathbf{V} \mathbf{a}}{\mathbf{k} \cdot \mathbf{T}}} - 1 \right)$	2) $It = Jt \cdot AJ$

Valiable	Description	Units
Jt	total junction current density	1_A/m^2
Jo	saturation current density	1_A/m^2
Va	applied voltage	1_V
т	temperature	1_K
lt	total current	1_A
AJ	effective junction area	1_m^2
Dp	diffusion length of holes	1_m^2/s
ND	donor density	1_1/m^3
Lp	holes diffusion coefficient	1_m
Dn	electron diffusion coefficient	1_m^2/s
NA	acceptor density	1_1/m^3
Ln	diffusion length of electrons	1_m

Example: A pn junction is constructed with $1E18_cm^{-3}$ acceptors and $1E15_cm^{-3}$ donors. Find the current at 0.25_V forward bias at 300_K. Calculate the saturation current density, if the diffusion length for holes and electrons are 11.4_μ and 8.65_μ and the diffusion coefficients for electrons and holes are $35_cm^2/s$ and $12_cm^2/s$. The junction area is 0.025_cm^2 . Find the current.

Given	Result
Va = 0.25 V	$JT = 4.8877E-3_mA/cm2$
$T = 300 \ K$	$Jo = 3.0855E-4 \ \mu A/cm^2$
$AJ = 0.025 cm^2$	IT = 1.2220E-4 mA
$Dp = 12 cm^{2}/s$	_
$ND = 1E15 cm^{-3}$	
$Lp = 11.4 \mu$	
$Dn = 35 \text{ cm}^2/\text{s}$	

Unito

Equation Library

 $NA = 1E18_{cm}^{-3}$ Ln = 8.65_ μ

NMOS Transistors

These equations describe the behavior of voltage relations in an N channel MOS device. They assume that the physical geometry of the device is a rectangle and second order effects are ignored.

1) We = $W - 2 \cdot \delta W$ 2) Le = $L - 2 \cdot \delta L$

3)
$$Cox = \frac{\varepsilon ox \cdot \varepsilon o}{tox}$$

4)
$$\gamma = \frac{1}{Cox} \cdot \sqrt{2 \cdot \varepsilon o \cdot \varepsilon Si \cdot q \cdot NA}$$

5) $Vt = Vto + \gamma \cdot \left(\sqrt{2 \cdot ABS(\varphi p) + ABS(VBS)} - \sqrt{2 \cdot ABS(\varphi p)}\right)$

6)
$$\varphi p = \frac{-(k \cdot T)}{q} \cdot LN\left(\frac{NA}{ni}\right)$$

Variable	Description	Units
W	drawn width of a MOS transistor	1_m
L	drawn gate length	1_m
δW	width encroachment	1_m
δL	gate length encroachment	1_m
We	effective width	1_m
Le	effective length	1_m
tox	gate oxide thickness	1_Å
Cox	gate capacitance	1_F/m^2
Vt	threshold voltage	1_V
VBS	substrate voltage	1_V
γ	body factor	1_V ^ 1/2
NA	doping density	1_1/m^3
Vto	threshold at $VBS = 0$	1_V
ϕ p	Fermi potential	1 [¯] V
T	temperature	1 ⁻ K



Example: An NMOS device is fabricated with a 10_{μ} width and a 2_{μ} gate length. The lateral diffusion encroachment is 0.27_{μ} , and the gate oxide is 200_Å thick. If substrate doping is 1E15_cm⁻³, find the gate capacitance, Fermi potential, and effective gate length and transitor widths. What is the body coefficient?

Given	Result
$W = 10 \mu$	We = 9.46_ μ
$L = 2 \mu$	Le = 1.46 μ
$\delta W = 0.27 \mu$	$\gamma = 0.1064 V^{1/2}$
$\delta L = 0.27 \mu$	$\phi p = -0.2899 V$
tox = 200 Å	$Cox = 172656.6625 \text{ pF/cm}^2$
$NA = 1E15_{cm}^{-3}$	
T = 300 K	

Currents in NMOS Transistors

These equations describe the behavior of a silicon NMOS transistor. They use a two-port network model, include both linear and non-linear regions in the device characteristics, and are based on a gradual-channel approximation. (The electric fields in the direction of current flow are small compared to the electric fields in the direction perpendicular to current flow). The drain current and transconductance are calculated differently, depending on their region. The geometry of the device is rectangular.

1)
$$IDS = kn \cdot \left((VGS - Vt) \cdot VDS - \frac{VDS^2}{2} \right) \cdot (1 + \lambda \cdot VDS)$$

2) $gds = IDS \cdot \lambda$ 3) $Cox = \frac{\varepsilon o \cdot \varepsilon ox}{tox}$
4) $VDsat = VGS - Vt$

5)
$$gm = \left[Cox \cdot \mu n \cdot \left(\frac{We}{Le}\right) \cdot (1 + \lambda \cdot VDS) \cdot 2 \cdot IDS\right]^{\frac{1}{2}}$$

6)
$$kn = \frac{Cox \cdot \mu n \cdot We}{Le}$$

Variable	Description	Units
We	effective width	1 m
Le	effective length	1 ⁻ m
μn	electron mobility	1_m ^ 2/(V ⋅ s)
EOX	relative dielectric constant oxide	1
VDS	drain to source voltage	1_V
VGS	gate to source voltage	1_V
Vt	threshold voltage	1_V
gds	output conductance	1_S
gm	transconductance	1_A/V
λ	conductance parameter	1_1/V
IDS	drain current	1_A
tox	oxide thickness	1_Å
Cox	oxide capacitance	1_F/m ^ 2
VDsat	saturation voltage	1_V
kn	process constant	1_A/V^2

Example: An NMOS transistor has an effective width of 9.46 μ and a channel length of 1.46 μ . The electron mobility is 500 cm²/v-s. At a gate and drain voltage of 5 V, and at a threshold voltage of 0.75, find the output conductance and drain current. The conductance parameter is 0.1 V⁻¹ and the oxide permittivity is 3.9, and the gate oxide is 250 Å thick.

Given	Result
$\mu n = 500 \text{_cm}^2 \text{/V-s}$	IDS = 12.1241 mA
VDS = 5V	gds = 1.2124E-3_S
VGS = 5V	VDsat = 4.25V
Vt = 0.75 V	$kn = 4.4749E-4_A/V^2$
We = 9.46_ μ	$Cox = 138125.3300 \text{ pF/cm}^2$
Le = 1.46_{μ}	$gm = 4.0344E-3_A/V$
$\lambda = 0.1 V^{-1}$	
$\varepsilon ox = 3.9$	
$tox = 250 \AA$	

CMOS

These equations describe the circuit behavior of a CMOS inverter connected to a capacitive load.

1) $IDSp = kp \cdot (2 \cdot VDS \cdot (VGS - Vtp) - VDS^2)$ 2) $IDSn = kn \cdot (2 \cdot VDS \cdot (VGS - Vtn) - VDS^2)$ 3) $kp = \frac{Wp \cdot \varepsilon \circ \cdot \varepsilon \circ x \cdot \mu p}{2 \cdot Lp \cdot tox}$ 4) $kn = \frac{Wn \cdot \varepsilon \circ \cdot \varepsilon \circ x \cdot \mu n}{2 \cdot Ln \cdot tox}$ 5) $Vin = \frac{VDS - Vtp + Vtn \cdot \sqrt{kn/kp}}{1 + \sqrt{kn/kp}}$

Variable	Description	Units
IDSp	drain current in p device	1_A
kp	process constant p-MOS	1_A/V ^ 2
VDS	drain to source voltage	1_V
VGS	gate to source voltage	1_V
Vtp	p-channel threshold voltage	1_V
IDSn	drain current in n device	1_A
kn	process constant n-MOS	1_A/V ^ 2
Vtn	n-channel threshold voltage	1_V
Wp	width of P-MOS device	1_m
μ p	hole mobility	1_m ^ 2/(V ⋅ s)
Lp	gate length of n-MOS device	1_m
tox	gate oxide thickness	1_Å
Wn	width of n-MOS device	1_m
μn	electron mobility	1 [¯] m^2/(V⋅s)
Ĺn	gate length of p-MOS device	1_m
Vin	input voltage when IDSN = IDSP	1_V



Example: A CMOS inverter is designed with a p-channel threshold of -0.75_V and an n-channel threshold of 0.75_V. The transistor sizes for p and n are 10 x 2 and 4 x 2 in microns, respectively. Find the drain currents when the input voltage is 3_V and VDS = 5_V, and find the trip level.

Given VDS = 5_V VGS = 3_V Vtp = 0.75_V Vtn = 0.75_V Wp = 10_ μ μ p = 200_cm²/(V · s) Lp = 2_ μ tox = 200_Å Wn = 4_ μ μ n = 500_cm²/(V · s) Ln = 2_ μ

Result

$$\begin{split} IDSp &= 4.3703E\text{-}4_A\\ IDSn &= 4.3703E\text{-}4_A\\ kp &= 8.6328E\text{-}5_A/V^2\\ kn &= 8.6328E\text{-}5_A/V^2\\ Vin &= 2.50_V \end{split}$$

BJT- Ebers and Moll Equations

These equations describe the behavior of the NPN silicon bipolar transistor. They are based on the original large-signal model developed by J.J. Ebers and J.L. Moll.

1)
$$|E = -|ES \cdot \left(e^{-\left(\frac{q \cdot VBC}{k \cdot T}\right)} - 1\right) + \alpha R \cdot |CS \cdot \left(e^{-\left(\frac{q \cdot VBC}{k \cdot T}\right)} - 1\right)$$

2) $|C = -|CS \cdot \left(e^{-\left(\frac{q \cdot VBC}{k \cdot T}\right)} - 1\right) + \alpha F \cdot |ES \cdot \left(e^{-\left(\frac{q \cdot VBC}{k \cdot T}\right)} - 1\right)$
3) $|S = \alpha F \cdot |ES$ 4) $|S = \alpha R \cdot |CS$
5) $|B + |C + |E = 0$
6) $|CO = |CS \cdot (1 - \alpha F \cdot \alpha R)$ 7) $|CEO = \frac{|CO|}{1 - \alpha F}$

8) VCEsat =
$$\frac{k \cdot T}{q} \cdot LN \left(\frac{1 + \frac{1C}{1B} \cdot (1 - \alpha R)}{\alpha R \cdot \left(1 - \frac{1C}{1B} \cdot \left(\frac{1 - \alpha F}{\alpha F}\right)\right)} \right)$$

Variable	Description	Units
IE	total emitter current	1_A
IES	emitter-to-base saturation current	1_A
VBE	base-to-emitter voltage	1_V
αR	reverse common-base current gain	1
ICS	collector-to-base saturation current	1_A
VBC	base-to-collector voltage	1_V
IC	total collector current	1_A
αF	forward common-base current gain	1
IS	transistor saturation current	1_A
ICO	collector current	1_A
ICEO	CBopen collector current	1_A
VCEsat	collector-to emitter saturation voltage	1_V
Т	temperature	1_K
IB	total base current	1_A



Example: A bipolar transistor has a base current of 10_mA and a collector current of 11_mA. If the forward and reverse common emitter gains are 0.95 and 0.05 respectively, find the saturation voltage at 300_K.

 Given
 Result

 $IB = 10_mA$ VCEsat = 9.7482E-2_V

 $IC = 11_mA$ $\alpha F = 0.95$
 $\alpha R = 0.05$ $T = 300_K$

Chapter 3 Constants Library

In This Chapter

The Constants Library is a collection of physical constants commonly used in electrical engineering. This chapter covers:

- □ Types of Constants
- Using the Constants Library
- □ Using the ECON Function
- Constants Library Softkeys

Types of Constants

The Constants Library lists the symbols, descriptions and SI units of four types of constants, shown below:

Universal Constants

R	Universal gas constant
NA	Avogadro's number
С	Velocity of light
h	Plank's constant
k	Boltzmann's constant
hb	Dirac's constant

Physical Constants

q	Charge of an electron
<i>E</i> 0	Permittivity in vacuum
me	Electron rest mass
re	Classical electron radius
mp	Proton rest mass
R∞	Rydberg's constant

α	Fine structure constant
ao	Bohr radius
μB	Bohr magneton
λ	Wavelength of 1eV quantum
λς	Compton's wavelength
σ	Stefan-Boltzmann's constant
c1	First radiation constant
c2	Second radiation constant
Vt	Thermal voltage at 300 K

Silicon Properties

N	Atoms/cm^3
AW	Atomic weight
Siden	Density
а	Lattice parameter
εSi	Relative permittivity
Nc	Eff density of states in conduction band
Nv	Eff density of states in valence band
ml	Longitudinal eff mass of electrons
mt	Transverse eff mass of electrons
mlh	Eff mass of light holes
mhh	Eff mass of heavy holes
ϕ	Electron affinity
Eg	Bandgap at 300 K
ni	Intrinsic carrier concentration
lphath	Linear coefficient of expansion
μn	Drift mobility of electrons
μh	Drift mobility of holes
MP	Melting point
BP	Boiling point
kth	Thermal conductivity
spht	Specific heat
hosi	Work function
<i>ɛ</i> 1	Critical field in PN junction
εοχ	Relative permittivity
rad	Radians

Magnetic Properties

μο	Permeability of vacuum
φo	Magnetic flux quantum
F	Faraday's constant
μe	Electron magnetic moment
μP	Proton magnetic moment

Using the Constants Library

Select CONSTANTS LIBRARY from the main menu screen. The Constants Library menu displays four classes of constants:

Cons +UNIVERSAL PHYSICAL C SILICON PR MAGNETIC	tants Libra CONSTANTS ONSTANTS OPERTIES PROPERTIES	iry
	FONT	UP

Example: Suppose you want to find the density of pure silicon. Use the cursor keys to move the pointer to SILICON PROPERTIES and press **ENTER** to display the following screen:

 Silicon Properties
N : ATOMS/CM^3
→SIDEN : DENSITY
A : LATTICE PARAM ESI : REI PERMITTIVITY
NC : DEN DE STATES-CB
NV : DEN DF STATES-VK ML : LONGITUDINAL MASS
MAIN VALUE >STK FONT UP

Move the pointer to SIDEN. Five softkeys are available at this level and are described at the end of this chapter. To view the value for the SIDEN constant, press the **VALUE** softkey. This results in the following display:



To place the value of SIDEN on the stack, press ENTER or the STR

softkey. The screen flashes a "Value to stack" message, places the value on the stack as a tagged object, then returns to the SILICON PROPERTIES menu. The value(s) you entered on the stack become available for calculation when you exit the Pac. To remove the tag once the value is on the stack

press PRG OBJ NXT DTAG

Using the ECON Function

You can extract the value of any constant without entering the Electrical Engineering Application Pac with the ECON() function. In all cases, the constant name must be prefixed with a '\$' symbol, accessed by \square \square \square For example, suppose you want to retrieve the speed of light:

User Program Method

Inside a user program, use the commands '\$c' ECON or 'ECON(\$c)' EVAL to call for the speed of light.

Stack Method

Type '\$c' into level 1 of the stack and press the **ECON** library softkey or type the letters ECON and press **ENTER**.

The constant value will have SI units if units are selected (i.e., if flag 60 is clear); otherwise, the value will have no units.

Constants Library Softkeys

- VALUE Displays the value of the constant with units on the screen. Press ENTER to return to the constants list.
- **STK** Places a copy of the selected constant on the calculator stack. Whether or not the value has units appended is controlled by the units key setting, which can be toggled at the Equation Library screen.

FONT Toggles between large and small display font.

- **UP** Moves up one level in the menu structure.
- MAIN Exits to the main menu.

Notes:

Chapter 4 Circuit Analysis Tools

In This Chapter

This set of tools solves some common problems found in electrical engineering. The following sections should be read in order, since some topics common to all sections are discussed first.

- □ AC Circuit Analysis
- □ Fourier & Laplace Transforms/Gain and Phase Plots
- Ladder Network Analysis
- **Transmission Lines**
- □ Two-Port Networks

The first five topics or "tools" in this chapter are accessed directly from the main menu. The screen below shows the options available on the main menu; the 'resume solving' option only appears if you've been using the equation library previously.

EL →RESU EQUA AC C FOUR LADD TRAN TWD CONS	ECTI ME SO ITION L IRCUITS IER/LA ER NE ISMISSI -PORT TANTS	RICA LYING IBRAR PLACE TWORK ION LII NETWO LIBRA	L EN TRANS ANAL NES IRKS BY	IG AF	P
				FONT	QUIT

Softkeys to access gain plot and phase plot functions are available at the opening screen, prior to starting the EE Application Pac program.

{ HOME	}
4:	
3:	
2:	
1:	\downarrow \downarrow
EENPP E	ON DEREC DERUE GPLD PHPLD

AC Circuit Analysis

Conversions between wye and delta, single phase to three phase, combinations of series and parallel impedance and admittance, and power analysis are provided under this topic.

AC Circuit Performance (Z)

A simple AC circuit can be modeled as a source voltage Vs and a source impedance Zs which appears in series with the voltage source. This source drives a load impedance ZL.



When these three variables are specified, a variety of circuit performance properties can be calculated. These values can be complex numbers, which complicates the calculations when they're done by hand. Since the HP 48SX handles complex numbers directly, much of the tedium of working these problems is eliminated.

IMPORTANT!

When entering values for AC voltage, it's essential to decide what input values you will use, and then to be consistent throughout the calculation. In the following examples, the assumption is made that all AC values are entered in volts RMS, DC values are in volts, and impedances are in ohms.

There is no check on the consistency of units in these sections; the HP 48SX does not provide that feature when working with complex numbers.

Entering Data

Voltages, currents and impedances may be either real or complex values. In electrical circuits, real numbers represent the resistive component of a voltage or current, and complex numbers represent the reactive (inductive or

capacitive) component. Most real-world values contain both real and reactive components.

These complex numbers can be entered two ways, as a real part and imaginary part, or as a scalar vector and phase angle. In electrical engineering, complex numbers are shown in a notation like 5 + j22, where 5 is the real part, and 22 is the complex part. The letter 'j' represents the square root of -1, commonly known as 'i' by mathematicians. Engineers use 'j' because 'i' is usually used to denote a current. Complex numbers may also be shown as a 'phasor', such as $22.6 \angle 77$, meaning a magnitude of 22.6 at an angle of 77 degrees from the horizontal.

Both rectangular (5,22) and polar (22.6, \angle 77) modes are supported in the HP 48SX calculator. To enter a complex number in rectangular notation, press and enter the real part, followed by the imaginary part. Separate the real and imaginary parts with either a space or a comma. To enter the number in polar notation, press (1) and enter the magnitude, then the \angle symbol, then the phase angle in degrees.

The calculator knows about the square root of -1, which it calls 'i', instead of 'j'. When looking at examples in the calculator owner's manual, keep this difference in mind.

A simple example; the hair dryer

Let's try a simple example to get the feel of it: A hair dryer plugged into a wall socket.

A →¥S: ZS: ZL:	СС	kt P	erf.	(Z)	
CALC				FONT	UP

From the main menu, move to AC CIRCUITS and press ENTER. Press ENTER again at AC CKT PERFORMANCE (Z).

Press ENTER a third time to set Vs:

{ HOME EEAPPD }	PRG
Enter Vs:	
110	
€SKIP SKIP→ €DEL DEL→ INS ■	ASTK

Enter 110 volts for Vs, (standard U.S. wall-plug voltage). The phase angle of Vs needs to be referenced somewhere, so consider it zero. Since it's zero, just enter 110, and then press **ENTER**. It's a real number, so you don't see an imaginary part.

{ HOME EEAPPD }	PRG
Enter Zs:	
(<u>.5∡5)</u>	
(€SKIP SKIP→ €DEL DEL→	INS • ASTR

Zs is the source impedance, in this case the impedance looking into the wall plug. Probably about .5 Ω , with maybe 5 degrees or so of inductive reactance. With the pointer at Zs, press **ENER**. To set polar display mode, press **G** MODES **ENTR**. You'll see a square appear next to R∠Z, indicating that the calculator will now display complex numbers in polar mode.

To enter .5 Ω at 5 degrees, press $\square \square$.5 $\square \angle 5$ [MIF]. The data can be entered in either rectangular or polar coordinates, independent of the display mode. The data is displayed by the calculator in the requested mode, but it is always stored internally in the rectangular coordinate system.

Now the pointer is set to ZL, the load impedance. It's a heating coil and a motor in parallel, so it has both resistive and inductive components. The coil is about 7 Ω and the motor is probably about $1 + j100\Omega$, if it's driving a big fan. In parallel, this works out to (6.961,0.4869).

R∡Z { Home Eeappd }	PRG
Enter ZL:	
(6.961.4869) Eskipiskipe foel (del + ins d:	↑STK

Press ENTER then (1) 6.961 .4869 ENTER. You'll see the value expressed in polar form as $(6.9780, \angle 4.0011)$.

F ¥S: : ≯ZS: i ZL: i	AC Ckt 110.0000 0.5000.2 6.9780.2	Perf 5.0000) 4.0011)	. (Z)	
	C)	ilC	FONT	UP

Press CALC to set the calculator to work:

(→YL IL : P PF PFA ZLX	kt (14. 150 150 150 105 105 0.9 6 60 60 5	Perf 2.6461, 7099,4 9.9175 6.2374 .3566 376 73.1100 6000,4-	Parar 2-0.0668) -4.0679) 5.0000)	ńS
			→STK FON	T UP

Examining the Results

Voltage across the load impedance.
Current through the load.
Volt-ampere product in the load (apparent power).
Real power (the part that causes smoke).
Reactive power.
Power factor, a ratio of real power to apparent power.
Theoretical maximum power deliverable from this source.
Load impedance for maximum power transfer from source.

VL is about 102 volts, since part of the voltage is dropped across the source impedance. IL is almost 15 amps, close to the limit for a single residential circuit. VI, the volt-ampere product, is 1509 volt-amperes. P, the real power, is 1506 watts, a truly impressive hair dryer.

RP, the reactive power, is 105 VARS. PF, the power factor, is .9976, indicating an almost-entirely resistive load. Not surprising, since it's a heating coil. Pmax is 6073 watts, representing the maximum power available from this wall-outlet source. This would be available into a load of $(.5, \angle -5)$, which is the value given by ZL*, the complex conjugate of Zs. If you enter the value given by ZL* for Zs and recalculate, you'll find IL to be 110 amps.

Inline Computation

You can also do in-line computations when entering values. For example, enter the combination of 7 Ω and 1+j100 Ω for ZL. Press **UP** to back out to the data entry screen. Select ZL and press **ENTER**, then press **DEL** or **ATH** to clear the line.

R∡Z { Home Eeappd }	PRG
Enter ZL:	
7 / 1(1 100) / +	/
+SKIP SKIP→ +DEL DEL→ INS ■ /	NSTK.

Now enter the equation 1/(1/7 + (1/(1+j100))) for the two impedances in parallel by typing $1 \ 1 \ 7 \ \vdots \ 1 \ 100$ $\blacktriangleright \ \vdots \ + \ \vdots \ ENTER$. If this seems

confusing, study the HP 48SX manual regarding Reverse Polish Notation. The result should be $(6.9781, \angle 4.0010)$.

In general, computations can be performed at any of the data entry screens except for those which request a list to be entered. If an error is made, the calculator will return an error message and request the data again.

Display Modes

You may want to change the display mode to show the results in a different notation, such as fixed-point, scientific or engineering. To switch to a different notation, press **ENER** at any data entry line (except for list entry). At the end of the line of data, add a command, like '2 FIX' (fixed point, 2 digits after decimal) or '3 ENG' (engineering notation, 3 digits after decimal).



The default is whatever the calculator was set for when you started the EE application pack software.



AC Circuit Performance (Y)

Admittance is the reciprocal of impedance, and it's a convenient unit to use with parallel loads. The admittances of parallel loads simply add together, giving you the equivalent load of all the parallel elements.

As in the preceding example, three variables must be specified. The source current is Is and the source admittance is Ys. These drive a load admittance, YL. The SI unit for admittance is the Siemens, but many textbook still use 'mho' (ohm, spelled backwards).


For example, imagine a current source of 1_mA with a source admittance of .0001_mho (a 10_k Ω resistor in parallel) and three loads; a 2.7_k Ω resistor, a .1_ μ F capacitor and a 1_mH inductor. Assume a frequency of 10_kHz, and the capacitor becomes j6.283E-03_mho, and the inductor is -j15.92E-03_mho. The resistor admittance is 370.4E-06_mho. Enter the sum of these three values for YL.

Press ENTER at AC Ckt Perf. (Y) to start that topic. Press ENTER at Is and type .001 (remember, it's in amps) then press ENTER. Move the pointer to Ys and enter .0001. Move down to YL and enter the following:

370.4E-06	(0,-15.92E-03)	(0,6.283E-03)	+ + ENTER
(resistor)	(inductor)	(capacitor)	(add together)

The result should be $(9.6441\text{E}-3, \angle -87.7990)$ in polar coordinates or (3.704E-4, -9.637E-3) expressed in rectangular terms.



Press CALC. As before, the results for the combined load are displayed:

AC Ckt Perf. (Y)
→YL : (0.0051.0.1035)
YI : 0.0001
P : 3.9788E-6
PF : 0.0001
PMAX: 0.0025
YLX: (0.0001,0.0000)
I →STK FONT UP

VL is the load voltage, about 100_mV.

IL is the load current, about 1_mA.

VI is the apparent power in the load, about .1_mW.

P, the real power is about $4_{\mu}W$, and reactive power is about $100_{\mu}W$.

PF, the power factor is about .04, so the vast majority of the 'power' is reactive, through the inductor and capacitor. Their admittances are much greater than that of the $2.7_K\Omega$ resistor, so it makes sense.

Pmax is the maximum power from this source; about 2.5_mW.

YL* is the load admittance that would produce maximum power transfer from the source, given its source admittance Ys. Power transfer is maximized when the real parts of Ys and YL are equal, and their imaginary parts are opposite in sign; that is, YL* is the complex conjugate of Ys.

Z's (Impedances) in Series

This calculation determines the voltage drop across each impedance in a series string. It takes two arguments; a voltage V and a list of impedances. Let's plug in four impedances and see what we get.

A stereo amplifier has an output voltage of 15_V, for example, and it's driven with a 2_kHz signal for test purposes. On the output is 100 feet of two-conductor wire, an 8_{Ω} "tweeter" speaker and a 10_µF capacitor to block the low frequencies.

 $\begin{array}{l} 100_ft \mbox{ wire } x .005_\Omega/ft = .5_\Omega \\ tweeter = 2 + 2 \cdot \pi \cdot 2000 \cdot 600_\mu H = (2 + j7.54)_\Omega \\ capacitor = .01 + 1/(2 \cdot \pi \cdot 2000 \cdot 10_\mu F) = (.01 - j7.96)_\Omega \\ 100_ft \mbox{ wire } x .005_\Omega/ft = .5_\Omega \end{array}$

First enter V, 15. Next, enter the four impedances inside the brackets. { .5 (2,7.54) (.01,-7.96) .5 }



Press **CALC** to see the voltage across each component. The four voltages V1 to V4 correspond to the four impedances in the order they were entered:



From the results in the screen above, it's apparent that the wires leading to the speaker cause a substantial voltage drop (V1 and V4). In a real stereo system, low-resistance wires would improve performance. The voltage drop across the wires represents wasted power and lower output level.

The speaker (V2) and the capacitor (V3) are splitting the rest of the voltage drop about equally. At lower frequencies, the capacitor bears most of the load, and at higher frequencies the speaker takes over. In a real sound system, multiple sizes of speakers are used, along with a 'crossover' network which routes low and high frequencies to the appropriate-sized speaker.

Y's (Admittances) in Parallel

Series calculations are easiest to handle with impedances; they're just added up. Similarly, parallel admittances add up too, so they work out well for parallel kinds of problems. This calculation figures the current through each admittance in a parallel set.



Imagine a 10 mA current source driving three parallel load resistances; 100, 270 and 910 Ω . What's the current through each load? Take the reciprocal of each impedance to get the admittance.

1/100 = .01 1/270 = .0037 1/910 = .0011

First, enter the current Is (10E-3). Next, enter the admittances. Since this is a list, you can't do in-line calculations, so you need to already know admittance. If you had calculated the values outside of the Electrical Engineering Application Pac, you could leave them on the stack in a list. Then, enter this topic, and retrieve the list you left on the stack by pressing **SIK ECHO** and **M**. You may need to edit out an extra set of brackets. For the purpose of this example, just type in the numbers: { .01 .0037 .0011 }



Press **CALC**, and three currents are the result, one for each admittance:



It's no surprise that the 100_{Ω} (.01_mho) admittance gets most of the current.

Z's (Impedances) in Parallel

This works like the last section, but the loads are expressed in impedance terms (ohm) instead of admittance. This saves you from having to convert to admittance if the original data is for impedance. Try the same example as in the previous section. The results should come out the same.



Phase Conversions, 3ϕ -1 ϕ and 1ϕ -3 ϕ

Power systems often use three-phase power for the generation and transmission of electricity, and industrial plants commonly use it in large motors and machines. Many analyses are simplified if only one phase is considered, but this requires that load conditions be re-computed for a single-phase-equivalent load.

3ϕ to 1ϕ Conversion

The 3ϕ - 1ϕ conversion requires two arguments; line-to-line voltage, and phase current.

3ø → 1 →yøa: 240 Iøa: (6,2)	ØC	onver	-sio	Π
	CALC	F	ONT	UP

As an example, enter a value of 240 volts line-to-line for V ϕ 3, and (6,2) amps for I ϕ 3. This implies a capacitive load, since current is leading voltage (the reactive component is positive, and the voltage is referenced to zero).

Press the **CALC** softkey to compute the following results:



Examining the Results

VARS	Total volt-ampere product in the load.
PF	Power factor, where 1 is purely resistive and 0 is purely reactive.
V1	Equivalent line voltage that this load would see if connected to a single-phase source. This is the same as the line-neutral voltage for a Y-connected load or line-line voltage for a Δ -connected load.
I1	Equivalent load current that this load would carry if connected to a single phase source. This is the same as the line current for a Y-connected load, or the line current for a Δ -connected load divided by $\sqrt{3}$.

1 ϕ -3 ϕ Conversion

The 1ϕ - 3ϕ conversion requires two arguments; load voltage and load current.

Use the hair dryer example again. What would the conditions be for three hair dryers connected as a balanced load in both Y and Δ ?



Let's plug in 110 volts for $V\phi 1$, and (14.6,-1) for $I\phi 1$. Press **CALC**. The results are shown in the screen below:



Examining the Results

VARS	Volt-amperes reactive, is 4829. That's three hair dryers' power, since these are configured as a balanced three-phase load.
PF	Power factor is nearly 1. Given the relatively small reactive component to IL, that makes sense.
V3	Line-to-line voltage, if these loads were Y-connected. The load current would remain the same, (14.6,-1). The voltage is higher because in Y-connection the voltage is impressed across multiple loads simultaneously.
13	Phase current, if these loads were Δ -connected. Line-to-line voltage would still be 110 volts. The current is higher because in Δ -connection the current splits between two loads.

Impedance conversion, Y- Δ and Δ -Y

Circuit analysis sometimes requires that a Y-connected set of impedances be converted to their equivalent in a Δ -connection, or vice versa. These calculations take the Y or Δ -connected impedances and transform them to the other form.



The transformation between Y and Δ is such that from the outside terminals A, B and C it is not possible to tell which way the loads are connected. Since they are equivalent, circuits can often be simplified by substituting one for the other.



In a Y connection, Z1 is 15 Ω , Z2 is j30 Ω and Z3 is 25-j20 Ω . The equivalent Δ parameters are:

 $\begin{array}{l} {\sf ZA} \ = \ 65.0 \ + \ j60.0 \ \Omega \\ {\sf ZB} \ = \ 30.0 \ - \ j32.5 \ \Omega \\ {\sf ZC} \ = \ 6.22 \ + \ j41.0 \ \Omega \end{array}$



Try these values in a Δ -Y conversion to get Z1, Z2 and Z3 again.

Fourier and Laplace Transforms

The section on Fourier and Laplace transforms has tabular data on common transforms. Pole/zero entry can be used to derive a transfer function, and gain/phase plots can be created based on the derived function. A finite

Fourier transform is provided to create a transfer function from data points, and the inverse function can be used to recreate a set of data from a known transfer function.

Laplace Transform Pairs

This section lists transform pairs in s-plane and time domain forms. Use **G** We see the rest of the line if it extends off the end of the screen. These transforms are used to reduce complex differential equations in the time-domain into simple algebraic expressions in the frequency domain. Although there are many Laplace transforms, these are the ones most commonly used for circuit analysis.

+Lap	lace	Tr	anst	orm	Pa
→1 //s~	₂┊┨				
1750	N : T^N -A) :	I-1/(EXP(N-1)! A¥T)		
178	-Å)^N : ^2+A^2)	T^Q	(-1)XE	XP(AXT) Za)/(N·
ŝżč	~Ž+Å~Ž)	÷ ĝ	SCANT		
		• 31	÷S1K	FONT	UP

Inverse Transfer Functions

This section lists selected transfer function pairs, with the s-domain function on the left, and the equivalent time-domain expression on the right. These transforms are used when returning from the frequency domain to the time domain after analyzing the response of a circuit.



Pole-Zero Analysis

In this section a transfer function can be derived from the poles and zeros of an s-parameter function. A constant multiplier is entered (1 is typical), along with a list of poles and a list of zeros, mapped in the s-plane. This transfer function can be used as input to the phase and gain plot functions.

As an example, consider the equation below

 $F(s) = \frac{10(s+5)(s+3-j4)(s+3+j4)}{s(s+10)(s+6-j8)(s+6+j8)}$

The poles of this function are 0, -10, -6 + j8 and -6 - j8. The zeros are -5, -3 + j4 and -3 - j4. To get a transfer function, first enter a constant (1 is fine), and then a list of zeros and a list of poles. See the screen below:



Press **CALC** to compute the transfer function.



The function is automatically stored in the HOME EEAPPD directory, ready for use by the GPLOT and PHPLOT functions.

To plot phase and gain, exit the application pack by pressing UP UP

UP QUIT. Press the **GPLO** menu key to start the gain plot program.

(HOM	E }				
4:					
2:					1
1:				\downarrow	\downarrow
EEHPP	ECON	DEREC	DERUB	GPLO	PHPLO

The first screen asks whether the existing picture should be cleared before plotting a new function. Ordinarily the answer is YES, unless you want to overlay multiple plots to look at how a function changes.



Next, select a horizontal range appropriate to the function you want to plot. It may be necessary to experiment a bit to get the right value. For this example,

use .1 4 as the horizontal range, representing a frequency range from $10^{.1}$ to 10^{4} Hz (3_Hz to 10_kHz). The X-axis represents frequency, and the result is a Bode plot with frequency plotted logarithmically, and gain plotted linerarly.

{ HOME E	EAPPD }	PRG
Enter for s <min></min>	horizontal (no units) (Max)	range :
.14 Cessices	P+ +DEL DEL+ II	NS O MSTK

Vertical range is selected next, and it can be auto-scaled to the horizontal range if desired. The range will be selected such that it fills the screen. Just press **ENTER** to select auto-ranging.



The plot program will autoscale the function, then begin plotting. It may take a few minutes, depending on the function to be plotted.



Labels for the axes, and the function will be printed on the display. To remove these, and clear up the display, press the **COORD** and **LABEL** softkeys. For more information on the plotting softkeys, read Chapter 18 of the *HP 48SX Owner's Manual*.



The phase plot program uses the same sequence of operations, and is started by pressing the **PHPLO** menu key at the opening screen.

Fourier Transform Pairs

Like the earlier Laplace section, selected transform pairs are listed.



Fourier Coefficients

A summary of coefficients are listed for common waveforms.

+Fou	rie	r Co	effi	cie	nts
→RECT	PULSI	E - AN	ŞXAX	T0/TX3	INCC
SYM	TRIAN	GULAR	- AN	AXTO/	T¥(SI
SYM	TRAPE	ZDIDAL	- BN - AN	O ZXAX(T0+T
	CINEL		- BN	0	
NULL	SINC	-	BNO	XIUTIS	K (2 4 1
			⇒STK	FONT	UP

Finite Fourier Transform

For a single cycle of an arbitrary waveform, a set of data points may be taken at discrete intervals. A Fourier series may be obtained by analysis of these data points, representing the frequency components present in this single cycle of the signal. Frequency components up to 1/2 the sampling frequency may be obtained.

The data is entered as a list; the assumption is made that the data samples were taken over one full cycle of the waveform, and that the sampling rate remained constant over the sampling interval.

To use the FFT, enter a list of data points sampled over a single cycle of a waveform as shown below. These are just the amplitudes; the time interval is assumed to be constant between samples.



After entering the list of sample data, press **CAUC** to compute the coefficients of the Fourier series.



To see all of the coefficients, press 🗩 🕅.



If the list is extremely long, it won't all fit on one screen. If that's the case, use the **STK** menu key to place the coefficient list on the stack, then exit the application pack and examine the list using the stack editing functions. Go ahead and press **STK** to save the list on the stack, for use in the next example.

Inverse FFT

This transform takes the Fourier series coefficients as input and returns a list containing an evenly spaced set of data points over one cycle of a repetitive signal.



Enter the coefficients you just calculated by retrieving them from the stack. Press ENTER to enter coefficients, then the **STK** menu key to get to the stack. Press the **ECHO** key to copy the level 1 stack contents back to the data entry line. Press ON to leave the stack. You'll need to edit out an extra set of brackets, and the ':Coefficients:' label, but it's better than typing in all the numbers by hand.

Now that the numbers are in, press **CALC** to compute values for the data points. Neglecting round-off errors, you should get some data points that look the same as the ones you typed in originally, except for round-off errors and the fact that all data is now converted to complex numbers.



Ladder Network Analysis

Ladder network analysis interactively constructs a network from the load-impedance end. Load conditions are computed when the ladder section is complete. Illustrated below are the various kinds of elements that can be used to form a ladder network.





This routine provides a method of reducing multi-element ladder networks to a single equivalent impedance. It constructs a matrix as each element is added to the network. When a calculation is requested, the matrix is evaluated and the transfer characteristics are computed. The Zin which results is substituted for the original ZL, and another network can then be added using the old network as the new load impedance. From this point, the transfer characteristics for the next ladder network are computed using the previous network as a load, and the original load impedance ZL no longer appears. The assumption is made that the operating frequency is specified first and not changed for the remainder of the analysis.

A ladder network can be composed of many different kinds of devices, placed in series or parallel with the load. These may include passive components such as resistors, capacitors and inductors, or active devices like gyrators and current sources. The following table shows the different ladder elements and the data which must be entered for each.

Series or Parallel Resistor	Enter resistor value in Ω .
Series or Parallel Capacitor	Enter capacitor value in F.
Series or Parallel Inductor	Enter inductor value in H.
Series or Parallel RL	Enter resistor and inductor values.
Series or Parallel LC Tank	Enter the inductor and capacitor values.
Series or Parallel RC	Enter resistor and capacitor values.
Transformer	Enter turns ratio n, $(10:1 = 10)$.
Gyrator	Specify the gyration resistance, α .
VCIS, Voltage-Controlled	Specify gm, the transconductance, and $r_{\rm b}$,
Current Source	the input resistance.

ICIS, Current-Controlled	Specify β (or h_{fe}), the current gain, and r_{b} ,
Current Source	the input resistance.
Transmission Line, Open	Specify the stub length Θ in degrees and
Stub and Shorted Stub	the characteristic impedance Z_0 .

To evaluate a ladder network, begin at the output by entering the initial load impedance and the operating frequency. Press **CALC** to load these initial conditions and bring up a list of ladder elements. Next, add a single element by selecting it from the list and entering its value. Do not try to enter more than one element; the first one encountered will be used. Press **NXIE** to combine the new element into the network.

For each subsequent item, select it, enter its value and press **NATE**. Repeat the process for each element, working from the load down to the input of the ladder. When finished, press **CALC** to compute the input impedance, current and voltage gains, transconductance, and power transferred. To return to the previous level to restart the ladder analysis, press **CTD**.

Transmission Lines

Transmission line calculations find propagation constants, phase velocity, VSWR, and other factors given the physical characteristics of the line. 'Smith chart' calculations replace the traditional graphic methods of determining line impedances with the more-accurate method of direct computation. The equations provide input, output, short-and open circuit impedances and VSWR without the need for paper charts.

Transmission lines are key elements in every electronic system. They may be used for power transmission, strung between towers and separated by many feet, or they might be twisted pairs carrying telephone conversations, packed by the hundreds into a cable measuring an inch in diameter.

The uses of transmission lines are so varied that it is difficult to cover every specific case, but a generalized form often serves to approximate the real-life situation. This model assumes two wires, separated by some insulating medium and carrying a signal at some fixed frequency for a fixed distance. The properties that must be known are the inductance, capacitance and leakage conductance between the two wires, and the series resistance per unit length.

Transmission Line Parameters

For a unit length, the following parameters are entered:

- rresistancelinductance between conductor pairginter-conductor conductance (leakage)
- c capacitance between conductor pair

The general conditions for the line are then entered:

f	operating frequency
d	length of line (in terms of unit length)
ZL	terminating impedance

For two conductors in a flat ribbon cable, for example, the values are:

r	.067 Ω/ft
1	2E-6H/ft
g	1E-9_mho/ft
c	18E-12_F/ft
f	1E6 Hz
d	150_ft
ZL	100 [_] Ω

When data entry is complete, you should see a screen something like this:

Tr: →R: 6 C: 1 C: 1 F: 1 D: 1 D: 1 D: 1 D: 1	000E-1 000E-1 000E-1 000E-1 000E-1 000E6 000E6 000E6	ission I Iz	line	са
		CALC	FONT	UP

Press CALC to get the results shown below:

Transmission Line →α: 100,7E-6 Ø: 37,70E-3 VPH: 166,7E6 ØZD: -152,5E-3 MAG ZD: 333,3E0 VSWR: (538,5E-3,4179,9E0)	ра
+STK FONT	UP

Examining the Results

- α is the attenuation constant, in nepers per unit length. The signal is reduced in strength as it travels along the line. It's about 100E-6 ft.
- ϕ is the phase constant in radians per unit length. It represents the amount of phase shift which takes place as the signal propagates down the line. It's about $38E-3_r/ft$.

- V_{ph} is the phase velocity, in this case feet per second. The speed of light is about 982.1E6_ft/sec, and in this case V_{ph} is 167E6_ft/s. This phase velocity represents the speed at which a signal propagates along this transmission line. It is possible to get values greater than the speed of light, if the values you choose for L and C are physically impossible. The limits for L and C (in the real world) are the permeability and permittivity of free space.
- ϕ Zo is the phase angle of the characteristic impedance in radians.
- Mag Zo is the magnitude of the line's characteristic impedance.
- VSWR is the voltage standing wave ratio. It's a measure of how well the terminating impedance on the line matches the characteristic impedance. If the two are identical, the VSWR is 0. Try repeating the calculation with a terminating resistance of 333_Ω , and see how the VSWR changes.

For non-zero values, energy in the signal is reflected back along the line to the source. In an application such as a broadcast station or amateur radio transmitter, minimizing this figure is critical to achieving maximum signal strength and preventing damage to the transmitter.

Smith Chart Impedance Calculations

The Smith chart is a polar representation of normalized resistance and reactance curves. The traditional method of calculating open- and short-circuit conditions for a transmission line has been through graphical construction on a Smith chart, with the results read from the chart and interpolated. The equations provided in this section use the same data that would be entered on the chart in graphic form, but they solve for the results directly.

Tr: →R: 6 L: 2 G: 1 F: 1 F: 1 D: 1 ZL:	ANSM 7.00E- 000E- 000E- 000E- 8.00E- 8.00E- 8.00E- 8.00E- 50.0E0	ission I Iz	line	ca
		CALC	FONT	UP

Using the same data as in the previous section, try the problem again. It computes the input, output, open-circuit and short-circuit impedances for a transmission line, as well as VSWR, the voltage standing wave ratio.



Two-Port Networks

The two-port network section does performance calculations for a two- port with known z, h or y parameters, given the characteristics of the source and load. Conversions are provided between z, h, a and y parameters.

Combinations of two-port networks can be evaluated in series, parallel and hybrid topologies. The combined networks can be reduced to an equivalent set of z-parameters which can then be combined with additional networks to form complete systems.



In this section a variety of tools are provided for evaluating two-port networks. The two-port concept is widely used to simplify electronic circuits or to model a subcircuit too complex to evaluate directly. The parameters can be measured on the bench, and the results plugged in to predict performance in an electronic system.

The two-port network is used as a model for many different kinds of systems. Transistors are modeled as two-ports; in fact, h-parameters such as h_{fe} are commonly listed in data sheets for transistors. Operational amplifiers can also be modeled in this way, as well as very complex systems whose internal design may not even be known. As long as the required parameters can be

measured from the outside, the internal configuration of the two-port is irrelevant.

Two-port networks do have to follow some basic rules in order for an analysis to be valid.

- Energy storage (like a battery) is not permitted within the circuit.
- Independent sources of voltage or current are not permitted within the two-port, although dependent sources are allowed.
- Current into a port must equal current out of the port.
- All connections between the two ports must be made internally; an external connection between port 1 and port 2 is not permitted.
- Currents are positive when flowing IN to a port, and negative when flowing OUT. Negative results for impedances are a sure sign that you forgot which way a current was going.

Given these restrictions, and the four two-port parameters, any number of these blocks may be connected together and analyzed as a complete system.

Measuring Two-Port Parameters

Listed below are the formulas for z, y, h and a parameters.

$$z_{11} = \frac{V_1}{I_1} \left| z_{12} = \frac{V_1}{I_2} \right|_{I_1=0} z_{21} = \frac{V_2}{I_1} \left| z_{22} = \frac{V_2}{I_2} \right|_{I_1=0}$$

$$y_{11} = \frac{I_1}{V_1} \bigg|_{V_{2=0}} y_{12} = \frac{I_1}{V_2} \bigg|_{V_{1=0}} y_{21} = \frac{I_2}{V_1} \bigg|_{V_{2=0}} y_{22} = \frac{I_2}{V_2} \bigg|_{V_{1=0}}$$

$$h_{11} = \frac{V_1}{I_1} \bigg|_{V_{2=0}} h_{12} = \frac{V_1}{V_2} \bigg|_{I_{2=0}} h_{21} = \frac{I_2}{I_1} \bigg|_{V_{2=0}} h_{22} = \frac{I_2}{V_2} \bigg|_{I_{1=0}}$$

$$a_{11} = \frac{V_1}{V_2} \bigg|_{I_{2=0}} a_{12} = -\frac{V_1}{I_2} \bigg|_{V_{2=0}} a_{21} = \frac{I_1}{V_2} \bigg|_{I_{2=0}} a_{22} = -\frac{I_1}{I_2} \bigg|_{V_{2=0}}$$

As an example, z_{11} is the voltage across port 1, divided by the current into port 1 (port 1 input impedance), given that current into port 2 is zero; that is, port 2 is open-circuit. It's clear that these parameters can be measured with a meter, for any random collection of circuitry, if it follows the rules listed above.

Electrical Engr. Pac

Conversions Between Parameters

A variety of conversions are provided to make it easy to get z, y, a or h parameters if any of the types are known. Select a conversion from the type you have to the type you want, and enter the values.

For example, let's calculate the [z] parameters for a simple two-port network; a wall-plug power supply, like the sort used to power calculators or modems. Such a supply might have characteristics something like this:

Input, 110_VAC rms, 30_mA maximum (3.3_W). If the output is open-circuit, input current is about 1_mA. When the output is shorted, input current rises to 30_mA. In normal operation, it's somewhere in between these extremes.

Output, 6_VAC rms nominal, 7.5_V open circuit, 400_mA short-circuit current. The output voltage will drop to about 6_V under load, with the output voltage varying significantly as the load current changes. No regulation or other control is assumed.

$a_{11} = \frac{V_1}{V_2} \Big _{I_{2=0}} = \frac{110}{7.5} = 14.67$	$\left a_{12} = -\frac{V_1}{I_2}\right _{V_{2=0}} = -\frac{110}{400} = 275.0$
$a_{21} = \frac{I_1}{V_2} \Big _{I_{2=0}} = \frac{.001}{7.5} = 133.3E - 6$	$a_{22} = -\frac{I_1}{I_2}\Big _{V_{2=0}} = -\frac{.030}{400} = .075$

Select CONVERT $A \rightarrow Z$ and enter the values given above. The calculations can be done on the input line, or you can just type in the values.

Press CALC and the converted parameters will be displayed.



The z-parameters should be:

- z11 110,000 (input impedance).
- z12 7975 (transfer impedance).
- z21 7500 (reverse transfer impedance).
- z22 562.5 (ouput impedance).

If you got a negative result for any of the impedances, go back a moment and think about the sign convention for current (particularly I2).

We'll use these results in a moment to examine how this power supply would work under load. Use the cursor keys and the **STK** menu key to place each z-parameter value on the stack.

Circuit Performance

The circuit performance topic provides three ways to enter the two-port parameters; as impedance, Z; as admittance, Y; or as the hybrid h-parameters.

After entering the appropriate parameters, the source voltage Vg, source impedance Zs and load impedance ZL are entered. Let's use the previous results to see how the power supply works with a 20 ohm resistor attached as a load.

Select CIRCUIT PERFORMANCE (Z SPEC) and retrieve the four z-parameters from the stack. To do that, press ENTER at each z-parameter, then the **STK** menu key. Use the cursor keys to point to the right value (it's tagged!) and press **ECHO**. Press ENTER and then use the **SKIP** and **EDEL** keys to skip over the numeric value and delete the tag.

Enter 110 for Vg, the 'generator' voltage.

Enter zero for Zs, the source resistance (it's a wall outlet, and we'll assume its resistance is negligible).

Finally, enter 20 for Zl, the load resistance on the output side of the two-port.

Circuit →211: 110.0E3 212: 7.975E3 221: 7.500E3 222: 562.5E0 VG: 110.0E0 25: 0.000E0 2L: 20.00E0	performance
CĤ	C FONT UP

Press CALC to see the results.

Ci →ZIN: I2: - VT: : ZT: : I2/I1 V2/V V2/V	7.3186 7.3186 193.56 7.50060 18.7560 18.7560 1. 35.1 6: 35.1 6: 35.1	it p 3 -3) 9880 198-3 198-3	erfo	ormar	nce
			⇒stκ	FONT	UP

Examining the Results

- **Zin is the input impedance, which is influenced by the load.** It's a little over $7_k \Omega$. This is the impedance which the wall outlet looks into.
- I2 is the current flowing in the load, about 200_mA; negative, since it's flowing out of the two-port and into the load.
- Vt is the Thevenin voltage seen at the output of the network into an open circuit. It's exactly 7.5_V, as expected from the initial assumptions made about this power supply.
- Zt is the Thevenin impedance seen at the output of the network into a short circuit. From the load's point of view, this power supply looks like a 7.5_V source with an 18.75_Ω resistor in series.
- I2/I1 is the current gain. It's a measure of the current attenuation or amplification which takes place through the network. As expected, it's negative, since current flows IN (+) the input and OUT (-) the output, and it's just under 13, so this network amplifies current (no surprise, it's a power supply!).
- V2/V1 is the voltage gain, analogous to current gain, measured across the terminals of the network. This gain figure neglects the effect of source impedance on the total gain across the network.
- V2/Vg is the voltage gain, measured from the voltage source to the load impedance. Multiply this value by the generator voltage Vg to get the actual voltage across the load resistance. The voltage gain figures are small, since this supply steps down a high voltage to a lower one. If a significant source resistance Zs had been included, the overall gain V2/Vg would be lower, due to losses in the source resistance.

If it's more convenient to enter h- or y-parameters, select the appropriate topic from the menu and enter the data in the same way.

Connections of Two-port Networks

Two-port networks can be connected together in a variety of topologies. The ability to combine two networks into a single one lets you stack small sections together to create a complex system. The sketch below illustrates the connections.



Select the kind of connection needed, and enter the Z-parameters for each of the networks. If you don't have the Z-parameters, use the conversions to solve for them first. Although the textbook methods specify different parameters, depending on the connection type, these calculations uniformly use Z-parameters and convert internally to do the calculations.

Once the eight parameters are entered, press **CALC** to get the equivalent parameters for the connected networks. These results can be plugged into the next combination to solve a large network.

Circuit Analysis Tools

Notes:

Appendix A Warranty and Service

Pocket Professional Support

You can get answers to your questions about using your Pocket Professional card from Sparcom. If you don't find the information in this manual or the *HP 48SX Owner's Manual*, contact us in writing, at 897 N.W. Grant, Corvallis, OR 97330, U.S.A., or by calling us at 503-757-8416.

Limited One-Year Warranty

What Is Covered

The Pocket Professional is warranted by Sparcom Corporation against defects in material and workmanship for one year from the date of original purchase. If you sell your card or give it as a gift, the warranty is automatically transferred to the new owner and remains in effect for the original one-year period. During the warranty period, we will repair or replace (at no charge) a product that proves to be defective, provided you return the product and proof of purchase, shipping prepaid, to Sparcom.

What Is Not Covered

This warranty does not apply if the product has been damaged by accident or misuse or as the result of service or modification by other than Sparcom.

No other warranty is given. The repair or replacement of a product is your exclusive remedy. ANY OTHER IMPLIED WARRANTY OF MERCHANTABILITY OR FITNESS IS LIMITED TO THE ONE-YEAR DURATION OF THIS WRITTEN WARRANTY. IN NO EVENT SHALL SPARCOM CORPORATION BE LIABLE FOR CONSEQUENTIAL DAMAGES. Products are sold on the basis of specifications applicable at the time of manufacture. Sparcom shall have no obligation to modify or update products, once sold.

If the Card Requires Service

Sparcom will repair a card, or replace it with the same model or one of equal or better functionally, whether it is under warranty or not. There is a service charge for service after the warranty period. Cards are usually serviced and reshipped within five working days.

Send the card to Sparcom Corporation, 897 N.W. Grant, Corvallis, OR 97330, U.S.A.

Service Charge

Contact Sparcom for the standard out-of-warranty repair charges. This charge is subject to the customers local sales or value-added tax wherever applicable.

Cards damaged by accident or misuse are not covered by the fixed charges. These charges are individually determined based on time and material.

Shipping Instructions

If your card requires service, ship it to Sparcom.

- Include your return address and a description of the problem.
- Include proof of purchase date if the warranty has not expired.
- Include a purchase order, along with a check, or credit card number and expiration date (VISA or MasterCard) to cover the standard repair charge.
- Ship your card postage prepaid in adequate protective packaging to prevent damage. Shipping damage is not covered by the warranty, so we recommend that you insure the shipment.

Environmental Limits

The reliability of the Pocket Professional depends upon the following temperature and humidity limits:

- Operating temperature: 0 to 45 °C (32 to 113 °F).
- Storage temperature: -20 to 60 °C (-4 to 140 °F).
- Operating and storage humidity: 90% relative humidity at 40 °C (104 °F) maximum.

Warranty and Service

NOTES

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Appendix C Questions and Answers

Questions Commonly Asked

- **Q.** I can't find the **EEAPP** subdirectory in the Library menu. How can I verify that the card and the calculator are functioning properly?
- A. There are several possibilities:

a. Check to make sure that the card is properly seated in the calculator port.

b. Turn the calculator off and on.

c. The calculator checks the application card when it turns on. If an "Invalid Card Data" or a "Port Not Available" message is displayed, the card may require service.

- **Q**. What do three dots (...) mean at the end of a display line?
- A. The three dots indicate that the object is too long to show on one line. To view the complete object, use the cursor keys to move the arrow to the object and press WS. Pressing ENTER or returns you to the original display of the item.
- **Q.** I'm using the Equation Library to solve a problem. After selecting the equations and entering values for the variables, the calculator displays "Too many unknowns." What's wrong?
- **A.** Not enough variables were specified to completely solve the problem. You will have to specify more values and solve again.
- **Q.** I'm using the Equation Library to solve a problem. After selecting the equations, I'm ready to enter values for my variables. I find that some of the variables have values already displayed. What's wrong?
- A. The variables with values displayed indicate that these variable names have been used in solving another equation. To start with a clean slate of values, you can use **CLEAR** to reset the values of all variables to 0.

- **Q**. While using the Equation Library, I turned units off and all the numbers changed. What's wrong?
- A. In no-units mode, the Equation Library assumes that all values are SI in order for the equations to solve correctly. Therefore, when units are turned off, all values are first converted to SI units, then the unit tags are eliminated.
- **Q.** While using the Equation Library to solve an equation set, intermediate answers are given. Why?
- **A.** The Sparcom's equation solver engine has the ability to solve a set of equations in a systematic fashion. The result of computation from each equation is reported, to keep you informed of the solver's progress.
- **Q**. The calculator displays "Bad Guess(es)" while running the Equation Library. What's wrong?
- A. The HP 48SX root finder encountered variable values or units that prevented a solution. You may need to start the root finding process by providing a "guess" value. See Chapter 1 for details.
- **Q**. While solving for an angle, I got an answer that was too large: For example, 8752 degrees instead of the expected answer of 112 degrees.
- A. The calculated result may be offset by integer multiples of 360 degrees. By entering a "guess" value, or by solving in no-units mode, you should be able to avoid this problem.
- **Q.** I solved a problem some time ago, and I'm trying to recall those calculated values for a problem I'm working on now. The values from the past calculation have changed. What's wrong?
- **A.** Most likely, the same variable name was used in solving another equation, so you will not be able to recall the old values.
- **Q**. While searching a list of information, I used the alpha key, but the search function didn't work. Why?
- **A.** Since the search function is case-sensitive, you most likely entered the letters in the wrong case.

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