SUBSTANTIAL PROGRAMS

FOR THE

HP-11C AND 15C

(And HP-12C, HP-41)

Thomas W. Beers -- 1987

SPECIAL NOTE TO THE HP-15C USER

While the ten basic programs described in this manual are primarily for the HP-11C they will also work for the HP-15C, assuming the appropriate instructions are keyed. The pertinent steps to be changed (for 15C use) are indicated in the program listings by an asterisk, and the proper keys shown in parentheses. With regard to these programs there are four main differences between the 11C and 15C which cause these changes:

1. Location of the summation registers.
   11C: registers 0 through 5;
   15C: registers 2 through 7.

2. Number and location of conditional tests.
   11C: 8, located on the keyboard directly;
   15C: 2 directly on the keyboard (g X≤Y, g X=0), and
   10 indirectly available by g TEST n.

3. Syntax of ISG (DSE is similar).
   11C: f ISG uses register I as the index register;
   15C: f ISG n assumes the user specified
   register n as the index register.

4. Location of π key.
   11C: instruction is f π (code= 42,16);
   15C: instruction is g π (code= 43,26).

SUBSTANTIAL PROGRAMS FOR THE HP-11C

(AND HP-12C, HP-15C, and HP-41)

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INTRODUCTION

The collection of major programs described in this book relates primarily to the Hewlett-Packard HP-11C, a hand held, programmable calculator. Although the supply of available "canned" programs for the HP-11C is sparse (compared to the HP-41*, for example) this calculator, and its companion the HP-15C, remain favored choices of the scientific clientele who are tuned to the efficiency afforded by the RPN (reverse Polish notation) system of problem solving.

Having survived the trauma of the HP-34C being replaced by the HP-11C the author recognizes that to spend significant time assembling programs dedicated to a specific model of calculator practically assures that the introduction of a refined model is imminent. Nevertheless, I have put together several years of part-time efforts in the present work. This bold move (in light of the eventual obsolescence of the HP-11C) is felt justified since the background of each program is thoroughly described, providing the user with the formulas and perhaps the solution format for writing programs appropriate for the HP-11C replacement.

The reader will note that there are ten programs for the HP-11C (all of which were consciously made compatible to the HP-15C), one program for the HP-12C, and that starting with program 11F006, companion programs utilizing the HP-41 appear**.

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*"HP-41" is used throughout, however the programs so indicated herein will run on the 41C, 41CV or 41CX models.

**Analogous HP-41 programs also have been prepared for the first five 11C programs (SLR, LVC, PLS, ABCD, and SIMSTRAT), however they are not included in this publication. See page 210 for their description and availability.

This "companion" practice emerged in the chronology of programs when it was realized that the same (or very similar) program-use description could be employed for programs written for the different models. For the HP-41 programs, printed listings and program barcode are provided.

Several "narrative" sections are included in this book to make it more of an educational tool than simply a collection of programs. Appendix A part of program FICALC-11/15 discusses the "Concept of Time and Money". This section is copied, with permission, from Hewlett-Packard literature in order to keep an excellent tutorial from "getting lost" as new products and manuals are introduced. It was written in an HP-12C framework*, but most sample problems can be solved using the FICALC-11/15 or AMORT78-11 programs and the HP-11C (or appropriate HP-41 program). Also included (with the FICALC-11/15 program as Appendix B) is "An Elaboration of a Compound Interest Problem...", which shows in some detail the nature of compound interest and several ways of using the calculator to solve common problems.

Another narrative section, briefly describing amortization schedules and the rule of 78s is included just prior to the AMORT78-11 program.

In the next section brief descriptions are given in order by program number. The only significance of the number is that it indicates the order in which the programs were written. Since this chronological order was followed, the subjects covered (statistical, forestry and surveying related, and financial) occur randomly. A group of "amortization" type programs and separate introduction are included at the end of the collection. In all programs written for the HP-41, printed results can be obtained when a printer (HP-82143A or HP-82162A) is connected.

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*Actually the material was written for the predecessor of the HP-12C, the HP-38C.

ANNOTATED DIRECTORY OF PROGRAMS

Program Number	Program Name	Program Page	Purpose and Special Notes
11F001*	SLR	10	This SIMPLE LINEAR REGRESSION program provides for calculator initialization to accept and process two-variable data, providing: the slope and Y-intercept, their standard errors and t tests; the slope forced through the origin and its standard error; the correlation coefficient, and; point and interval estimates for arbitrary predicted mean Y and individual Y.
11F002	LVC	18	The program, LOG VOLUME CALCULATIONS, calculates either board-foot volumes (by four different log rules) or cubic-foot volume (by Huber, Smalian or Newton formulas) for logs of specified length and diameter.

*The original significance of this numbering scheme was: "11" indicates the calculator model; "F" implies forestry related (in the broad sense); "001" is the program's sequence in the category.

Program Number	Name	Page	Purpose and Special Notes
11F003	PLS	26	The program, **P**ARABOLIC **L**EAST **S**QUARES, calculates the gross and corrected (for the mean) sums of squares and products as well as the regression coefficients for the model: $Y=b_0+b_1X_1+b_2X_2$. The traditional parabolic fit is achieved if X_2 is defined as the square of X_1. Optionally, X_1 and X_2 can be considered independent, leading to the usual multiple linear regression fit.
11F004	ABCD	33	The program, **A**ZIMUTH OR **B**EARING **C**ORRECTION FOR **D**ECLINATION, achieves the conversion of angles measured from the true north meridian to angles measured from magnetic north and vice versa. Angles can be expressed as bearings (e.g., S 25 E) or in azimuth units from north (e.g., 125 degrees) and can be in either the degrees, minutes, and seconds or the degree, decimal degree format.

Program Number	Program Name	Program Page	Purpose and Special Notes
11F005	SIMSTRAT	47	This **SIM**PLE AND **STRAT**IFIED RANDOM SAMPLING program is used to summarize data from either a simple or stratified sample, obtaining within stratum and overall estimates of the means and standard errors. Optionally, arbitrary confidence interval estimates can be obtained. Either the finite or the infinite population case can be processed.
11F006	FICALC-11/15	55	The **FI**NANCIAL **CALC**ULATIONS ON THE HP-**11**C OR HP-**15**C program is an attempt to simulate the HP-12C in the solution of the basic financial formula relating n (number of periods), I (periodic interest rate), PV (present value), PMT (periodic payment), and FV (future value). When four of the variables are provided, the unknown can (in most cases) be obtained by simple one-key depression. Conversion between annual percentage rate (APR) and effective annual rate (EAR) is also provided.

Program Number	Name	Page	Purpose and Special Notes
41F041	FICALC	113	The program, FINANCIAL CALCULATIONS, was written for the HP-41 calculator to serve as a close companion to FICALC-11/15, making it feasible (with minor changes) to use the same set of instructions and examples. A refinement was added to provide for a "12-multiplier" or "12-divisor" key similar to the HP-12C.
11F007	AREA-11/15	117	The program, AREA CALCULATIONS ON THE HP-11C OR HP-15C, was written to calculate the area of a closed traverse, given input in the form of straight-side distances and directions either as azimuths or as bearings and quadrants. Area is calculated in square units corresponding to the distance units and optionally in acres or hectares. The linear error of closure is calculated and displayed if desired. The program can also be used as a bearing-to-azimuth converter. Note that there is no provision to "balance" the traverse; if this is necessary it must be done separately.

Program Number	Name	Page	Purpose and Special Notes
41F039	AREA41	126	The program, **AREA** CALCULATIONS ON THE HP-**41**, serves as a companion to AREA-11/15. Additions to the program include the provision to calculate the precision of the survey, audio signals to indicate needed user input, and the availability of printed output.
41F040	AREA-S	134	The program, **AREA**-**S**HORTENED VERSION, provides the same output as AREA41 but chooses the most likely input options from thos available in that program, thus reducing the code from 262 to 172 steps.
11F008	MUGO-11	138	**M**EANS, WITH **U**NGROUPED AND **G**ROUPED **O**PTIONS ON THE HP-**11**C was written to calculate, for a set of univariate data, the arithmetic, quadratic, geometric, and harmonic means, and the standard deviation, coefficient of variation, and standard error. Data can be ungrouped, or grouped using either true frequencies or weights other than frequencies.
41F042	MUGO41	147	The program, **MUGO** ON THE HP-**41**, was written as a companion to MUGO-11. Printed output can be obtained.

Program Number	Name	Page	Purpose and Special Notes
11F009	AMORT78-11	159	The program, AMORTIZATION SCHEDULES AND SIMILAR FINANCIAL CALCULATIONS USING THE "RULE OF 78s" WITH THE HP-11C, provides the usual entries in a traditional amortization schedule: payment number, interest paid, reduction in principal, and outstanding balance. With slightly different input, a similar schedule can be obtained assuming the rule of 78s procedure is used for interest allocation.
11F010	FINC78-11	170	FINANCIAL CALCULATIONS ASSUMING THE RULE OF 78S WITH THE HP-11C serves as a companion to AMORT78-11. For the user solely interested in the rule of 78s procedure, interest allocation, reduction in principal remaining balance and appropriated summations are obtained much faster than in AMORT78-11. Additionally, the "rebate" due after a particular payment is provided.
12F001	AMORT-12	180	This program, AMORTIZATION SCHEDULES WITH THE HP-12C, provides the same amortization schedule entries as AMORT78-11, but because of the AMORT function on the 12C results are obtained much faster. An addendum to the program is suggested to calculate an effective periodic interest rate when the payment and compounding frequencies are not the same.

Program			
Number	Name	Page	Purpose and Special Notes
41F043	AMORT	188	The program, normal **AMORT**IZATION SCHEDULES WITH THE HP-41, provides the same schedule entries as AMORT78-11, but calculations are faster, input is prompted for, and output is identified.
41F044	AM7841	201	This program, **AM**ORTIZATION SCHEDULES AND SIMILAR FINANCIAL CALCULATIONS USING THE RULE OF **78**s WITH THE HP-**41**, represents a direct analogy to AMORT78-11, but provides advantages cited in the previous program (AMORT).

Calculator: HP-11C

Program Name: Simple Linear Regression

Author: Thomas W. Beers

Purpose: To initialize the calculator to accept two-variable data, process data which are input by the Σ+ operation, and provide the statistics listed below:

a, the Y-intercept, its standard error, s_a, and t_a to test the hypothesis: $\alpha = 0$

b, the slope, its standard error, s_b, and t_b to test the hypothesis: $\beta = 0$

b', the slope forced through the origin, and its standard error, $s_{b'}$

r, the correlation coefficient

for specified X_o and Student's t, $\hat{Y}_o$, the predicted Y for X_o is provided as well as confidence interval estimates assuming mean Y and assuming individual Y.

Note: the usual calculation of a and b, $\hat{Y}$ and r, and $\overline{X}$, $\overline{Y}$, by depression of the L.R., $\hat{y}$, r and $\overline{X}$ keys is still possible, and does not interfere with the program.

A. Storage assignments

Register	15C	Use
I		external storage of appropriate Student's t
0-5	(2-7)	statistical: n, ΣX, ΣX^2, ΣY, ΣY^2, ΣXY
6	(0)	$\Sigma x^2 = \Sigma(X_i - \overline{X})^2 = \Sigma X^2 - n\overline{X}^2$, corrected sum of squares for X

A. Storage assignments *(continued)*

Register	15C	Use
7	(1)	$\Sigma y^2 = \Sigma(Y_i - \overline{Y})^2 = \Sigma Y^2 - n\overline{Y}^2$, corrected sum of squares for Y
8		$\Sigma xy = \Sigma(X_i - \overline{X})(Y_i - \overline{Y}) = \Sigma XY - n\overline{X}\overline{Y}$, corrected sum of products
9		s_{YX}, the standard error of estimate
.0		X_o, the current X for which $\hat{Y}$ and confidence intervals are being calculated and the intermediate answer, C, used in confidence interval calculations

B. Labels

Name	Use
[1]*	program start, clears registers and display
[A]	calculates and displays a; by R/S also s_a and t_a
[B]	calculates and displays b; by R/S also s_b and t_b
[C]	calculates and displays r
[D]	predicted Y and confidence interval calculation and display
[E]	main processor; calculates and stores $\overline{X}$, $\overline{Y}$, Σx^2, Σy^2, Σxy, and s_{YX}; must preceed [D] and optional parts of [A] and [B]
[0]	calculates and displays b' (slope forced through the origin) and $s_{b'}$
2	confidence interval calculation in label D

*brackets indicate meaningful external accessibility

C. Flags-- none used

D. Program procedure and example

I. In PRGM mode, load program

II. In RUN mode, proceed as given below; assume the following data, in USER mode and FIX 2:

Y	X
7	5
5	3
9	8

	Instructions			*Example*		
Step	Input	Key	Output	Input	Key	Output
1. Initialize	-	GSB 1	0.00	-	GSB 1	0.00
2. Data input	$Y_1 \uparrow X_1$	Σ+	1.00	7↑5	Σ+	1.00
	(repeat for all data pairs)			5↑3	Σ+	2.00
				9↑8	Σ+	3.00
3. Process data	-	E	s_{YX}	-	E	0.32

a. Optional: sums, gross and corrected sums of squares and products can now be recalled from registers 0 through 8.

	15C		
-	RCL 0	(2)	3.00(n)
-	RCL 1	(3)	16.00(ΣX)
-	RCL 2	(4)	98.00(ΣX^2)
-	RCL 3	(5)	21.00(ΣY)
-	RCL 4	(6)	155.00(ΣY^2)
-	RCL 5	(7)	122.00(ΣXY)
-	RCL 6	(0)	12.67(Σx^2)
-	RCL 7	(1)	8.00(Σy^2)
-	RCL 8		10.00(Σxy)

4. a statistics	-	A	a	-	A	2.79
	-	R/S	s_a	-	R/S	.52
			t_a			5.35
5. b statistics	-	B	b	-	B	0.79
	-	R/S	s_b	-	R/S	0.09
			t_b			8.66
6. r statistics	-	C	r	-	C	0.99
	-	gX^2	r^2	-	gX^2	0.99

D. Program procedure and example *(continued)*

Step	Input	Key	Output	Input	Key	Output
	Instructions			*Example*		
7. predicted Y and confidence intervals (assume t = 2 and X_o = 4, then X_o = 6)						
a.	t	STO I	t	2	STO I	2.00
	X_o	D	$\hat{Y}_o$	4	D	5.95
			c.i. low } for mean Y			5.50
			c.i. high			6.39
b. (optional)	-	R/S	c.i. low } for ind. Y	-	R/S	5.16
			c.i. high			6.74
c.	next X_o	D	$\hat{Y}_o$	6	D	7.53
			c.i. low			7.13
			c.i. high			7.92
d. (optional)	-	R/S	c.i. low	-	R/S	6.77
			c.i. high			8.29
8. b' statistics	-	GSB 0	b'	-	GSB 0	1.24
	-	R/S	$s_{b'}$	-	R/S	0.13

III. Comments

1. Note that the order of program execution suggests that depression of E immediately follow the last data input, but thereafter the order of pressing A, B, C, D, and GSB 0 is immaterial, and each can be repeated.

2. After the data have been input (and before or after E) the keyboard functions $\overline{X}$, s, $\hat{y}$, r, and L.R. can be used to obtain their appropriate output.

3. Correction for erroneous input data (key stroke blunders!) can be made by use of the $\Sigma -$ key as described in the manual.

E. Program statements and code

Step	Statement	Code	Step	Statement	Code	Step	Statement	Code
01	[f LBL 1] 1/	42 21 1	31	-	30	61	f PSE	42 31
02	f CLEAR REG	42 34	32*	RCL 0 (2)	45 0	62	÷	10
03	g CLX	43 35	33	2	2	63	R/S	31
04	R/S	31	34	-	30	64	[f LBL C]	42 21 13
05	[f LBL E]	42 21 15	35	÷	10	65	f $\hat{y}$, r	42 48
06	g $\bar{X}$	43 0	36	$\sqrt{X}$	11	66	$X \gtrless Y$	34
07	STO 8	44 8	37	STO 9	44 9	67	R/S	31
08	$X \gtrless Y$	34	38	R/S	31	68	[f LBL D]	42 21 14
09	STO X 8	44 20 8	39	[f LBL A]	42 21 11	69	STO .0	44 .0
10	gX^2	43 11	40	f L.R.	42 49	70	f $\hat{y}$, r	42 48
11*	STO 7 (1)	44 7	41	R/S	31	71	f PSE	42 31
12	$X \gtrless Y$	34	42*	RCL 2 (4)	45 2	72	g $\bar{X}$	43 0
13	gX^2	43 11	43*	RCL 0 (2)	45 0	73	R↓	33
14*	STO 6 (0)	44 6	44*	RCL 6 (0)	45 6	74	$X \gtrless Y$	34
15*	RCL 0 (2)	45 0	45	X	20	75	g R↑	43 33
16	CHS	16	46	÷	10	76	RCL .0	45 .0
17*	STO X 6 (0)	44 20 6	47	$\sqrt{X}$	11	77	-	30
18*	STO X 7 (1)	44 20 7	48	RCL 9	45 9	78	gX^2	43 11
19	STO X 8	44 20 8	49	X	20	79*	RCL 6 (0)	45 6
20*	RCL 2 (4)	45 2	50	f PSE	42 31	80	÷	10
21*	STO + 6 (0)	44 40 6	51	÷	10	81*	RCL 0 (2)	45 0
22*	RCL 4 (6)	45 4	52	R/S	31	82	1/X	15
23*	STO + 7 (1)	44 40 7	53	[f LBL B]	42 21 12	83	+	40
24*	RCL 5 (7)	45 5	54	f L.R.	42 49	84	RCL .0	45 .0
25	STO + 8	44 40 8	55	$X \gtrless Y$	34	85	$X \gtrless Y$	34
26*	RCL 7 (1)	45 7	56	R/S	31	86	STO .0	44 .0
27	RCL 8	45 8	57	RCL 9	45 9	87	(f LBL 2)	42 21 2
28	gX^2	43 11	58*	RCL 6 (0)	45 6	88	$\sqrt{X}$	11
29*	RCL 6 (0)	45 6	59	$\sqrt{X}$	11	89	R↓	33
30	÷	10	60	÷	10	90	$X \gtrless Y$	34

1/ Brackets imply meaningful external accessibility, parentheses indicate internal use.

E. Program statement and code *(continued)*

Step	Statement	Code	Step	Statement	Code
91	g R↑	43 33	121	1	1
92	RCL I	45 25	122	-	30
93	X	20	123	÷	10
94	RCL 9	45 9	124*	RCL 2 (4)	45 2
95	X	20	125	÷	10
96	-	30	126	$\sqrt{X}$	11
97	f PSE	42 31			
98	g LST X	43 36			
99	2	2			
100	X	20			
101	+	40			
102	R/S	31			
103	X≷Y	34			
104	f $\hat{y}$, r	42 48			
105	g LST X	43 36			
106	1	1			
107	RCL .0	45 .0			
108	+	40			
109	GTO 2	22 2			
110	[f LBL 0]	42 21 0			
111*	RCL 5 (7)	45 5			
112*	RCL 2 (4)	45 2			
113	÷	10			
114	R/S	31			
115*	RCL 5 (7)	45 5			
116	X	20			
117	CHS	16			
118*	RCL 4 (6)	45 4			
119	+	40			
120*	RCL 0 (2)	45 0			

g MEM leads to

P-00 r-.0

Program memory completely full, since $R_{.0}$ is needed for computations

*Indicates statements that must be changed for the HP-15C. Suggested or required change is shown in parentheses. Although the "code" for these statements will also change this is left for the 15C user to alter.

F. Formulas used

I. In LBL E

1. means, $\overline{X}$ and $\overline{Y}$, calculated by g $\overline{X}$ function.

2. corrected sums of squares and products (lower case letters):

$$\Sigma x^2 = \Sigma X^2 - n\overline{X}^2$$

$$\Sigma y^2 = \Sigma Y^2 - n\overline{Y}^2$$

$$\Sigma xy = \Sigma XY - n\overline{X}\overline{Y}$$

3. s_{YX}, standard error of estimate:

$$s_{YX} = \sqrt{\frac{\Sigma y^2 - (\Sigma xy)^2/\Sigma x^2}{n-2}}$$

II. In LBL A

1. a, the Y-intercept, calculated by f L.R. function.

2. s_a, the standard error of a:

$$s_a = s_{YX}\sqrt{\frac{\Sigma X^2}{n\Sigma x^2}}$$

3. t_a, the calculated test statistic:

$$t_a = \frac{a}{s_a}\text{, to test } H_o\text{: } \alpha = 0\text{, (d.f.} = n-2\text{)}$$

III. In LBL B

1. b, the slope, calculated by the f L.R. function.

2. s_b, the standard error of b:

$$s_b = s_{YX}\sqrt{\frac{1}{\Sigma x^2}} = \frac{s_{YX}}{\sqrt{\Sigma x^2}}$$

3. t_b, the calculated test statistic:

$$t_b = \frac{b}{s_b}\text{, to test } H_o\text{: } \beta = 0\text{, (d.f.} = n-2\text{)}$$

F. Formulas used *(continued)*

IV. In LBL 0

1. b', the slope forced through the origin:

$$b' = \frac{\Sigma XY}{\Sigma X^2}$$

2. $$s_{b'} = s'_{YX}\sqrt{\frac{1}{\Sigma X^2}} = \frac{s'_{YX}}{\sqrt{\Sigma X^2}}$$

where $$s'_{YX} = \sqrt{\frac{\Sigma Y^2 - b'\Sigma XY}{n-1}}$$

3. Though not done in the program, one can test H_o: $B'=1$, (i.e., Y=X) by calculating $t = \frac{b'-1}{s_{b'}}$, (d.f. = n - 1).

V. In LBL C

1. r, the correlation coefficient, calculated by the f $\hat{y}$, r function.

2. r^2, the coefficient of determination can be calculated by gX^2.

3. t_r, the calculated test statistic, was not calculated in the program; however it must equal t_b, therefore Label B then R/S can be used.

VI. In LBL D

1. $\hat{Y}_o$, predicted Y for a specific value of X (i.e., X_o), calculated by f $\hat{y}$, r function.

2. confidence intervals about the predicted mean Y given X_o:

$$\hat{Y}_o \pm t\, s_{YX}\sqrt{C}$$

where
$$C = \frac{1}{n} + \frac{(X_o-\overline{X})^2}{\Sigma x^2}$$

and t = the user supplied value of Student's t assuming the proper degrees of freedom (n-2) and desired confidence (1-probability) level

3. confidence intervals about an <u>individual</u> predicted Y given X_o:

$$\hat{Y}_o \pm t\, s_{YX}\sqrt{1+C}$$

Calculator: HP-11C

Program Name: Log Volume Calculations

Author: Thomas W. Beers

Purpose: To calculate the board-foot volume of logs by Doyle, Scribner, International 1/4" and International 1/8" log rules and to calculate cubic-foot volume using Huber, Smalian, and Newton formulas.

Board-foot calculations assume the scaling diameter and log length are input as $d_u \uparrow L$, whereas the cubic-foot calculations assume the following input

for Huber: $D_m \uparrow L$

for Smalian: $D_b \uparrow D_u \uparrow L$

for Newton: $D_b \uparrow D_m \uparrow D_u \uparrow L$

where: D_b, D_m, and D_u are log diameters in inches at the base, midpoint and upper end,

L = log length in feet,

and d_u = diameter inside bark at the small end of the log

A. Storage assignments

Register	Use
0	Log length, L
1	L/16, used in board-foot calculations
2	d_u, scaling diameter for bd. ft. calculations
3	temporary storage of Int. 1/8" volume

B. Labels

Name	Use
[E]*	initialization for board-foot calculations, clears flag 0 and sets number of decimals to zero.
[A]	Doyle volume; calculation and display

B. Labels *(continued)*

Name	Use
[B]	Scribner volume; calculation and display
[C]	International 1/4" volume calculation and display; Int. 1/8" volume obtained by R/S depression
[D]	all board-foot volumes calculated and displayed in the above order; R/S between displays
[1]	Huber's formula cubic-foot volume; identified by 1 decimal place
[2]	Smalian's formula cubic-foot volume; identified by 2 decimal places
[3]	Newton's formula cubic-foot volume; identified by 3 decimal places.
0,6,7,8	used in label C to calculate Int. 1/4" and Int. 1/8" volumes
9	subroutine called by labels A,B,C, and D to store L, L/16, and d_u
	*brackets indicate meaningful external accessibility

C. Flags

Number	Use
0	in labels D and 9 to control the execution of label 9
1	in label C, flag is set to control the subroutine execution of label 7 the first time through the loop

D. Program procedure and example

I. In program mode, key in the program

II. In RUN and USER mode, proceed as given below. The example will use the data for log 1 given in the following table.

	diameters				length in	
log no.	base (D_b)	mid (D_m)	upper (D_u)	scaling (d_u)	feet (L)	Volumes
1	16.6	15.4	14.5	14	16	100 (Doyle) 123 (Scribner) 136 (Int. 1/4") 150 (Int. 1/8") 20.7 (Huber) 21.20 (Smalian) 20.863 (Newton)
2	12.4	10.0	9.1	9	14	22,37,43,48 7.6,9.03,8.101
3	24.5	24.0	23.8	23	4	90,92,91,100 12.6,12.73,12.620
4	18	17.5	17	16	8.2	74,85,87,96 13.7,13.71,13.700

					Example	
Step	Input	Key	Output	Input	Key	Output
1. Initialize[1/]	-	E	0.	-	E	0.
2. Bd.ft.calc.[2/]						
a.	$d_u \uparrow L$	A	Doyle	14↑16	A	100.
b.	$d_u \uparrow L$	B	Scribner	14↑16	B	123.
c.	$d_u \uparrow L$	C	Int. 1/4"	14↑16	C	136.
d.	-	R/S	Int. 1/8"	-	R/S	150.
e.	$d_u \uparrow L$	D	Doyle	14↑16	D	100.
	-	R/S	Scribner	-	R/S	123.
	-	R/S	Int. 1/4"	-	R/S	136.
	-	R/S	Int. 1/8"	-	R/S	150.

[1/] This initialization step is necessary only at the start of the first set of board-foot calculations or after any cu. ft. calculations.

D. Program procedure and example *(continued)*

[2/] The order or choice of steps 2a through 2e is immaterial except that 2c must precede 2d.

	Step	Input	Key	Output	Example Input	Example Key	Example Output
3.	Cu.ft.calc.[3/]						
	a.	$D_m \uparrow L$	GSB 1	Huber	15.4↑16	GSB 1	20.7
	b.	$D_b \uparrow D_u \uparrow L$	GSB 2	Smalian	16.6↑14.5↑16	GSB 2	21.20
	c.	$D_b \uparrow D_m \uparrow D_u \uparrow L$	GSB 3	Newton	16.6↑15.4↑14.5↑16	GSB 3	20.863

[3/] The order or choice of steps 3a through 3c is immaterial.

4. Go to step 1 if board-foot volumes are needed for the next log, or continue with step 3 for more cubic-foot volumes.

E. Program statements and code

Step	Statement	Code	Step	Statement	Code	Step	Statement	Code
1	[f LBL A]4/	42 21 11	31	RCL 0	45 0	61	.	48
2	GSB 9	32 9	32	4	4	62	7	7
3	RCL 2	45 2	33	÷	10	63	1	1
4	4	4	34	g INT	43 44	64	X	20
5	-	30	35	STO 4	44 4	65	-	30
6	g X^2	43 11	36	g LST X	43 36	66	g F? 1	43 6 1
7	RCL 1	45 1	37	f FRAC	42 44	67	g RTN	43 32
8	X	20	38	GSB 7	32 7	68	STO + 3	44 40 3
9	g RTN	43 32	39	g CF 1	43 5 1	69	RCL 4	45 4
10	[f LBL B]	42 21 12	40	X	20	70*	g X>0 (g TEST 1)	43 20
11	GSB 9	32 9	41	STO 3	44 3	71	GTO 6	22 6
12	RCL 2	45 2	42	RCL 4	45 4	72	(f LBL 0)	42 21 0
13	g X^2	43 11	43	g X=0	43 40	73	RCL 3	45 3
14	.	48	44	GTO 0	22 0	74	.	48
15	7	7	45	(f LBL 6)	42 21 6	75	9	9
16	9	9	46	1	1	76	0	0
17	X	20	47	STO - 4	44 30 4	77	5	5
18	RCL 2	45 2	48	(f LBL 7)	42 21 7	78	X	20
19	2	2	49	RCL 4	45 4	79	R/S	31
20	X	20	50	2	2	80	RCL 3	45 3
21	-	30	51	÷	10	81	R/S	31
22	4	4	52	RCL 2	45 2	82	GTO D	22 14
23	-	30	53	+	40	83	(f LBL 9)	42 21 9
24	RCL 1	45 1	54	STO 1	44 1	84	g F? 0	43 6 0
25	X	20	55	g X^2	43 11	85	g RTN	43 32
26	g RTN	43 32	56	.	48	86	STO 0	44 0
27	[f LBL C]	42 21 13	57	2	2	87	1	1
28	GSB 9	32 9	58	2	2	88	6	6
29	g SF 1	43 4 1	59	X	20	89	÷	10
30	(f LBL 8)	42 21 8	60	RCL 1	45 1	90	STO 1	44 1

4/Brackets imply meaningful external accessibility, parentheses indicate internal use.

E. Program statements and code *(continued)*

Step	Statement	Code
91	X≷Y	34
92	STO 2	44 2
93	g RTN	43 32
94	[f LBL D]	42 21 14
95	g CF 0	43 5 0
96	GSB 9	32 9
97	g SF 0	43 4 0
98	GSB A	32 11
99	R/S	31
100	GSB B	32 12
101	R/S	31
102	GTO C	22 13
103	[f LBL E]	42 21 15
104	f FIX 0	42 7 0
105	g CF 0	43 5 0
106	g CLX	43 35
107	g RTN	43 32
108	[f LBL 1]	42 21 1
109	f FIX 1	42 7 1
110*	f Π (g Π)	42 16
111	X	20
112	5	5
113	7	7
114	6	6
115	÷	10
116	X≷Y	34
117	g X^2	43 11
118	X	20
119	R/S	31
120	GTO 1	22 11
121	[f LBL 2]	42 21 2
122	f FIX 2	42 7 2
123*	f Π (g Π)	42 16
124	X	20
125	1	1
126	1	1
127	5	5
128	2	2
129	÷	10
130	X≷Y	34
131	g X^2	43 11
132	g R↑	43 33
133	g X^2	43 11
134	+	40
135	X	20
136	R/S	31
137	GTO 2	22 2
138	[f LBL 3]	42 21 3
139	f FIX 3	42 7 3
140	R↓	33
141	g X^2	43 11
142	R↓	33
143	g X^2	43 11
144	X≷Y	34
145	g X^2	43 11
146	g R↑	43 33
147	+	40
148	X≷Y	34
149	4	4
150	X	20
151	+	40
152	X	20
153*	f Π (g Π)	42 16
154	X	20
155	3	3
156	4	4
157	5	5
158	6	6
159	÷	10
160	R/S	31
161	GTO 3	22 3

g MEM leads to

P-00 r-5

*Indicates statements that must be changed for the HP-15C. Suggested or required change is shown in parentheses. Although the "code" for these statements will also change this is left for the 15C user to alter.

F. Formulas used

I. Board foot volumes

In the following formulas,

V = bd. ft. volume of the log,

d_u = diameter inside bark at the small end, i.e., scaling diameter in inches,

and L = Log length in feet

a. Doyle scale (in label A)

$$V = (d_u - 4)^2 \left(\frac{L}{16}\right)$$

b. Scribner scale (in label B)

$$V = (.79d_u^{\,2} - 2d_u - 4)\left(\frac{L}{16}\right)$$

c. International 1/4 inch scale (in label C)

V = .905 (Int. 1/8" volume)

d. International 1/8 inch scale (in label C)

The Int. 1/8" volume was calculated using the algorithm described in Forest Mensuration, 3rd. edition by Husch, Miller and Beers. 1982. John Wiley & Sons, N.Y.

II. Cubic foot volumes

In the following formulas

V = cu. ft. volume

D_b = diameter in inches at log base

D_m = diameter in inches at log mid-point

F. Formulas used *(continued)*

D_u = diameter in inches at upper end of log

L = log length in feet

a. Huber formula (in label 1)

$$V = \left(\frac{\Pi D_m^{\ 2}}{576}\right)(L)$$

b. Smalian formula (in label 2)

$$V = \left(\frac{\Pi}{576}\right)\left(\frac{L}{2}\right)(D_b^{\ 2} + D_u^{\ 2})$$

c. Newton formula (in label 3)

$$V = \left(\frac{\Pi}{576}\right)\left(\frac{L}{6}\right)(D_b^{\ 2} + 4D_m^{\ 2} + D_u^{\ 2})$$

Calculator: HP-11C

Program Name: Parabolic Least Squares

Author: Thomas W. Beers

Date: January, 1983

Purpose: To calculate gross and corrected sums of squares and products and the regression coefficients for the least squares fit to model: $\hat{Y} = b_0 + b_1X_1 + b_2X_2$, with the following optional definition of the X's:

1. X_1 and X_2 are non-functionally related independent variables
2. $X_2 = X_1^2$

A. Storage assignments

Register	15C	Use
0	(2)	n
1	(3)	ΣX_1 then $\overline{X}_1$ then b_1
2	(4)	ΣX_1^2 then Σx_1^2
3	(5)	ΣY then $\overline{Y}$ then b_0
4	(6)	ΣY^2 then Σy^2
5	(7)	ΣX_1Y then Σx_1y
6	(0)	ΣX_2 then $\overline{X}_2$ then b_2
7	(1)	ΣX_2^2 then Σx_2^2
8		ΣX_1X_2 then Σx_1x_2
9		ΣX_2Y then Σx_2y
.0		X_2 then Determinant, $D = \Sigma x_1^2\Sigma x_2^2-(\Sigma x_1x_2)^2$
I		control number for automatic display (LBL D)

B. Labels

Name	Use
[E]*	program start; initializes calculator for new data
[A]*	cumulative calculation of sums, and gross sums of squares and products; A is pressed after each data set is keyed

B. Labels *(continued)*

Name	Use
[C]*	after A, calculation of corrected sums of squares and products and the regression coefficients
[D]*	routine to automatically display contents of rgisters 0 through 10 (.0)
4	incrementing part of label D
5	misc. calculating routine used in label C
	*brackets indicate meaningful external accessibility

C. Flags -- none used

D. Program procedure and example

I. In PRGM mode, load program.

II. In RUN mode, select option 1 or 2 and proceed.

1. Option 1 (X_1 and X_2 "unrelated") assumes:

USER mode

input format is: Y,ENTER,X_1,ENTER,X_2 ($Y \uparrow X_1 \uparrow X_2$)

sample data:

Y	X_1	X_2
2	3	5
5	6	12
9	8	15

	Instructions			*Example*		
Step	Input	Key	Output	Input	Key	Output
a. Initialize.		E	zero		E	0.000000000
b. Data input	$Y_1 \uparrow X_{11} \uparrow X_{21}$	A	X_{21}^2	2↑3↑5	A	25.00
	$Y_2 \uparrow X_{12} \uparrow X_{22}$	A	X_{22}^2	5↑6↑12	A	144.00
	$Y_3 \uparrow X_{13} \uparrow X_{23}$	A	X_{23}^2	9↑8↑15	A	225.00
	(repeat for all data triplets)					

c. (optional) To display sums, gross sums of squares and products use any of the following:

(1) manual display:			15C
	RCL 0 → n	= 3.00	ΣX_2 = 32.00
	RCL 1 → ΣX_1	= 17.00	ΣX_2^2 = 394.00
	RCL 2 → ΣX_1^2	= 109.00	n = 3.00
	RCL 3 → ΣY	= 16.00	ΣX_1 = 17.00

D. Program procedure and example *(continued)*

RCL	4 → ΣY^2	=	110.00
RCL	5 → $\Sigma X_1 Y$	=	108.00
RCL	6 → ΣX_2	=	32.00
RCL	7 → ΣX_2^2	=	394.00
RCL	8 → $\Sigma X_1 X_2$	=	207.00
RCL	9 → $\Sigma X_2 Y$	=	205.00
RCL	.0 → X_{2n}	=	15.00

15C
ΣX_1^2 = 109.00
ΣY = 16.00
ΣY^2 = 110.00
$\Sigma X_1 Y$ = 108.00

(2) automatic display: press D and the display will be:

register number
(pause)
register contents

(3) semi-automatic display (having the display stop after each register rather than pause) can be achieved as follows:

(a) replace program step 119 (h PSE) with an R/S statement (R/S)

(b) press D

(c) press R/S after each register display

d. Calculate.

Instructions			Example		
Input	Key	Output	Input	Key	Output
	C	b_0		C	-3.40
	R↓	b_1		R↓	3.80
	R↓	b_2		R↓	-1.20

e. (optional) To display the regression coefficients and corrected sums of squares and products proceed as in c above; for example using automatic display:

press D and read:

0.	(pause)	3.00	=	n
1.	(pause)	3.80	=	b_1
2.	(pause)	12.67	=	Σx_1^2
3.	(pause)	-3.40	=	b_0
4.	(pause)	24.67	=	Σy^2
5.	(pause)	17.33	=	$\Sigma x_1 y$
6.	(pause)	-1.20	=	b_2
7.	(pause)	52.67	=	Σx_2^2
8.	(pause)	25.67	=	$\Sigma x_1 x_2$
9.	(pause)	34.33	=	$\Sigma x_2 y$
10.	(pause)	8.33	=	Determinant, D

15C
-1.20 = b_2
52.67 = Σx_2^2
3.00 = n
3.80 = b_1
12.67 = Σx_1^2
-3.40 = b_0
24.67 = Σy^2
17.33 = $\Sigma x_1 y$

D. Program procedure and example *(continued)*

2. Option 2 ($X_2 = X_1^2$) assumes:

USER mode

input format is: Y,ENTER,X_1,ENTER,ENTER,X (Y ↑ X_1 ↑ ↑ times key)

sample data:

Y	X_1
2	3
5	6
9	8

	Instructions			*Example*		
Step	Input	Key	Output	Input	Key	Output
a. Initialize		E			E	0.000000000
b. Data input	Y_1↑X_{11}↑↑	X, A	X_{21}^2	2↑3↑↑	X, A	81.00
	Y_2↑X_{12}↑↑	X, A	X_{22}^2	5↑6↑↑	X, A	1296.00
	Y_3↑X_{13}↑↑	X, A	X_{23}^2	9↑8↑↑	X, A	4096.00

c. (optional) To display sums, gross sums of squares and products use any of the three options cited in 1c.

	Input	Key	Output	Input	Key	Output
d. Calculate		C	b_0		C	2.60
		R↓	b_1		R↓	-0.80
		R↓	b_2		R↓	0.20

e. (optional) To display regression coefficients and corrected sums of squares and products proceed as in 1c.

press D and read:

					15C
0.	(pause)	3.00	=	n	0.20 = b_2
1.	(pause)	-0.80	=	b_1	1512.67 = Σx_2^2
2.	(pause)	12.67	=	Σx_1^2	3.00 = n
3.	(pause)	2.60	=	b_0	-0.80 = b_1
4.	(pause)	24.67	=	Σy^2	12.67 = Σx_1^2
5.	(pause)	17.33	=	$\Sigma x_1 y$	2.60 = b_0
6.	(pause)	0.20	=	b_2	24.67 = Σy^2
7.	(pause)	1512.67	=	Σx_2^2	17.33 = $\Sigma x_1 y$
8.	(pause)	137.33	=	$\Sigma x_1 x_2$	
9.	(pause)	192.67	=	$\Sigma x_2 y$	
10.	(pause)	300.00	=	Determinant	

E. Program statements and code

Step	Statement	Code	Step	Statement	Code	Step	Statement	Code
1	[f LBL A][1/]	42 21 11	31*	RCL 5 (7)	45 5	61	X	20
2	f FIX 2	42 7 2	32*	RCL 1 (3)	45 1	62	CHS	16
3	STO .0	44 .0	33*	RCL 3 (5)	45 3	63*	RCL 5 (7)	45 5
4	R↓	33	34	GSB 5	32 5	64*	RCL 7 (1)	45 7
5	(f LBL 2)	42 21 2	35*	STO 5 (7)	44 5	65	X	20
6	Σ+	49	36	RCL 8	45 8	66	+	40
7	g LST X	43 36	37*	RCL 1 (3)	45 1	67	RCL .0	45 .0
8	RCL .0	45 .0	38*	RCL 6 (0)	45 6	68	÷	10
9	X	20	39	GSB 5	32 5	69	RCL 8	45 8
10	STO + 8	44 40 8	40	STO 8	44 8	70*	RCL 5 (7)	45 5
11	X≷Y	34	41	RCL 9	45 9	71	X	20
12	RCL .0	45 .0	42*	RCL 3 (5)	45 3	72	CHS	16
13*	STO + 6 (0)	44 40 6	43*	RCL 6 (0)	45 6	73	RCL 9	45 9
14	X	20	44	GSB 5	32 5	74*	RCL 2 (4)	45 2
15	STO + 9	44 40 9	45	STO 9	44 9	75	X	20
16	RCL .0	45 .0	46*	RCL 7 (1)	45 7	76	+	40
17	g X^2	43 11	47*	RCL 6 (0)	45 6	77	RCL .0	45 .0
18*	STO + 7 (1)	44 40 7	48	ENTER ↑	36	78	÷	10
19	g RTN	43 32	49	GSB 5	32 5	79*	RCL 0 (2)	45 0
20	[f LBL C]	42 21 13	50*	STO 7 (1)	44 7	80*	STO ÷ 3 (5)	44 10 3
21*	RCL 4 (6)	45 4	51	RCL 8	45 8	81*	STO ÷ 1 (3)	44 10 1
22*	RCL 3 (5)	45 3	52	g X^2	43 11	82*	STO ÷ 6 (0)	44 10 6
23	ENTER ↑	36	53	CHS	16	83	R↓	33
24	GSB 5	32 5	54*	RCL 2 (4)	45 2	84*	RCL 6 (0)	45 6
25*	STO 4 (6)	44 4	55*	RCL 7 (1)	45 7	85	X≷Y	34
26*	RCL 2 (4)	45 2	56	X	20	86*	STO 6 (0)	44 6
27*	RCL 1 (3)	45 1	57	+	40	87	X	20
28	ENTER ↑	36	58	STO .0	44 .0	88	CHS	16
29	GSB 5	32 5	59	RCL 8	45 8	89	X≷Y	34
30*	STO 2 (4)	44 2	60	RCL 9	45 9	90*	RCL 1 (3)	45 1

[1/] Brackets imply meaningful external accessibility, parentheses indicate internal use.

E. Program statements and code *(continued)*

Step	Statement	Code	Step	Statement	Code
91	X≷Y	34	121	GTO 4	22 4
92*	STO 1 (3)	44 1	122	g RTN	43 32
93	X	20	123	[f LBL E]	42 21 15
94	CHS	16	124	f CLEAR REG	42 34
95	+	40	125	0	0
96*	STO + 3 (5)	44 40 3	126	f FIX 9	42 7 9
97*	RCL 6 (0)	45 6			
98*	RCL 1 (3)	45 1			
99*	RCL 3 (5)	45 3			
100	g RTN	43 32			
101	(f LBL 5)	42 21 5			
102	X	20			
103*	RCL 0 (2)	45 0			
104	÷	10			
105	-	30			
106	g RTN	43 32			
107	[f LBL D]	42 21 14			
108	.	48			
109	0	0			
110	1	1			
111	STO I	44 25			
112	(f LBL 4)	42 21 4			
113	RCL I	45 25			
114	g INT	43 44			
115	f FIX 0	42 7 0			
116	f PSE	42 31			
117	RCL (i)	45 24			
118	f FIX 2	42 7 2			
119	f PSE	42 31			
120*	f ISG (f ISG I)	42 6			

NOTE: Program memory is completely full, since data registers are R_0 through $R_{.0}$ and g MEM leads to

P-00 r-.0

*Indicates statements that must be changed for the HP-15C. Suggested or required change is shown in parentheses. Although the "code" for these statements will also change this is left for the 15C user to alter.

F. Formulas used:

I. Corrected sums of squares and products (designated by lower case letters) were calculated by:

$$\Sigma x^2 = \Sigma X^2 - \frac{(\Sigma X)^2}{n}, \text{ similarly for } \Sigma y^2$$

$$\Sigma xy = \Sigma XY - \frac{(\Sigma X)(\Sigma Y)}{n}, \text{ similarly for } \Sigma x_1 x_2$$

II. The least squares normal equations were assumed in "deviations from the mean" form, that is:

$$b_1\ \Sigma x_1{}^2 + b_2\ \Sigma x_1 x_2 = \Sigma x_1 y$$

$$b_1\ \Sigma x_1 x_2 + b_2\ \Sigma x_2{}^2 = \Sigma x_2 y$$

The regression coefficients, b_1 and b_2, were found using Cramer's Rule making use of a ratio of two determinants, thus:

$$b_1 = \frac{\Sigma x_1 y\ \Sigma x_2{}^2 - \Sigma x_2 y\ \Sigma x_1 x_2}{[\Sigma x_1{}^2\ \ \Sigma x_2{}^2 - (\Sigma x_1 x_2)^2] = D}$$

$$b_2 = \frac{\Sigma x_1{}^2\ \ \Sigma x_2 y - \Sigma x_1 x_2\ \Sigma x_1 y}{D}$$

then the Y-intercept, b_0, was found by:

$$b_0 = \overline{Y} - b_1\overline{X}_1 - b_2\overline{X}_2$$

Calculator: HP-11C

Program Name: ABCD--Azimuth or Bearing Correction for Declination

Author: Thomas W. Beers

Date: 1983

Purpose: To achieve the conversion of angles measured from the true north meridian to angles measured from magnetic north and vice versa. Angles can be expressed as bearings (e.g., S 25 E) or in azimuth units from north (e.g., 125 azimuth degrees) and can be in the degrees, minutes, seconds (dms) format (dd.mmss) or the decimal degrees (decimal) format (dd.dddddd).

A. Storage assignments

Register	Use
0	angle of declination: + meaning East and - meaning West
1	the quadrant in which a bearing lies: 1 = NE, 2 = SE, 3 = SW, 4 = NW
2	used variously to store the angle to be converted, an intermediate answer, or the final converted answer
3	the constant, -1, used in various formula solutions

B. Labels

Because of the complex logic of the problem, a number of label "names" are used more than once. To make the name unique in the following list, the program step where the label occurs is also given when such duplication exists.

Name	Use
[D]*	program start; generates and stores the -1 constant and stores the keyed-in declination angle (+ = E, - = W) after first converting the angle to decimal format if necessary
[A]	converts a keyed-in azimuth from magnetic to true (if flag 0 is clear) or from true to magnetic (if flag 0 is set)
[B]	separates and stores the quadrant number (Q) and the bearing (dd.mmss or dd.dd...), assuming the input angle format was Qdd.mmss (F1 clear) or Qdd.dd... (F1 set)

B. Labels *(continued)*

Name	Use
[9]	stores 1 for quadrant number (NE) and stores keyed-in bearing; leads to answer
[6]	stores 2 for quadrant number (SE) and stores keyed-in bearing; leads to answer
[5]	stores 3 for quadrant number (SW) and stores keyed-in bearing; leads to answer
[8]	stores 4 for quadrant number (NW) and stores keyed-in bearing; leads to answer
	NOTE: if labels 9, 6, 5, or 8 are used, the bearing input format does not require Q as is needed if label B is used, therefore the format is either dd.mmss or dd.dd..., as appropriate; the output (the answer), however, is in the "Q and angle" format.
[E]	clears flags 0 and 1 and as an indicator, changes the decimal fix to 2
	*brackets indicate meaningful external accessibility
2(step 27) 3(step 30)	used in label A to achieve the desired true to magnetic or magnetic to true conversion for azimuths
4(step 32)	generates a minus sign to indicate that the output is an azimuth, and converts answer to dd.mmss format if necessary (F1 clear)
0(step 77)	subroutine used to convert angles to decimal format and to change the display "fix" appropriately (FIX 4 = dd.mmss format) (FIX 6 = dd.dd... format)
2(step 86)	primary routine in the conversion of bearings from magnetic to true and vice versa and to direct control to appropriate output label: 3(step 127) if "answer" < 0 4(step 154) if "answer" > 90 1(step 105) if $0 \leq$ "answer" ≤ 90
1(step 105)	assembles the output bearing into the format Qdd.mmss or Qdd.dddddd as appropriate
0(step 117) 7(step 122)	called in label 2(step 86) to adjust the exponent in the conversion formula
3(step 127)	calculates bearing and quadrant number if bearing "answer" < 0

B. Labels *(continued)*

Name	Use
5(step 146) 6(step 150)	used by label 3(step 127) to complete the calculations
4(step 154)	calculates bearing and quadrant number if bearing "answer" >90

C. Flags

Number	Use
0	set or cleared externally to indicate the desired direction of conversion: F0 clear indicates magnetic to true F0 set indicates true to magnetic
1	set or cleared externally to indicate the desired input and output format: F1 clear indicates dd.mmss or Qdd.mmss and achieves a pause to observe keyed input F1 set indicates dd.dd... or Qdd.dd...; note that as an indicator, input is not displayed prior to the answer

D. Program procedure and examples.

I. In program mode, key in the program.

II. Proceed according to the following steps.

1.a. Ensure that USER mode is activated, and set or clear flags 0 or 1 as needed; the direction of conversion and input-output format is achieved as follows:

		Flag 1	
		Clear	Set
Flag 0	Clear	mag. to true, dms format	mag. to true, decimal format
	Set	true to mag., dms format	true to mag., decimal format

b. Pressing E will clear both flags and indicate such by a FIX 2 display.

D. Program procedure and examples *(continued)*

2.a. Key the declination, using the convention of positive number for East and negative for West declination; use the prevailing input format.

b. Press D and observe the keyed-in value (if F1 is clear) and then the equivalent angle in decimal format.

3. If the angle to be converted is a bearing proceed as in either option a, or b; if an azimuth go to step 4.

a. Key the bearing in as a number composed of the quadrant followed by the bearing itself; for example, if in dms format
for N 45° 30' 15" E, key 145.3015
for S 4° 10' 00" W, key 304.1

Then, press B and observe the input (if F1 is clear) then the answer in the same Q and angle format:
243.3030 means S 43° 30' 30" E
403.1830 means N 3° 18' 30" W

b. Alternatively, key the bearing directly, then indicate the quadrant by pressing GSB followed by one of the numbers 9, 6, 5, or 8, indicating quadrants 1, 2, 3, and 4, respectively.

(visualize these keys as

8=NW	9=NE
5=SW	6=SE

on the keyboard).

Then observe the input as it was keyed (if F1 is clear), then the answer in the Q and angle format described in 3a.

4. If the angle is an azimuth, key the number and press A, observe the keyed number (if F1 is clear) then the answer as an azimuth with a minus sign preceding it; the only purpose for the minus is to indicate the answer is an azimuth.

5. If declination remains the same, repeat steps 3 or 4 for the next angle, otherwise go to step 2.

6. To change the direction of conversion or the input-output format alter the flags as described in step 1.

III. Examples (assume USER mode)

A. Declination = 2° 30' E; convert the following magnetic bearing to true bearings (CF 0), using Qdd.mmss input format (CF 1 and use key B):

S 58° 40' E = ? (ANS. = S 56° 10' E)
N 1° 35' 30" W = ? (ANS. = N 0° 54' 30" E)

D. Program procedure and examples *(continued)*

Step	Input	Keys	Output	Comment
1.	-	E	X.XX	clear F0 and F1
2.	2.3	D	2.3000	input check
		then	2.500000	D in dec. format
3a.	258.4	B	258.4000	input check
		then	256.1000	answer = S 56° 10' E
	401.353	B	401.3530	input check
		then	100.5430	answer = N 0° 54' 30" E

B. Declination = 2° 30' E; convert the following true bearings to magnetic bearings (SF 0) using the dms format (CF 1) and the GSB 9, 6, 5, 8 procedure to indicate quadrant number:

S 56° 10' E = ?   (S 58° 40' E)
N 0° 54' 30" E = ?   (N 1° 35' 30" W)

Step	Input	Keys	Output	Comment
1.	-	g SF 0	-	set flag 0
		g CF 1	-	clear flag 1
2.	2.3	D	2.3000	input check
		then	2.500000	D in dec. format
3b.	56.1	GSB 6	56.1000	input check
		then	258.4000	answer = S 58° 40' E
	.543	GSB 9	0.5430	input check
		then	401.3530	answer = N 1° 35' 30" W

C. Declination = 8.25° W; noting that the decimal format is assumed (SF 1), convert the following magnetic bearings to true bearings (CF 0). Use B key and Qdd.dddddd input format for the first angle and the GSB 9, 6, 5, 8 approach for the second one. Furthermore convert the answer to dms format.

S 1.5° E = ?   (S 9.75° E = S 9° 45' E)
N 2.123° E = ?   (N 6.127° W = N 6° 7' 37.2" W)

Step	Input	Keys	Output	Comment
1.	-	g CF 0	-	clear flag 0
		g SF 1	-	set flag 1

D. Program procedure and examples *(continued)*

Step	Input	Keys	Output	Comment
2.	8.25	CHS, D	-8.250000	
3a.	201.5	B	209.750000	= S 9.75° E
	-	f → HMS	209.450000	= S 9° 45' E
3b.	2.123	GSB 9	406.127000	= N 6.127° W
	-	f → HMS	406.073720	= N 6° 7' 37.2" W

D. Declination = 18° 50' 30" W; convert the following magnetic azimuths to true azimuths (CF 0), assuming dms format (CF 1). Furthermore, convert the answers to decimal format.

12° 30' = ? (353° 39' 30" = 353.6583°)
285° 25' 15" = ? (266° 34' 45" = 266.5792°)

Step	Input	Keys	Output	Comment
1.	-	E	X.XX	clear F0 and F1
2.	18.503	CHS, D	-18.5030	input check
		then	-18.841667	D in dec. format
4.	12.3	A	12.3000	input check
		then	-353.3930	= 353° 39' 30"
	-	g→H	-353.6583	= 353.6583°
	285.2515	A	285.2515	input check
		then	-266.3445	= 266° 34' 45"
	-	g→H	-266.5792	= 266.5792°

E. A survey composed of magnetic bearings made in 1812 when the declination was 10° 30' W is to be re-run in 1983 when the declination is determined to be 11° 45' 15" W. Given below are the first two bearings; calculate the true bearings (CF 0), then the 1983 magnetic bearings (SF 0), assuming dms format (CF 1).

1912 magnetic bearing	true bearing	1983 magnetic bearing
N 8° 15' 30" E	? (N 2° 14' 30" W)	? (N 9° 30' 45" E)
S 85° 26' 15" E	? (N 84° 3' 45" E)	? (S 84° 11' E)

D. Program procedure and examples *(continued)*

Step	Input	Keys		Output		Comment
1.	-	E		X.XX		clear F0 and F1
2.	10.3	CHS, D		-10.3000		input check
				-10.500000		D in dec. format
3b.	8.153	GSB 9		8.1530		input check
			then	402.1430	=	N 2° 14' 30" W
	85.2615	GSB 6		85.2615		input check
			then	184.0345	=	N 84° 3' 45" E
1.	-	g SF 0		-		SF 0 for true to magnetic conversion
2.	11.4515	CHS, D		-11.4515		input check
			then	-11.754167		D in dec. format
3b.	2.143	GSB 8		2.1430		input check
			then	109.3045	=	N 9° 30' 45" E
	84.0345	GSB 9		84.0345		input check
			then	284.1100	=	S 84° 11' E

E. Program statements and code

Step	Statement	Code	Step	Statement	Code	Step	Statement	Code
01	[f LBL D][1]	42 21 14	31	+	40	61	STO 2	44 2
02	1	1	32	(f LBL 4)	42 21 4	62	2	2
03	CHS	16	33	CHS	16	63	STO 1	44 1
04	STO 3	44 3	34	g F? 1	43 6 1	64	GTO 2	22 2
05	R↓	33	35	g RTN	43 32	65	[f LBL 5]	42 21 5
06	GSB 0	32 0	36	f FIX 4	42 7 4	66	GSB 0	32 0
07	STO 0	44 0	37	f →HMS	42 2	67	STO 2	44 2
08	g RTN	43 32	38	g RTN	43 32	68	3	3
09	[f LBL A]	42 21 11	39	[f LBL B]	42 21 12	69	STO 1	44 1
10	GSB 0	32 0	40	GSB 0	32 0	70	GTO 2	22 2
11	STO 2	44 2	41	EEX	26	71	[f LBL 8]	42 21 8
12	RCL 0	45 0	42	2	2	72	GSB 0	32 0
13	STO + 2	44 40 2	43	÷	10	73	STO 2	44 2
14	g F? 0	43 6 0	44	g INT	43 44	74	4	4
15	STO - 2	44 30 2	45	STO 1	44 1	75	STO 1	44 1
16	g F? 0	43 6 0	46	g LST X	43 36	76	GTO 2	22 2
17	STO - 2	44 30 2	47	f FRAC	42 44	77	(f LBL 0)	42 21 0
18	3	3	48	EEX	26	78	f FIX 6	42 7 6
19	6	6	49	2	2	79	g F? 1	43 6 1
20	0	0	50	X	20	80	g RTN	43 32
21	RCL 2	45 2	51	STO 2	44 2	81	f FIX 4	42 7 4
22 *	f X>Y (g TEST 7)	42 20	52	GTO 2	22 2	82	f PSE	42 31
23	GTO 2	22 2	53	[f LBL 9]	42 21 9	83	g →H	43 2
24 *	g X<0 (g TEST 2)	43 10	54	GSB 0	32 0	84	f FIX 6	42 7 6
25	GTO 3	22 3	55	STO 2	44 2	85	g RTN	43 32
26	GTO 4	22 4	56	1	1	86	(f LBL 2)	42 21 2
27	(f LBL 2)	42 21 2	57	STO 1	44 1	87	RCL 2	45 2
28	-	30	58	GTO 2	22 2	88	RCL 0	45 0
29	GTO 4	22 4	59	[f LBL 6]	42 21 6	89	RCL 3	45 3
30	(f LBL 3)	42 21 3	60	GSB 0	32 0	90	g F? 0	43 6 0

[1] Brackets imply meaningful external accessibility, parentheses indicate internal use.

E. Program statements and code *(continued)*

Step	Statement	Code	Step	Statement	Code	Step	Statement	Code
91	GSB 0	32 0	121	g RTN	43 32	151	2	2
92	RCL 1	45 1	122	(f LBL 7)	42 21 7	152	STO 1	44 1
93	Y^X	14	123	1	1	153	GTO 1	22 1
94	X	20	124	STO - 1	44 30 1	154	(f LBL 4)	42 21 4
95	-	30	125	R↓	33	155	2	2
96	STO 2	44 2	126	g RTN	43 32	156	X	20
97	g F? 0	43 6 0	127	(f LBL 3)	42 21 3	157	RCL 2	45 2
98	GSB 7	32 7	128	g ABS	43 16	158	-	30
99 *	g X<0 (g TEST 2)	43 10	129	STO 2	44 2	159	STO 2	44 2
100	GTO 3	22 3	130	RCL 1	45 1	160	RCL 1	45 1
101	9	9	131	2	2	161	RCL 3	45 3
102	0	0	132 *	f X=Y (g TEST 5)	42 40	162	RCL 1	45 1
103 *	f X≤Y (g X≤Y)	42 10	133	GTO 5	22 5	163	Y^X	14
104	GTO 4	22 4	134	R↓	33	164	-	30
105	(f LBL 1)	42 21 1	135	3	3	165	STO 1	44 1
106	RCL 1	45 1	136	RCL 1	45 1	166	GTO 1	22 1
107	EEX	26	137 *	f X=Y (g TEST 5)	42 40	167	[f LBL E]	42 21 15
108	2	2	138	GTO 6	22 6	168	g CF 0	43 5 0
109	X	20	139	RCL 3	45 3	169	g CF 1	43 5 1
110	RCL 2	45 2	140	X≷Y	34	170	f FIX 2	42 7 2
111	+	40	141	Y^X	14			
112	g F? 1	43 6 1	142	X	20			
113	g RTN	43 32	143	-	30			
114	f FIX 4	42 7 4	144	STO 1	44 1			
115	f →HMS	42 2	145	GTO 1	22 1			
116	g RTN	43 32	146	(f LBL 5)	42 21 5			
117	(f LBL 0)	42 21 0	147	3	3			
118	1	1	148	STO 1	44 1			
119	STO + 1	44 40 1	149	GTO 1	22 1			
120	R↓	33	150	(f LBL 6)	42 21 6			

g MEM leads to
P - 05 r - 3

* Indicates statements that must be changed for the HP-15C. Suggested or required change is shown in parentheses. Although the "code" for these statements will also change this is left for the 15C user to alter.

F. Background and formulas used.

I. Background.

In the author's experience (admittedly limited), instructions for the conversion of magnetic to true or true to magnetic bearings frequently degenerate to "use logic" and/or "draw yourself a picture". If one does not make such conversions very often, logic becomes fuzzy and what picture to draw is not confidently apparent. In this approach it is imperative that one recall exactly what "magnetic bearing", "true bearing", and "declination" mean-- the following statements, extracted from Beers and Miller (1973), may help:

"The direction of a line is generally indicated by the angle between the line and some line of reference. The line of reference is generally a true meridian or a magnetic meridian.

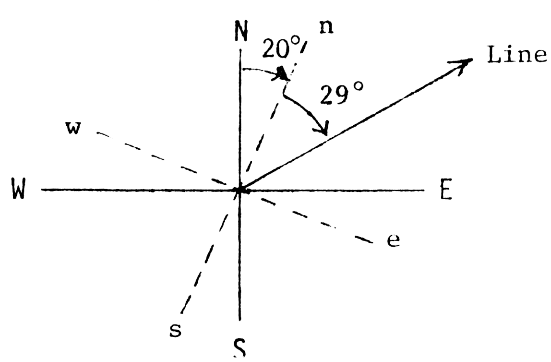
The axis on which the earth rotates is an imaginary line cutting the earth's surface at two points: the north geographic pole and the south geograpic pole. The true meridian at any place is the great circle drawn on the earth's surface passing through both poles and the place.

If a magnetized steel bar such as a compass needle is allowed to rotate on a pivot, the bar will take very nearly the same direction at any given place. The direction of the line so indicated is the magnetic meridian at the place. The magnetic meridian has the general direction of the true meridian, but varies from the true meridian by different angles at different locations.

Except in a few places, the magnetic meridian through a point on the earth's surface does not coincide with the true meridian at the point. In other words, the compass needle does not point toward the geographic poles of the earth.

The magnetic declination, D, is the angle between the true meridian and the magnetic meridian, and is considered east if magnetic north is east of true north, and west if magnetic north is west of true north. When it is desired to attach a sign to declination, east declination is considered positive, and west negative."

By combining the above concepts and using azimuth (from North) one can define the true and magnetic directions of a line as the angle measured clockwise from true or magnetic North, respectively, to the line in question. Furthermore, magnetic declination is the angle between true and magnetic north as shown in the sketch below, where the magnetic compass points are shown in lower case letters:



Declination = 20° E (=+20)

Direction of line:

magnetic azimuth = 29°
true azimuth = 49°

F. Background and formulas used *(continued)*.

When one needs to convert magnetic to true or true to magnetic directions (using azimuths) the following procedure can be deduced from the above picture:

$$\text{true} = \text{magnetic} + \text{declination} \quad (1)$$
$$\text{and magnetic} = \text{true} - \text{declination} \quad (2)$$

where "declination" is + if East and - if West.

However, when one expresses the direction of a line as a bearing (i.e., the angle is measured clockwise or counterclockwise from North or South, as needed, to maintain the angle less than 90°) the conversion process becomes more difficult to visualize. The nearest to a definitive procedure that I have found proceeds as follows (Society of American Foresters, 1955):

> *"A rule of thumb for the conversion of magnetic to true bearings is suggested. To obtain true bearings, if the declination is east, add declination to compass reading in the NE and SW quadrants and subtract declination from compass reading in the SE and NW quadrants.*
>
> *If the declination is west, subtract declination from compass reading in the NE and SW quadrants and add declination to compass reading in the SE and NW quadrants. To obtain magnetic bearings from true bearings reverse the above procedure."*

Implicit in this "rule of thumb" is the fact that declination is always a positive number (whether E or W) and one must first specify the quadrant before conversion can take place. When to add and when to subtract the declination can be indicated by memorization of , or reference to, the following "crutch":

<u>Magnetic to True</u>

<u>West Declination</u>

+	-
-	+

<u>East Declination</u>

-	+
+	-

<u>True to Magnetic</u>

<u>reverse the signs above</u>

F. Background and formulas used *(continued)*.

Since this type of solution does not lend itself to efficient programming procedures, an algorithm was developed to achieve bearing conversions assuming the declination (D) is specified as + for East and - for West, and the quadrant number (Q) is specified (NE=1, SE=2, SW=3, NW=4) for the bearing to be converted from magnetic (M) to true (T) or vice versa:

magnetic to true: $$T = M + (-1)^{Q+1}(D) \quad (3)$$

true to magnetic: $$M = T + (-1)^{Q}(D) \quad (4)$$

or, alternatively,

$$T = M - (-1)^{Q}(D) \quad (5)$$

$$M = T - (-1)^{Q+1}(D) \quad (6)$$

Just a little thought regarding the algorithm (3) and (4) or (5) and (6) will reveal that the "answer" (i.e., the after-conversion bearing may change quadrants and the answer may be greater than 90° or less than 0°, both of which can lead to unsatisfactory bearing representations. Therefore, additional relationships are needed to indicate the final <u>after</u> adjustment quadrant (Q_a) and bearing (B_a).

Let's assume that the "answer" from equations (3) and (4) or (5) and (6) is designated by β and the input quadrant as (Q_b); then three cases need addressed:

1. $0 \leq \beta \leq 90$ leads to $B_a = \beta$ (7)

and $Q_a = Q_b$ (8)

i.e., the bearing has not changed quadrants as a result of the conversion.

2. $\beta > 90$ (i.e., the bearing has "crossed" the East-West line) leads to

$$B_a = 180 - \beta \quad (9)$$

$$\text{and} \quad Q_a = Q_b + (-1)^{Q_b+1} \quad (10)$$

$$\text{or} \quad Q_a = Q_b - (-1)^{Q_b} \quad (11)$$

F. Background and formulas used *(continued)*.

3. $\beta<0$ (i.e., the bearing has crossed the North-South line) leads to

$$B_a = |\beta| \tag{12}$$

and

$$\text{if } Q_b = 1,\ Q_a = 4, \tag{13}$$

$$\text{if } Q_b = 4,\ Q_a = 1 \tag{14}$$

$$\text{or } Q_a = Q_b - 3(-1)^{Q_b} \tag{15}$$

otherwise

$$Q_a = Q_b + (-1)^{Q_b} \tag{16}$$

The reader who has not yet tired of this discussion, and who has retained a perspective on the whole conversion process might wonder: converting bearings seems too complicated, so why not convert azimuths (by the simple equations (1) and (2)) then change the azimuths to bearings? The fact is that this approach is feasible and may or may not be the desired programming approach-- the ultimate decision depends on the scope of the program and the configuration of the calculator (or computer) to be used. The necessary algorithm(s) for this approach are described below, assuming one inputs quadrant number and bearing and desires the same format in the output (the converted bearing):

1. Change the input bearing (B) to an azimuth (A) by:

if Q =	formulas is	
1 (NE)	$A = B$	(17)
2 (SE)	$A = 180 - B$	(18)
3 (SW)	$A = 180 + B$	(19)
4 (NW)	$A = 360 - B$	(20)

2. Convert from true to magnetic or vice versa using equations (1) or (2).

3. Change the converted azimuth (A) back to a bearing (B) by:

if azimuth (A) is	bearing (B) is	and	Q is	
$A<0$	$B = \|A\|$		4 (NW)	(21)
$0<A\leq 90$	$B = A$		1 (NE)	(22)
$90<A\leq 180$	$B = 180 - A$		2 (SE)	(23)
$180<A\leq 270$	$B = A - 180$		3 (SW)	(24)
$270<A\leq 360$	$B = 360 - A$		4 (NW)	(25)
$A>360$	$B = A - 360$		1 (NE)	(26)

F. Background and formulas used *(continued)*.

II. Formulas used, and logic flow:

1. In label A-- assuming azimuth input and output:

   a. equations (1) and (2) were used to convert magnetic to true (F0 clear) or true to magnetic (F0 set):

      output = input + D, if F0 clear
      output = input - D, if F0 set

   b. then:

      if output > 360; answer = 360 - output
      if output < 0 ; answer = 360 + output
      otherwise; answer = output

2. In label B-- assuming quadrant and bearing input and output:

   a. the "packed" input was first separated and stored as quadrant number and bearing.

   b. equations (5) and (6) were then used to convert from magnetic to true (F0 clear) or true to magnetic (F0 set) bearings.

   c. then:

      if output < 0, equation (12), equation (15) and a modification of equation (16) were used to calculate converted bearings and quadrants;

      if output > 90, equations (9) and (11) were used; otherwise, equations (7) and (8) apply.

   d. the final converted bearings and quadrants were then assembled and packed into the Q and angle format for display.

3. In externally accessible labels 9, 6, 5, and 8, the key depressed after GSB determines the input quadrant, enabling the bearing to be keyed as a bearing, not in packed format. Since the unpacking is not necessary, logic proceeds as in label B (above) with the unpacking step omitted.

G. Literature cited.

Beers, T.W. and C.I. Miller. 1973. Manual of Forest Mensuration. T&C Enterprises, P.O. Box 2196 West Lafayette, IN 47906.

Society of American Foresters. 1955. Forestry Handbook. Ronald Press, New York, NY.

Program No. 11F005

Calculator: HP-11C

Program Name: SIMSTRAT -- Simple and Stratified Random Sampling

Author: Thomas W. Beers

Date: January 1984

Purpose: To summarize data from either a simple or stratified sample, obtaining within stratum and overall estimates of the mean, standard error, and optionally, arbitrary confidence intervals.

A. Storage assignments

Register	15C	Use
I		Student's t used for confidence interval calculations (user input, but default value = 2.00)
0-5	(2-7)	Statistical: n, ΣX, etc.
6	(0)	N_h, population size, stratum h
7	(1)	$\overline{X}_h$, sample mean, stratum h then $\overline{X}_{st}$, overall sample mean for stratified sample.
8		$\Sigma N_h \overline{X}_h$
9		ΣN_h
.0		s_h
.1		$s_{\overline{X}_h}$ then $s_{\overline{X}_{st}}$, overall standard error; both corrected for finite population if appropriate.
.2		$\Sigma N_h^2 s^2_{\overline{X}_h}$ used in overall standard error calculation.

B. Labels

Name	Use
[A]*	program start and initialization, clears all registers and display to zero

B. Labels *(continued)*

Name	Use
[B]	after each stratum is summarized, calculates and displays stratum mean and standard error and accumulates necessary sums
0	internal use when population is finite
[1]	calculates and displays overall mean and standard error for the stratified sample
[2]	calculates and displays limits of confidence interval estimate using stored Student's t or 2.00 if no other number is stored in I
3	loop to summarize basic data
4	calculates and displays $s_{\bar{X}}$ for each stratum and performs calculations for overall $s_{\bar{X}}$ for the stratified sample
[5]	calculates and displays $s_{\bar{X}}$ (%) for individual strata

**brackets indicate meaningful external accessability.*

C. Flags

Number	Use
0	if clear, a finite population correction is applied; if set, a finite population correction is <u>not</u> applied

D. Program procedure and example

I. In PRGM mode, load program

II. In RUN mode, proceed as given below; assume the following stratified sample chosen from a finite population of size 100.

	Stratum 1	Stratum 2	Stratum 3	Total
Data:	3,0,2	12,8,15,13	18,22	
N_h	30	50	20	100
$\bar{X}_h$	1.7	12.0	20.0	$10.5 = \bar{X}_{st}$
s_h	1.53	2.94	2.83	
$s_{\bar{X}_h}$	0.84	1.41	1.90	$0.84 = s_{\bar{X}_{st}}$

D. Program procedure and example *(continued)*

1. Stratified Sampling assuming a finite population correction -- proceed as follows: (assume FIX 2 initially)

	Input	Key	Output	Example Input	Example Key	Example Output
a.	-	g CF 0	-	-	g CF 0	-
b.	-	A	0.00	-	A	0.00
	X_{11}	R/S	1.00	3	R/S	1.00
	X_{21}	R/S	2.00	0	R/S	2.00
	(Repeat for all data in stratum 1)			2	R/S	3.00
c.	N_1	B	$\overline{X}_1$ (pause)	30	B	1.7 (pause)
			$s_{\overline{X}_1}$			0.84

d. (optional) calculate the confidence interval estimate for mean of the stratum, by GSB 2; if a value of t other than 2.00 is used (say 4.303) following step is needed:

Input	Key	Output	Example Input	Example Key	Example Output
t	STO I	t	4.303	STO I	4.30

We'll here assume the default value of t, therefore simply press GSB 2:

Input	Key	Output	Example Input	Example Key	Example Output
-	GSB 2	lower (pause) upper limit	-	GSB 2	-0.007 (pause) 3.340

e. After steps b and c (and optionally, d) depress R/S (Not A!) and repeat data entry part of b and repeat step c for all strata:

Using the sample data:

	Input	Key	Output
	-	R/S	0.000
stratum 2 data	12	R/S	1.000
stratum 2 data	8	R/S	2.000
stratum 2 data	15	R/S	3.000
stratum 2 data	13	R/S	4.000
	50	B	12.0 ($\overline{X}_2$)
			1.41 ($s_{\overline{X}_2}$)

D. Program procedure and example *(continued)*

		Input	Key	Output
f.		-	R/S	0.00
	Stratum 3 data	18	R/S	1.00
		22	R/S	2.00
		20	B	20.0 ($\overline{X}_3$)
				1.90 ($s_{\overline{X}_3}$)

g. Press GSB 1 to obtain overall estimates of mean and standard error, R/S to obtain std. error as a %, then GSB 2 for confidence interval estimates for the stratified sample:

Input	Key	Output	Example Input	Example Key	Example Output
-	GSB 1	$\overline{X}_{st}$ (pause)	-	GSB 1	10.5 (pause)
		$s_{\overline{X}_{st}}$			0.84
-	R/S	$s_{\overline{X}_{st}}$ (%)	-	R/S	8.0 (%)
-	GSB 2	upper (pause)	-	GSB 2	8.820*
		lower limit			12.180*

*These are .95 confidence limits, since the default value of t (= 2.00) is used. For .99 confidence limits, store 2.6 in register I prior to pressing GSB 2.

2. Stratified sampling assuming no finite population correction is applied--set flag 0 and proceed as in step 1.

3. Simple random sampling-- proceed as in step 1 or 2 as appropriate, and assume only one stratum. But, note the following:

   a. GSB 1 is no longer appropriate, but $s_{\overline{X}}$(%) can be calculated and displayed by GSB 5.

   b. If no f.p.c. is to be used (i.e., Flag 0 set) keying in population size, N, (step 1 c) is meaningless, therefore simply press B after all the X values have been processed.

D. Program procedure and example *(continued)*

c. GSB 2 can be used to calculate confidence intervals.

Using stratum 2 as an example: (in FIX 2) *Example*

With Flag 0 clear:

Input	Key	Output
-	A	0.00
12	R/S	1.00
8	R/S	2.00
15	R/S	3.00
13	R/S	4.00
50	B	12.0 = $\overline{X}$
		1.41 = $s_{\overline{X}}$
-	GSB 5	11.8 = $s_{\overline{X}}$ (%)
3.182	STO I	3.2
-	GSB 2	7.507 = lower
		16.493 = upper

or with Flag 0 set:

Input	Key	Output
-	A	0.00
12	R/S	1.00
8	R/S	2.00
15	R/S	3.00
13	R/S	4.00
-	B	12.0 = $\overline{X}$
		1.47 = $s_{\overline{X}}$
-	GSB 5	12.3 = $s_{\overline{X}}$ (%)
3.182	STO I	3.2
-	GSB 2	7.316 = lower
		16.684 = upper

E. Program statements and code

Step	Statement	Code	Step	Statement	Code	Step	Statement	Code
01	[f LBL A]1/	42 21 11	31	÷	10	61	RCL .2	45 .2
02	f CLEAR REG	42 34	32	(f LBL 4)	42 21 4	62	RCL 9	45 9
03	2	2	33	X	20	63	g X^2	43 11
04	STO I	44 25	34	$\sqrt{X}$	11	64	÷	10
05	g CLX	43 35	35	f FIX 2	42 7 2	65	$\sqrt{X}$	11
06	(f LBL 3)	42 21 3	36	STO .1	44 .1	66	f FIX 2	42 7 2
07	R/S	31	37	g X^2	43 11	67	STO .1	44 .1
08	Σ+	49	38*	RCL 6 (0)	45 6	68	R/S	31
09	GTO 3	22 3	39	g X^2	43 11	69	[f LBL 5]	42 21 5
10	[f LBL B]	42 21 12	40	X	20	70	RCL .1	45 .1
11*	STO 6 (0)	44 6	41	RCL .2	45 .2	71*	RCL 7 (1)	45 7
12	STO + 9	44 40 9	42	+	40	72	÷	10
13	f FIX 1	42 7 1	43	STO .2	44 .2	73	EEX	26
14	g $\overline{X}$	43 0	44	f CLEAR Σ	42 32	74	2	2
15	f PSE	42 31	45	RCL .1	45 .1	75	X	20
16*	STO 7 (1)	44 7	46	R/S	31	76	f FIX 1	42 7 1
17*	RCL 6 (0)	45 6	47	g CLX	43 35	77	R/S	31
18	X	20	48	GTO 3	22 3	78	[f LBL 2]	42 21 2
19	STO + 8	44 40 8	49	(f LBL 0)	42 21 0	79*	RCL 7 (1)	45 7
20	g s	43 48	50	1	1	80	RCL I	45 25
21	STO .0	44 .0	51	GTO 4	22 4	81	RCL .1	45 .1
22	g X^2	43 11	52	[f LBL 1]	42 21 1	82	X	20
23*	RCL 0 (2)	45 0	53	2	2	83	-	30
24	÷	10	54	STO I	44 25	84	f FIX 3	42 7 3
25	g F? 0	43 6 0	55	RCL 8	45 8	85	f PSE	42 31
26	GTO 0	22 0	56	RCL 9	45 9	86	g LST X	43 36
27*	RCL 6 (0)	45 6	57	÷	10	87*	RCL 7 (1)	45 7
28*	RCL 0 (2)	45 0	58	f FIX 1	42 7 1	88	+	40
29	-	30	59*	STO 7 (1)	44 7	89	R/S	31
30*	RCL 6 (0)	45 6	60	f PSE	42 31	90	g CLX	43 35
						91	GTO 3	22 3

Note: g MEM leads to
P - 00 r - .5

1/Brackets and parentheses are not programmable; they are used to indicate externally meaningful or internal use labels, respectively.

*Indicates statements that must be changed for the HP-15C. Suggested or required change is shown in parentheses. Although the "code" for these statements will also change this is left for the 15C user to alter.

F. Formulas

I. Simple random sampling

a. mean = $\overline{X} = \frac{\Sigma X}{n}$

b. standard error

$$s_{\overline{X}} = \sqrt{\frac{s^2}{n}}$$

or

$$s_{\overline{X}} = \sqrt{\frac{s^2}{n} \left(\frac{N-n}{N}\right)}$$ if finite population is assumed

c. confidence interval

$\overline{X} \pm t\, s_{\overline{X}}$, value of t determines degree of "confidence"

d. $s_{\overline{X}}(\%) = \frac{s_{\overline{X}}}{\overline{X}}(100)$

II. Stratified sampling

a. a., b., c.,d. same as above for within-stratum estimates

d. overall mean, $\overline{X}_{st} = \frac{\Sigma N_h \overline{X}_h}{N}$

where N_h = population size for stratum h

$\overline{X}_h$ = sample mean for stratum h

and N = total population size = ΣN_h

e. overall standard error, $s_{\overline{X}_{st}} = \sqrt{\frac{1}{N^2}(\Sigma N_h^2 s_{\overline{X}_h}^2)}$

where

$s_{\overline{X}_h}^2$ = squared standard error for stratum h, corrected, if appropriate, for finite population

F. Formulas used: *(continued)*

f. confidence intervals commonly used

$t = 2$ for .05 probability level

and $t = 2.6$ for .01 probability level.

g. $$s_{\overline{X}_{st}}\ (\%) = \frac{s_{\overline{X}_{st}}}{\overline{X}_{st}}(100)$$

Program No. 11F006

Calculator: HP-11C (and HP-15C with minor changes)

Program Name: FICALC-11/15 FInancial CALCulations on the HP-11C or HP-15C

Author: Thomas W. Beers

Date: April 1985

Purpose: To solve for certain unknown values among the usual components of simple or compound interest problems. Specifically, with the following definitions:

n = number of periods over which compounding takes place
I = periodic interest rate expressed as a percent
i = periodic interest rate expressed as a decimal
PV = present value
PMT = periodic payments made for the n periods
FV = future value
NAR = the stated or "nominal" annual rate of interest expressed as a percent
APR = annual percentage rate; the same as NAR
EAR = effective annual rate in percent; the amount that \$100 would earn in one year with annual compounding (i.e., simple interest),

The program can provide the solutions listed below.

1. Assuming a simple interest framework-- any one of I, PV, or FV can be obtained when the other two of these components are supplied as input; n is here assumed to be 1, usually 1 year.

2. Assuming a compound interest framework with payments equal to zero-- any one of n, I, PV, or FV can be obtained when the other three components are supplied as input.

3. Assuming, additionally, that periodic payments are made, i.e., PMT $\neq$ 0-- any one of n, PV, PMT, or FV can be obtained when I <u>and</u> the other three components are supplied as input.

4. "Effective" compound rates of interest (EAR) on an annual basis can be obtained if n (here defined to be the number of compounding periods per year) and the nominal <u>periodic</u> rate ($\frac{NAR}{n}$) are properly located in the calculator.

A. Storage Assignments

Register	Use
I	the intermediate calculation B (see section G)
0	the intermediate calculation C (see section G)
1	n
2	i (note, this is interest rate as a decimal)
3	PV
4	PMT
5	FV
6	the intermediate calculation A (see section G)

B. Labels

Name	Use
[A]*	stores n keyed in, or calculates n
[B]	stores i when keyed in as I, or calculates I
[C]	stores PV keyed in, or calculates PV
[D]	stores PMT keyed in, or calculates PMT
[E]	stores FV keyed in, or calculates FV
[0]	initializes the registers (all cleared to zero except R1 which is set to 1), clears flag 0, and maintains the current status of flag 1 (payments made at BEG or END of period)
[1]	sets flag 1 (selecting BEG mode) then proceeds to label 0
[2]	clears flag 1 (selecting the END mode) then proceeds to label 0
3	solves for and displays n — if flag 0 is set
4	solves for i and displays I — if flag 0 is set
5	solves for and displays PV — if flag 0 is set
6	solves for and displays PMT — if flag 0 is set
7	solves for and displays FV — if flag 0 is set
8	displays π (3.14), indicating the end of the input phase, and optionally provides the calculation and display of EAR
9	sets flag 0, enabling the use of labels 3 to 7, and calculates and stores A, B, and C. (see section F)

*brackets indicate meaningful external (i.e., in RUN mode) accessibility.

C. Flags

Number	Use
0	clear indicates the input phase, i.e., a number just keyed will be stored appropriately in R1, R2, R3, R4, or R5 set implies the calculation phase, and when keys A through E are pressed, program control passes to labels 3 through 7, respectively, to solve for the appropriate component note: the status of flag 0 is program controlled and it is unnecessary to alter its state from the keyboard.
1	set (achieved by pressing GSB 1) implies that the BEG mode for payments is assumed; this condition is indicated visually by the presence of the GRAD annunciator in the display clear (achieved by pressing GSB 2) implies that the END mode for payments is assumed; here the GRAD annunciator is not displayed.

D. Program Procedure in General

1. INITIALIZE

Performed to clear all registers to zero (except R1 is made = 1), and prepare the calculator for the input of new data. This step is not always necessary, since minor changes in the problem can be made, such as changing only the interest rate, leaving the other data in tact. If, however, the input data are to be substantially changed, it is wise to perform one of the following, as appropriate:

a. GSB 0 — this initializes the calculator and maintains the current status of the BEG/END mode.

b. GSB 1 — this initializes as in a. but additionally sets flag 1 causing GRAD to appear in the display, indicating the BEG (of period) payment mode is active.

c. GSB 2 — this initializes as in a. but additionally clears flag 1 causing the GRAD annunciator to disappear from the display, indicating the END (of period) payment mode is active.

2. INPUT DATA

a. In USER mode, key the data given in the problem. Then, after each, press the appropriate label key indicated below; the data are stored by the program as shown:

	A	B	C	D	E
	n	I(%)	PV	PMT	FV
stored in:	R1	R2 (stored as i, a decimal)	R3	R4	R5

NOTE, the following constraints apply to the input data:

(1) n = number of compounding intervals; if payments are involved in the problem, the number of payments must be the same as the number of compounding intervals.

(2) I = % interest (periodic) is keyed as a percent although it is stored as a decimal (i),

(3) the usual + and - cash flow conventions are followed for PV, PMT, and FV (see specific examples)

(4) if PMT = 0, this implies a "single payment" problem, and any 3 of n, I, PV and FV must be input to find the fourth. Implication: interest (I) can be the solved-for unknown, just as n, PV, or FV can be.

(5) if PMT ≠ 0, this implies an "annuity problem" with or without an initial payment (PV ≠ 0 or PV = 0), and the interest rate must be one of the input variables plus any 3 of n, PV, PMT, and FV to find the unknown. Implication: interest (I) cannot be a solved-for unknown; an attempt to do so will lead to "ERROR 3"

3. SOLUTION

After sufficient data have been properly input:

a. Press R/S, and observe 3.14, which indicates that the key for the desired unknown component can now be pressed.

b. Press the "unknown" key and read the answer.

NOTE, "ERROR 3" means you tried to solve for I when PMT ≠ 0; not possible with this program.

c. (optional) To find EAR (effective annual rate) for compounding other than annual:

(1) GSB 0, or GSB 1, or GSB 2
(2) key n = compounding frequency per year (monthly = 12, quarterly = 4, daily = 365), and press A

(3) key nominal <u>periodic</u> rate (i.e., key annual rate, ENTER, key n, ÷, and press B
(4) press R/S and read 3.14
(5) press R/S and read the desired EAR as a percent shown to 3 decimal places.

4. NEW PROBLEM

If only minor changes are to be made in the input data, simply key the new data, press the appropriate label key and proceed with the SOLUTION steps.

If major changes are necessary, return to the INITIALIZE steps.

E. Examples

The following arbitrary examples are meant to introduce the user to the variety of problems which can be solved by FICALC-11/15. A certain understanding of "interest problems" is assumed. Cash flow diagrams are shown where appropriate and the user not familiar with this method of phrasing the problem is encouraged to study the diagrams to get a feel for their use. Such observation will make obvious that amounts received, such as borrowed money or accumulated savings are considered positive (upward line), amounts paid out, such as deposits into a savings account, into an IRA, or payments on a loan are negative (downward line); that the completed diagram must have at least one + and one - line; and that payments must be specified as to the beginning or end of the compounding period. It is also helpful to remember that each problem should be viewed from the standpoint of the lender/investor, but not both viewpoints in the same problem.

This program is meant primarily to solve compound interest problems, therefore those dealing with simple interest will be described last and in less detail. Consult the following directory (Table 1) to determine the exact nature of the examples.

Table 1. Nature of Problems Described in Examples Which Follow.

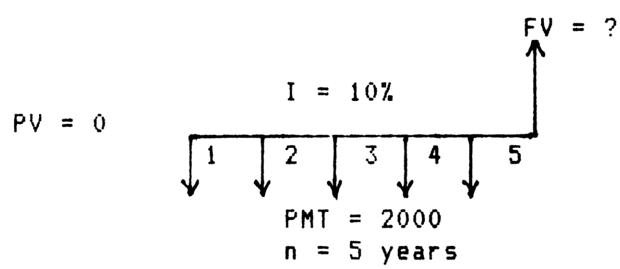
Example Number	Page	Type of Problem	Specific Nature
1.		IRAs and Saving Accounts	
a.	61	IRA	given n, I, PMT; find FV; annual compounding
b.	62	IRA	change n; find FV
c.	62	IRA	fix FV = 1,000,000; find n
d.	62	IRA	change I; find n
e.	62	savings account	given n, I, PV, PMT; find FV; monthly compounding
f.	63	savings account	find EAR for previous problem
g.	63	savings account	change compounding to daily; find EAR
h.	64	savings account	given n, I, PV, PMT; find FV; weekly compounding
2.		Loans	
a.	64	automobile	given n, I, PV, FV = 0; find PMT; monthly compounding
b.	65	automobile	change I; find PMT
c.	65	automobile	find exact FV after last payment
d.	65	automobile	given I, PV, PMT, FV = 0; find n and amount of interest
e.	66	automobile	given I with "simple interest"; better deal?
f.	66	automobile	find the corresponding compound interest, I
g.	67	home improvement	given n, I, PMT, FV = 0; find PV; monthly compounding
h.	68	home improvement	find loan balance after given number of months
i.	68	credit card	given n, I, PV, PMT = 0; find FV; monthly compounding
j.	69	credit card	find EAR in previous problem
k.	70	automobile	practice problems
3.		Home Purchase	
a.	70	regular mortgage	given n, I, PV, FV = 0; find PMT; monthly compounding
b.	71	regular mortgage	change n; find PMT
c.	71	regular mortgage	change n back, change I; find PMT
d.	71	regular mortgage	find exact FV after last payment
e.	72	contract sale	given n, I, PV, FV = 0; find PMT and "balloon payment"
f.	72	contract sale	find amount of interest paid in previous problem

Table 1. (continued)

Example Number	Page	Type of Problem	Specific Nature
4.		Effective Annual Rates	
a.	73	EAR, discrete compounding	given NAR, C (compounding frequency); find EAR
b.	75	EAR, continuous compounding	given NAR; find EAR
c.	75	EAR, 365/360 basis	given NAR; find EAR
d.	76	NAR, discrete compounding	given EAR, C; find NAR
e.	76	NAR, continuous compounding	given EAR; find NAR
5.		Cash Flow Analysis	
a.	77	net present value (NPV)	given n, I, series of cash flows (CF); find NPV
b.	78	modified internal rate of return (MIRR)	given n, two interest rates, and a series of cash flows; find MIRR
6.		Simple Interest	
a.	82	annual period	given n = 1, PV, I; find FV
b.	82	annual period	given n = 1, PV, FV; find I
c.	82	annual period	given n = 1, I, FV; find PV
d.	82	multi-year	given n, PV, I; find FV
e.	82	partial year	given no. of days, PV, I; find FV

1. IRAs and Savings Accounts

a. An IRA is to be established, depositing $2000 in an account on Jan. 1 for each of the next 5 years. Assuming 10 percent interest, compounded annually, what is the accumulated value (FV) at the end of the 5 years? NOTE, the payments are made at the beginning of each period.



SOLUTION

Input	Key(s)	Display	Comment
-	GSB,1	0.00	"GRAD" indicates BEG mode
5	A	5.00	# periods = n
10	B	0.10	I input, i displayed
2000	CHS,D	-2000.00	periodic payment
-	R/S	3.14	input complete
-	E	13,431.22	FV = future value

b. What would the accumulation be if such payments were made for 35 years?

SOLUTION (previous data still in place)

35	A	35.00	provide new n
-	R/S	3.14	
-	E	596,253.61	BIG BUCKS!

c. How long would it take to accumulate one million dollars?

SOLUTION (previous data still in place)

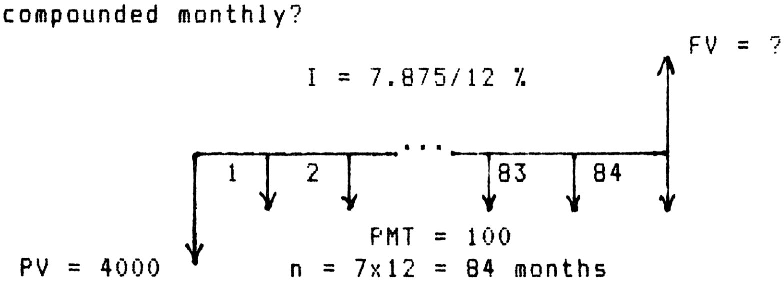
1000000	E	1,000,000.00	FV
-	R/S	3.14	
-	A	40.27	n = 40.27 years

d. If I managed to improve on the interest rate, say to 12 percent, how many years would it take?

SOLUTION (previous data still in place)

12	B	0.12	i = 0.12
-	R/S	3.14	
-	A	35.29	5 years sooner!

e. A credit union savings account present balance (start of month) is $4000. If I arrange for monthly deposits from my paycheck of $100 starting at the end of this month, how much would be accumulated after 7 years, if the nominal annual rate is 7.875%, compounded monthly?



SOLUTION

Input	Key(s)	Display	Comment
- 1/	GSB,2	0.00	END mode selected
84 or 7↑12X	A	84.00	n = 84
7.875 ↑ 12 ÷	B	0.01	actual i = 0.0065625
4000	CHS,C	-4000.00	PV
100	CHS,D	-100.00	monthly PMT
-	R/S	3.14	
-	E	18,088.03	FV

NOTE: Because of the way the problem is phrased, the last monthly payment will accumulate no interest, but this must be tolerated, due to the usual constraint that the number of payments and the number of compounding periods must be equal. If this were not the case, the program and the formulas cited in Section G do not apply.

f. In the previous problem the stated nominal annual rate was 7.875, compounded monthly. What is the effective annual rate (EAR)?

SOLUTION

-	GSB,0	0.00	BEG/END mode immaterial
12	A	12.00	n = compounding frequency per year
7.875↑12÷	B	0.01	i = 0.0065625 = monthly nominal rate
-	R/S	3.14	
-	R/S	8.166	EAR

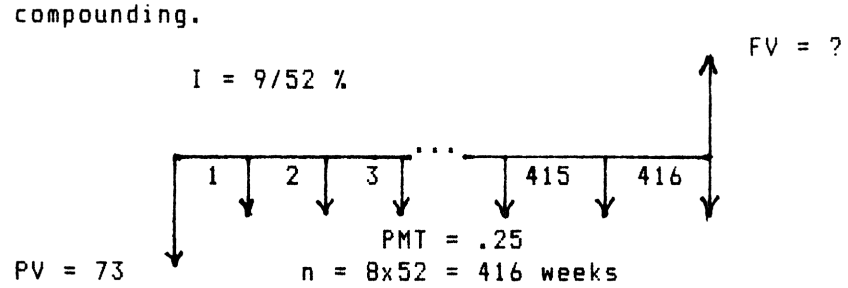
g. Would the effective annual rate change much if compounding took place daily?

SOLUTION

365	A	365.000	n = 365
7.875↑365÷	B	2.158 -04	i = 0.0002158 = daily nominal rate
-	R/S	3.142	
-	R/S	8.192	= EAR; not much change

1/ The symbol ↑ is used in this paper as short notation for the ENTER key.

h. Shortly after President Kennedy's assassination, an acquaintance of mine started collecting "Kennedy" dollar bills. He did this up until 1976 when the mood changed to collecting "Drummer-boy" quarters. In 1984, the collection box contained 73 dollar bills and 104 dollars in quarters (an average of one quarter per week for eight years). Ignoring any collecting value appreciation, what would the accumulation have been if the $73 had been deposited in a savings account in 1976 and one quarter per week added for eight years? Assume 9% nominal annual rate and <u>weekly</u> compounding.



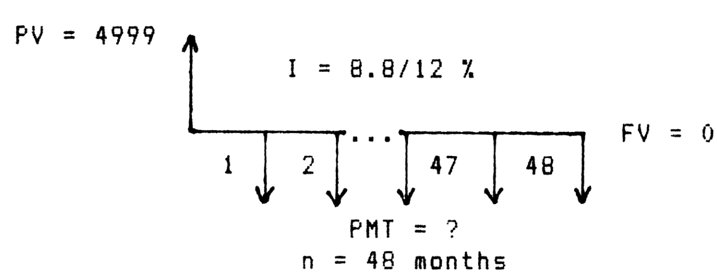
<u>SOLUTION</u>

Input	Key(s)	Display	Comment
-----	------	-------	-------
-	GSB,2	0.00	END mode
416	A	416.00	n = 416
9↑52÷	B	1.73 -03	actual i = 0.001731
73	CHS,C	-73.00	PV = 73
.25	CHS,D	-0.25	PMT = .25
-	R/S	3.14	
-	E	302.00	FV = 302.00

Since only $177 is in the box, my friend lost $125 by not putting the money in the account.

2. Automobile Purchase, Home Improvement Loans, Credit Cards.

a. A new car is advertised as being available for $4999.00 (after trade in and cash down payment). You can borrow this amount at 8.8% nominal annual rate and make payments over 48 months. Calculate the monthly payment. Note, you are paying off the loan gradually and at the end of the 48 months the amount you owe should be zero, i.e., FV = 0.



SOLUTION

Input	Key(s)	Display	Comment
-	GSB,2	0.00	END mode
48	A	48.00	n = 48
8.8 ↑ 12 ÷	B	0.01	actual i = 0.007333...
4999	C	4999.00	PV
-	R/S	3.14	
-	D	-123.93	monthly PMT

b. Say that another dealer is offering an 8.5% interest rate, how much do the payments change?

SOLUTION (previous data still in place)

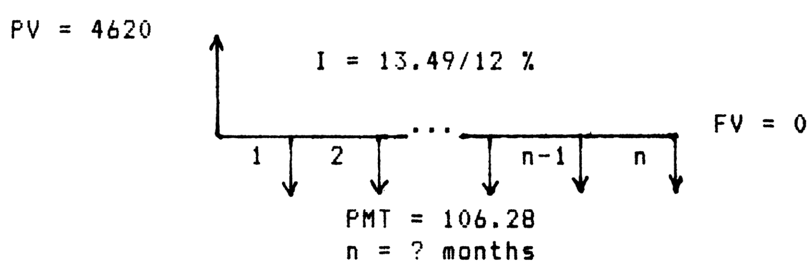
8.5 ↑ 12 ÷	B	0.01	
-	R/S	3.14	
-	D	-123.22	Not much change (.71 per month)

c. Assuming you go with the 8.5% loan, and make 48 payments of exactly 123.22, how much do you still owe? (FV = ?)

SOLUTION

123.22	CHS,D	-123.22	exact PMT
-	R/S	3.14	
-	E	0.18	probably added on to the last payment

d. Another car dealer (in the absence of manufacturer's subsidies) advertises a truck can be bought for $106.28 per month, by financing $4620.00 at 13.49 APR (annual percentage rate). How many months must you pay this and how much money in interest will you have paid?



SOLUTION

Input	Key(s)	Display	Comment
-	GSB,2	0.00	END mode
13.49↑12 ÷	B	0.01	actual i = 0.0112417
4620	C	4620.00	PV
106.28	CHS,D	-106.28	PMT
-	R/S	3.14	
-	A	60.00	n = 60 months

Since 60 x \$106.28 = \$6376.80 and you would borrow \$4620.00, the amount of interest paid is the difference: \$1756.80.

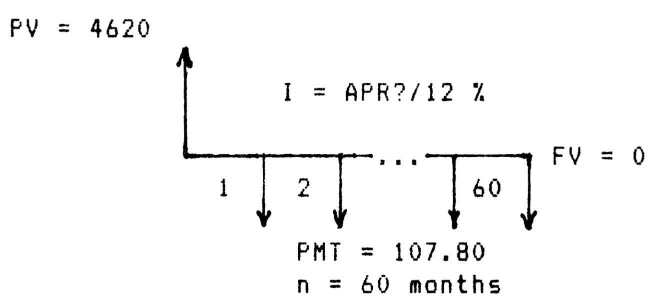
e. Yet another dealer offers the same deal at 8% "simple interest"; is this a better deal?

SOLUTION

Using simple interest, finance charges are calculated in this type of "deal", as 4620.00 x .08 x 5 years = \$1848.00. Therefore you would pay less interest under the previous plan (13.49% APR, with monthly compounding).

f. If the 8% simple interest dealer says you can pay the principal (\$4620.00) plus the interest (\$1848.00), total = \$6468.00, in easy monthly payments of 6468/60 = \$107.80, you should think!, and realize this is no longer a simple interest deal. Simple interest would mean that you make no payments until the end of the five years, therefore having use of the \$4620 for the full time of the loan. The next question that begs an answer is, if the deal is accepted, making the monthly payments of \$107.80, what is the actual corresponding compound interest rate (monthly compounding) being paid?

The cash flow diagram is



Unfortunately, as pointed out in the constraints cited in Section D, the program cannot directly solve for I (since PMT ≠ 0). However we can "zero-in" on the APR by trial and error as follows:

Comparing the PMT of $107.80 here with the PMT corresponding to APR = 13.49% in example d. ($106.28), let's try APR = 14%:

Input	Key(s)	Display	Comment
-	GBS,2	0.00	END mode
60	A	60.00	n = 60
14↑12÷	B	0.01	
4620	C	4620.00	PV
107.80	CHS,D	-107.80	PMT
-	R/S	3.14	
-	E	25.92	FV, too high

Since the goal is to find the APR that reduces FV to zero, try APR = 14.2

14.2↑12÷	B	0.01	
-	R/S	3.14	
-	E	-15.51	FV, too low

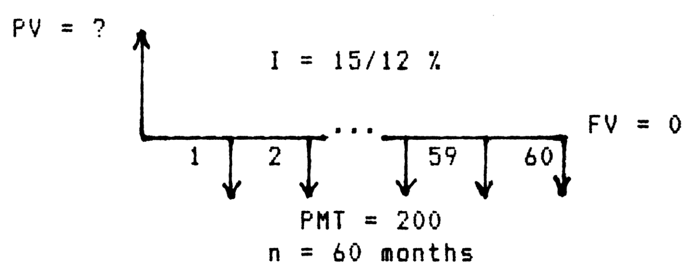
APR must be between 14.0 and 14.2, try 14.1

14.1↑12÷	B	0.01	
-	R/S	3.14	
-	E	5.27	FV, too high

Try APR = 14.12

14.12↑12÷	B	0.01	
-	R/S	3.14	
-	E	1.13	Close enough! (or continue, if preferred)

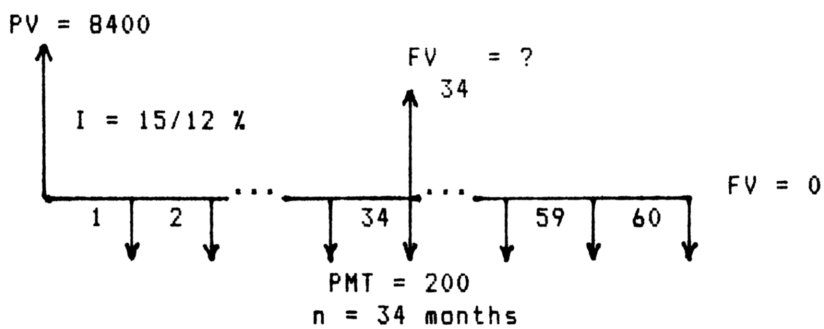
g. You want to add a new room to your house and can spring for $200 in monthly payments. If the local bank offers money at 15% compounded monthly, how much can you borrow if you want to pay off the note in 5 years?



SOLUTION

Input	Key(s)	Display	Comment
-	GSB,2	0.00	END mode
60	A	60.00	n = 60
15↑12÷	B	0.01	actual i = 0.0125
200	CHS,D	-200.00	PMT
-	R/S	3.14	
-	C	8406.92	PV = amount you can borrow

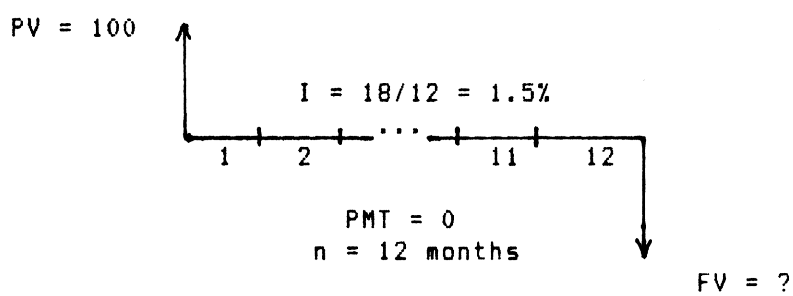
h. Say you hit the lottery just after the 34th payment and can pay the loan off. How much is needed? That is, what is the FV at the end of the 34th month; assume you actually borrowed $8400.



SOLUTION (previous data still in place)

Input	Key(s)	Display	Comment
34	A	34.00	n = 34
8400	C	8400.00	PV = amount borrowed
-	R/S	3.14	
-	E	-4405.70	FV_{34} = pay off amount

i. Your credit card bill carries the statement that you are charged 1.5% interest on the unpaid monthly balance. This is equivalent to the "18% annual rate" (since 18/12 = 1.5), which they are required to state also. If you have $100 balance at the beginning of Jan., how much do you owe at the end of Dec. that year?



SOLUTION

Input	Key(s)	Display	Comment
-	GSB,0	0.00	BEG/END mode immaterial
12	A	12.00	n = 12
1.5	B	0.02	actual i = 0.015
100	C	100.00	PV = 100
-	R/S	3.14	
-	E	-119.56	FV = amount due, end of Dec.

j. What is the effective annual interest rate (EAR) that this 1.5% monthly rate equates to?

SOLUTION (previous data still in place)

Input	Key(s)	Display	Comment
-	R/S	3.14	
-	R/S	19.562	EAR

Note that the 19.562% is the same as the number of dollars that the $100 would cost you for one year! That's the meaning of EAR in this context.

k. Need more practice? Verify the entries in the table below, solving either for PMT or PV (amount financed). Note that since these represent loans to be paid off, FV = 0; furthermore, the interest rate, I, cannot be solved for directly by the program since PMT $\neq$ 0.

8.8%

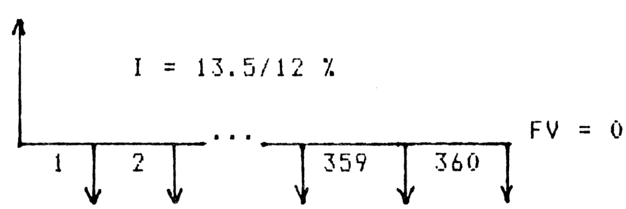
ON CAVALIERS AND S-10'S

AMOUNT FINANCED	MONTHLY PAYMENTS 48 MONTH CAVALIER	60 MONTH S-10
3,000	74.37	61.98
4,000	99.16	82.64
5,000	123.95	103.30
6,000	148.74	123.96
7,000	173.53	144.62
8,000	198.32	165.29
9,000	223.11	185.95
10,000	247.90	206.61

3. Home Mortgages and Contract Sales

a. You plan to purchase a house worth \$145,000 and can muster \$20,000 as a down payment, thus needing to borrow \$125,000. Assuming a 30-year mortgage (i.e., "amortized over 30 years"), monthly compounding, and annual interest rate of 13.5%, what are the monthly payments?

PV = 125,000



PMT = ?
n = 360 months

SOLUTIONS

Input	Key(s)	Display	Comment
-----	------	-------	-------
-	GSB,2	0.00	END mode
360	A	360.00	n
13.5↑12÷	B	0.01	actual i = 0.01125
125000	C	125,000.00	PV (note FV = 0 was achieved by GSB,2)
-	R/S	3.14	
-	D	-1431.77	PMT

b. How much would the payments change if a 40-year mortgage is available?

SOLUTION (previous data still in place)

40↑12X	A	480.00	revised n
-	R/S	3.14	
-	D	-1412.83	not much change in PMT amount

c. Reverting to the 30-year mortgage, what would the payments be if I = 12%?

SOLUTION (previous data still in place)

360	A	360.00	n
13↑12÷	B	0.01	actual i = 0.0108333
-	R/S	3.14	
-	D	-1382.75	PMT

So it looks like reducing the nominal annual interest rate from 13.5% to 13% saves more per month (1431.77 - 1382.75 = $49.02) than extending the loan by ten years (1431.77 - 1412.83 = $18.94)!

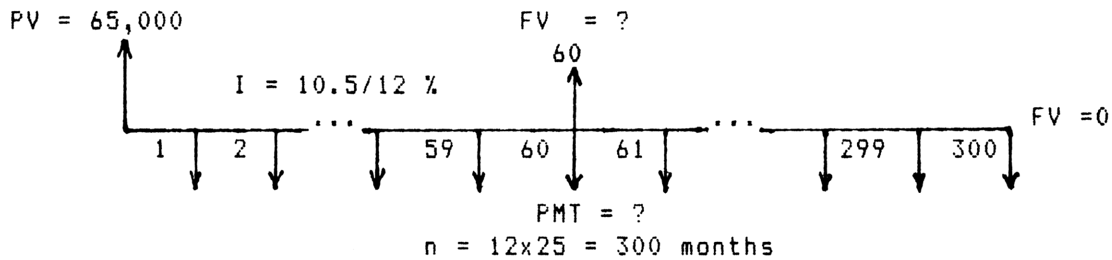
d. If our monthly payments were exactly 1382.75 as calculate in c. would the future value come out to exactly zero or would there be a final settlement? If so how much?

SOLUTION (previous data still in place)

1382.75	CHS,D	-1382.75	exact PMT
-	R/S	3.14	
-	E	2.62	FV

Therefore, after the 360th payment we would still owe $2.62, which would probably be added to that payment.

e. Say, on the same $145,000 house we can get a contract arrangement at 10.5% amortized over 25 years, can muster $80,000 down (leaving $65,000 to be paid on the contract) but we must pay the remaining balance after 5 years. What is the monthly payment? and What is the "balloon" payment due at the end of the 5 years?



SOLUTION

Input	Key(s)	Display	Comment
-	GSB,2	0.00	END mode
25↑12X	A	300.00	n
10.5↑12÷	B	0.01	actual i = 0.00875
65000	C	65,000.00	PV
-	R/S	3.14	
-	D	-613.72	PMT per month
613.72	CHS,D	-613.72	this step needed only to round PMT to exactly 613.72
60	A	60.00	n for the 5 yr. period
-	R/S	3.14	
-	E	-61,471.25	FV = balloon payment needed immediately after the 60th payment is made

f. Assuming you terminate the contract after five years by paying the balance due ($61,471.25), how much have you paid in interest over the five years?

The program will not directly answer this question but applying logic and knowledge as to where the various components are stored:

SOLUTION (previous data still in place)

-	-	-61,471.25	balance due is still in display
-	RCL 3	65,000.00	FV = amount of loan
-	+	3528.75	reduction in principal
-	RCL 4	-613.72	monthly payment

<u>SOLUTION</u> (previous data still in place, continued)

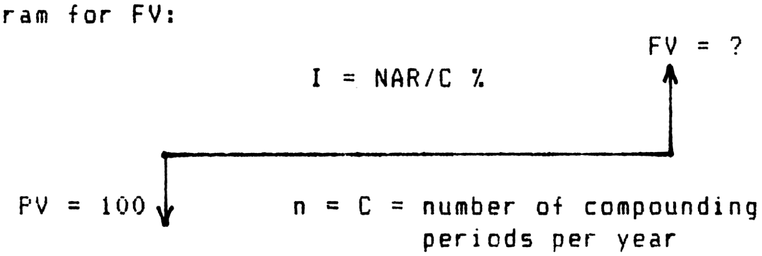
Input	Key(s)	Display	Comments
-	RCL 1	60.00	n = no. of payments
-	X	-36,823.20	amount of interest plus reduction in principal
-	+	-33,294.45	amount of interest paid over the 5 years.

The last operation (pressing + key) performed the addition of -$36,823.20 and the reduction in principal ($3528.75) which was still in the stack, where it was needed, the Y register.

4. Effective Annual Rates

When confronted with nominal annual rates of interest (NAR) and the frequency of compounding (C) it is occasionally of interest (no pun intended!) to determine the effective annual rate (EAR), which has been pointed out earlier to be the same as the amount that $100 would earn for one year assuming simple interest.

One <u>could</u> use the program and essentially solve the cash flow diagram for FV:



Then, EAR = FV - 100. However, because of certain intermediate calculations routinely made in the program, a simple procedure, cited in the next example (and also used in earlier examples), is recommended.

a. Complete the table below, finding the effective annual rates (answers are shown in the table).

Nominal Annual Rate (NAR)	Type of Compounding and the Resulting Compounding Frequency (C)	Effective Annual Rate (EAR)
%		%
7.875	semi-annual (2)	Ans. = 8.030
7.875	quarterly (4)	Ans. = 8.111
7.875	monthly (12)	Ans. = 8.166
8.250	monthly (12)	Ans. = 8.569
8.250	weekly (52)	Ans. = 8.593
9.000	daily (365)	Ans. = 9.416
9.000	"continuous"	Ans. = 9.417 (see example b.)

SOLUTION

Input	Key(s)	Display	Comment
-	GSB,0	0.00	BEG/END mode immaterial
2	A	2.00	n = C = 2
7.875↑2÷	B	0.04	actual i = 0.039375
-	R/S	3.14	
-	R/S	8.030	EAR = 8.030%
4	A	4.000	n = C = 4
7.875↑4÷	B	0.020	actual i = 0.0196875
-	R/S	3.142	
-	R/S	8.111	EAR = 8.111%
12	A	12.00	n = C = 12
7.875↑12÷	B	0.007	actual i = .0065625
-	R/S	3.142	
-	R/S	8.166	EAR = 8.166%
-	-	8.166	note, n is still 12
8.25↑12÷	B	0.007	actual i = 0.006875
-	R/S	3.142	
-	R/S	8.569	EAR = 8.569%
52	A	52.000	n = C = 52
8.25↑52÷	B	0.002	actual i = 0.001586538
-	R/S	3.142	
-	R/S	8.593	EAR = 8.593%
365	A	365.000	n = C = 365
9↑365÷	B	2.466 -04	actual i = 0.000246575
-	R/S	3.142	
-	R/S	9.416	EAR = 9.416% (actually 9.4162%)

(1) It is worth noting that all these answers could be obtained also by using the formula

$$EAR = 100[(1 + \frac{NAR}{100C})^{C} - 1]$$

For example, 9% NAR under daily compounding (in the above table) converts to

$$EAR = 100[(1 + \frac{9}{36500})^{365} - 1]$$

$$= 9.4162\%$$

(2) Yet another alternative procedure is suggested in the HP-12C Owner's Handbook (p. 179); that is, let the program solve for FV, after letting n = C,

$$I = \frac{NAR}{C}, \text{ and } PMT = -\frac{NAR}{C}$$

SOLUTION (using the previous example)

Input	Key(s)	Display	Comment
-	GSB,2	0.00	END mode
365	A	365.00	n = C = 365
9↑365÷	CHS,D	-0.02	PMT = NAR/C
-	CHS,B	2.47 -04	i = 0.000246575
-	R/S	3.14	
-	E	9.42	FV = EAR = 9.4162%

b. The theoretical maximum for effective annual rate computations occurs under "continuous" compounding. the FICALC-11/15 program will not calculate this EAR (call it EAR_c). However, the computational formula is simple:

$$EAR_c = 100[e^{NAR/100} - 1], \text{ therefore}$$

SOLUTION (for the last line in the above table)

key stroke sequence: 100, ↑, 9, ↑, 100, ÷, fe^x, 1, -, X
result = EAR_c = 9.4174284%

caution: fe^x was necessary since the calculator is still in USER mode (presumably).

A slightly shorter (and more abstruse!) key stroke sequence, making use of the % and Δ% functions would be:

1, ↑, 9, g%, fe_x, gΔ%

result = 9.417%, as before,

c. Occasionally there is reference to a compounding procedure which will lead to even higher rates than the theoretical maximum cited in b. It is the "365/360 basis of compounding". Effective annual rate is defined in this method as

$$EAR = 100[(1 + \frac{NAR}{36000})^{365} - 1]$$

thus, if NAR = 9%

$$EAR = 100[(+ \frac{9}{36000})^{365} - 1]$$

$$= 9.553\%$$

d. Sometimes there is a need to determine the nominal annual rate (NAR) if the effective annual rate (EAR) is given. For example, if the EAR is 8.569, what is the NAR assuming monthly compounding? (line 4 in the table given in example a.)

A fairly direct procedure for solving this problem is to use a procedure similar to that used in a. (2), also suggested in the HP-12C Owners Handbook (p. 180). That is, use the program to solve for the periodic I after letting FV = 100 + EAR, PV = 100, and n = C; then, NAR = (C)(I) = (n)(I).

SOLUTION (data from line 4 in Table in example a.)

Input	Key(s)	Display	Comment
-	GSB,0	0.00	BEG/END immaterial
12	A	12.00	n = C = 12
100	CHS,C,CHS	100.00	PV = -100, then sign changed to simplify next step.
8.569	+,E	108.57	FV = 108.57
-	R/S	3.14	
-	B	0.69	I = NAR/C = 0.6874835
12	X	8.25	NAR = 8.25%

e. If the assumption of continuous compounding prevails and one is given the effective annual rate (EAR_c), to find the NAR the inverse of the procedure used in example b. can be employed, the formula being, therefore

$$NAR = 100[\ln (\frac{EAR_c}{100} + 1)].$$

Find the NAR corresponding to EAR_c of 9.417% under the continuous compounding assumption.

SOLUTION

keystroke sequence: 9.417, ↑, 100, ÷, 1, +, gLN, 100, X

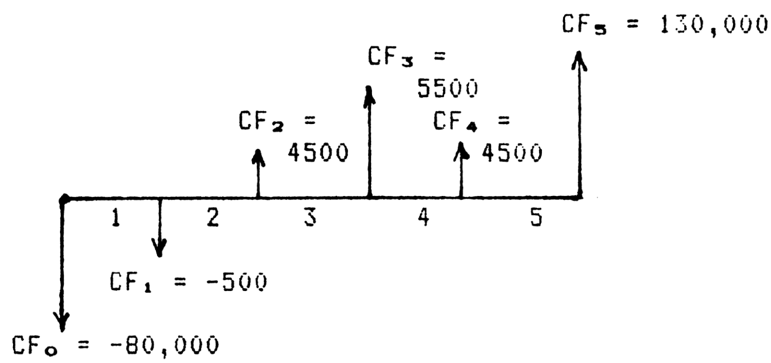
result = NAR = 9.00%

5. Cash Flow Analysis

Two common ways of making this type of analysis are called NPV (net present value) and IRR (internal rate of return). If one is faced with many such analyses he should find his way to the nearest HP-12C, HP-41C equipped with the Financial Module, or other such computer. For an occasional solution, however, the FICALC-11/15 program can be used repeatedly to discount future cash flows to the present, then add the present values so obtained to determine the NPV.

As an example, consider the problem below, extracted directly from the HP-12C Owners Handbook (p. 68).

a. An investor has an opportunity to buy a duplex for $80,000 and would like a return of at least 13% [compounded annually]. He expects to keep the duplex 5 years and then sell it for $130,000; and he anticipates the cash flows shown in the diagram below. Calculate NPV to determine whether the investment would result in a return or a loss.



SOLUTION

Using FICALC-11/15 the solution is obtained by considering each of the CF_1 through CF_5 to be future values (FV) and finding the corresponding present value (PV), taking into account the 13% annual compounding and the pertinent number of years for each. Then the PVs are added to the initial investment (CF_0) to determine profit (positive NPV) or loss (negative NPV).

CAUTION

The FICALC-11/15 program assumes the PV and FV to be opposite in signs, therefore one must either mechanically reverse the input or the visually reverse the output sign in order to achieve a meaningful NPV. It seems more logical to reverse the input FV sign and record the output PV as in the display.

This is the approach taken in the following table; the entries in the last column were obtained as shown below the table.

Year	Nature of Item	Amount	Present Value
0	initial investment (CF_0)	80,000	-80,000
1	payment (CF_1)	500	-442.48
2	income (CF_2)	4,500	3,524.16
3	income (CF_3)	5,500	3,811.78
4	income (CF_4)	4,500	2,759.93
5	sale of house (CF_5)	130,000	70,558.79
		NPV =	212.18

Input	Key(s)	Display	Comment
-	GSB 0	0.00	BEG/END mode immaterial
13	B	0.13	i = 0.13
500	E	500.00	CF_1, although a payment, is input as a positive FV; also note n = 1 is automatically achieved
-	R/S	3.14	
-	C	-442.48	PV of payment
2	A	2.00	n = 2
4500	CHS,E	-4500.00	CF_2 input as a negative FV
-	R/S	3.14	
-	C	3524.16	PV of income
3	A	3.00	n = 3
5500	CHS,E	-5500.00	CF_3 input as a negative FV
-	R/S	3.14	
-	C	3811.78	PV of income
4	A	4.00	n = 4
4500	CHS,E	-4500.00	CF_4 input as a negative FV
-	R/S	3.14	
-	C	2759.93	PV of income
5	A	5.00	n = 5
130000	CHS,E	-130,000.00	CF_5 input as a negative FV
-	R/S	3.14	
-	C	70,558.79	PV of income

Conclusion: since the sum of the PVs in the above table is positive ($212.18) the investment can be considered a success, i.e., realizing the 13% rate specified by the investor.

Question: Exactly what interest rate was realized? or, in other words, what rate would make the NPV = 0? This is what is called the internal rate of return (IRR), and is beyond the scope of the FICALC-11/15 program! To find IRR (using a calculator) one should seek out an HP-12C (or other earlier financial calculator) or a properly configured HP-41C or other computer. Yet another alternative is described in the next section.

b. Modified Internal Rate of Return (MIRR)

Another way of analyzing the cash flow, one which has certain advantages over the traditional internal rate of return, and which can be solved by the FICALC-11/15 program is the so-called modified IRR. The steps are as follows:

(1) Calculate the <u>future</u> values of all the <u>positive</u> cash flows using a specified "reinvestment" or "risk" interest rate; call the sum of these NFV_p.

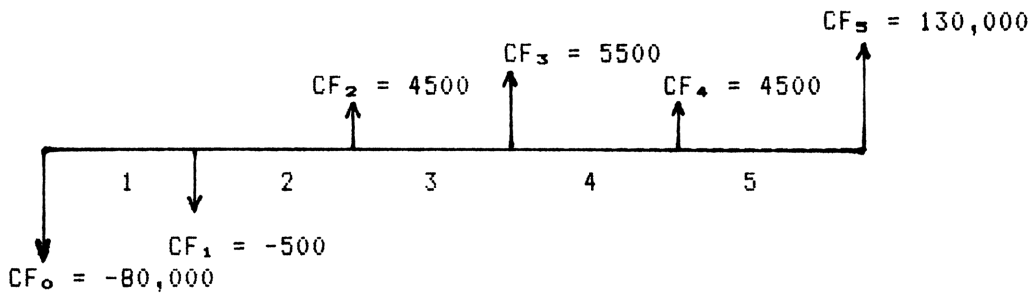
(2) Calculate the <u>present</u> values of all the <u>negative</u> cash flows using a specified "safe" interest rate; call the sum of these NPV_n.

(3) Knowing n, and letting PV = NPV_n and FV = NFV_p, use the program to solve for I, the "modified internal rate of return".

(4) Alternatively [to (3)], find MIRR from the formula

$$MIRR = 100\left[\left(\frac{NFV_p}{-NPV_n}\right)^{1/n} - 1\right] .$$

Using the same data given in the previous section dealing with the NPV cash flow analysis, except specifying I = 10% for positive cash flows and I = 6% for negative ones:



<u>SOLUTION</u>

With the program, complete the table below.

Input			Program Output	
<u>Years</u>	<u>I</u>	<u>CF_i</u>	<u>NPV</u>	<u>NFV</u>
0	-	-80,000	-80,000	-
1	6	-500	-471.70	-
3	10	4,500	-	5,989.50
2	10	5,500	-	6,655.00
1	10	4,500	-	4,950.00
0	-	130,000	-	130,000.00
Total			NPV_n = -80,471.70	147,594.50 = NFV_p

The steps required to do this are as follows, noting that the signs of the input cash flows are reversed to achieve the desired sign on the output:

Input	Key(s)	Display	Comment
-	GSB,0	0.00	BEG/END mode immaterial
6	B	0.06	i = 0.06, note n = 1 is achieved by the GSB initialization
500	E	500.00	FV
-	R/S	3.14	
-	C	-471.70	PV
3	A	3.00	n = 3
10	B	0.10	i = 0.10
4500	CHS,C	-4500.00	PV
-	R/S	3.14	
-	E	5989.50	FV
2	A	2.00	n = 2
5500	CHS,C	-5500.00	PV
-	R/S	3.14	
-	E	6655.00	FV
1	A	1.00	n = 1
4500	CHS,C	-4500.00	PV
-	R/S	3.14	
-	E	4950.00	FV

-	GSB,0	0.00	
5	A	5.00	n = 5
80471.7	CHS,C	-80,471.70	NPV_n = PV
147594.5	E	147,594.50	NPV_p = FV
-	R/S	3.14	
-	B	12.90	MIRR = 12.90%

Alternatively, the MIRR of 12.90 could be calculated from the table totals using the formula given earlier,

$$MIRR = 100\left[\left(\frac{147594.5}{-(-80471.7)}\right)^{1/5} - 1\right] = 12.90\ \%$$

by employing the keystroke sequence:

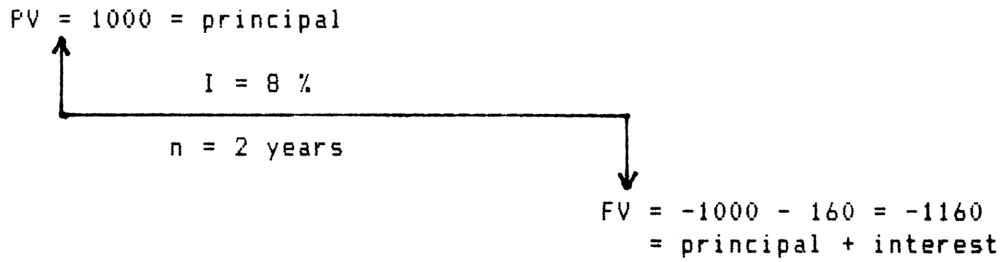
100, ↑, 147594.5, ↑, 80471.7, ÷, .2, fy^x , 1, -, X

6. Simple Interest.

Assuming simple interest, the principal (i.e., the original amount of money) earns interest for the entire life of the transaction. For example, if you borrow \$1000 at 8% simple interest for two years:

Amount of interest = principal x annual interest rate x no. years
= (\$1000) (.08) (2) = \$160.

A cash flow diagram would look like this:



The FICALC-11/15 program will not <u>directly</u> solve such problems. It can be used, however, in the same manner as for compound interest, if n = 1. Because of this need, the initialization of the program by GSB 0, GSB 1, or GSB 2 automatically sets n = 1, thus a simple interest scenario is always established, and inputting 1 for n is not needed.

When n is not equal to 1, as in the last two examples below, the program is used to calculate the FV (principal + interest) assuming one year; the amount of interest is then calculated and added to the principal to determine the actual FV.

The following examples should require no further description. The answers are obtained using the procedures indicated.

	Time	Amount Borrowed (+) or Invested (-) (PV)	Annual Interest Rate (I)	Total Amount Owed (-) or Accumulated (+) (FV)
a.	1 year	4200.00	12.5	? (Ans.=-4725.00)
b.	1 year	-4500.00	? (Ans.= 10.13)	4956.00
c.	1 year	? (Ans.=-5561.99)	7.875	6000.00
d.	3 years	-7000.00	10.5	? (Ans. = 9205.00)
e.	60 days	8000.00	11.5	? (Ans. = 8151.23)

SOLUTION

	Input	Key(s)	Display	Comment
a.[2]	-	BSB 0	0.00	BEG/END mode immaterial
	12.5	B	0.13	actual i = 0.125
	4200	C	4200.00	PV = principal borrowed
	-	R/S	3.14	
	-	E	-4725.00	FV = amount due
b.[2]	4500	CHS,C	-4500.00	PV = principal invested
	4956	E	4956.00	FV = total accumulation
	-	R/S	3.14	
	-	B	10.13	actual I = 10.1333%
c.[2]	7.875	B	0.08	actual i = 0.07875
	6000	E	6000.00	FV = accumulation
	-	R/S	3.14	
	-	C	-5561.99	PV = amount invested
d.	10.5	B	0.11	actual i = 0.105
	7000	CHS,C	-7000.00	PV = amount invested
	-	R/S	3.14	
	-	E	7735.00	FV = accumulation in one year
	-	+	735.00	Annual interest amount; note that -7000.00 was in the Y register, facilitating the calculation
	3	X	2205.00	3-year interest amount
	7000	+	9205.00	total accumulation
e.	11.5	B	0.12	actual i = 0.115
	8000	C	8000.00	PV = principal borrowed
	-	R/S	3.14	
	-	E	-8920.00	FV = annual amount due
	-	+	-920.00	annual interest amount
	60, ↑, 365		0.16	proportion of year money was borrowed
	-	X	-151.23	interest due
	-	CHS,+	8151.23	principal + 60-day interest due

----

[2] The perceptive and calculator-literate reader will note that examples a,b, and c can be solved easily using fundamental keyboard functions, that is:

a. 4200, ↑, 12.5, g%, + leads to 4725.00, as does 4200, ↑, 1.125, X;

b. 4500, ↑, 4956, gΔ% leads to 10.13; as does 4956, ↑, 4500, -, gLSTx, ÷, 100, X; and

c. 6000, ↑, 1.07875, ÷ leads to 5561.99.

F. Program statements and code

Step	Statement	Code	Step	Statement	Code	Step	Statement	Code
01	GTO 9	22 9	31	[f LBL C]	42 21 13	61	g CF 0	43 5 0
02	[f LBL 1][3/]	42 21 1	32	g F? 0	43 6 0	62	g RTN	43 32
03	g SF 1	43 4 1	33	GTO 5	22 5	63	(f LBL 4)	42 21 4
04	[f LBL 0]	42 21 0	34	STO 3	44 3	64	RCL 4	45 4
05	f FIX 2	42 7 2	35	g RTN	43 32	65*	g X ≠ 0 (g TEST 0)	43 30 0
06	f CLEAR REG	42 34	36	[f LBL D]	42 21 14	66	STO .9	44 .9
07	g CF 0	43 5 0	37	g F? 0	43 6 0	67	RCL 5	45 5
08	1	1	38	GTO 6	22 6	68	RCL 3	45 3
09	STO 1	44 1	39	STO 4	44 4	69	÷	10
10	g CLX	43 35	40	g RTN	43 32	70	g ABS	43 16
11	g F? 1	43 6 1	41	[f LBL E]	42 21 15	71	RCL 1	45 1
12	g GRD	43 9	42	g F? 0	43 6 0	72	1/X	15
13	g RTN	43 32	43	GTO 7	22 7	73	Y^X	14
14	[f LBL 2]	42 21 2	44	STO 5	44 5	74	1	1
15	g CF 1	43 5 1	45	g RTN	43 32	75	-	30
16	g DEG	43 7	46	(f LBL 3)	42 21 3	76	STO 2	44 2
17	GTO 0	22 0	47	RCL 0	45 0	77	EEX	26
18	[f LBL A]	42 21 11	48	RCL 5	45 5	78	2	2
19	g F? 0	43 6 0	49	-	30	79	X	20
20	GTO 3	22 3	50	RCL 0	45 0	80	g CF 0	43 5 0
21	STO 1	44 1	51	RCL 3	45 3	81	g RTN	43 32
22	g RTN	43 32	52	+	40	82	(f LBL 5)	42 21 5
23	[f LBL B]	42 21 12	53	÷	10	83	RCL 5	45 5
24	EEX	26	54	g LN	43 12	84	RCL 6	45 6
25	2	2	55	RCL 2	45 2	85	RCL 0	45 0
26	÷	10	56	1	1	86	X	20
27	g F? 0	43 6 0	57	+	40	87	+	40
28	GTO 4	22 4	58	g LN	43 12	88	CHS	16
29	STO 2	44 2	59	÷	10	89	RCL 6	45 6
30	g RTN	43 32	60	STO 1	44 1	90	1	1

[3/] Brackets and parentheses are not programmable; they are used to indicate externally meaningful or internal use labels, respectively.

F. Program statements and code (continued)

Step	Statement	Code	Step	Statement	Code	Step	Statement	Code
91	+	40	121	STO 5	44 5	151	2	2
92	÷	10	122	g CF 0	43 5 0	152	X	20
93	STO 3	44 3	123	g RTN	43 32	153	g CF 0	43 5 0
94	g CF 0	43 5 0	124	(f LBL 9)	42 21 9	154	f FIX 3	42 7 3
95	g RTN	43 32	125	g SF 0	43 4 0			
96	(f LBL 6)	42 21 6	126	RCL 2	45 2			
97	RCL 5	45 5	127	g X=0	43 40			
98	RCL 3	45 3	128	GTO 8	22 8			
99	RCL 6	45 6	129	1	1			
100	1	1	130	+	40			
101	+	40	131	RCL 1	45 1			
102	X	20	132	Y^X	14			
103	+	40	133	1	1			
104	CHS	16	134	-	30			
105	RCL 6	45 6	135	STO 6	44 6			
106	RCL I	45 25	136	0	0			
107	X	20	137	RCL 2	45 2			
108	÷	10	138	1/X	15			
109	STO 4	44 4	139	g F? 1	43 6 1			
110	g CF 0	43 5 0	140	1	1			
111	g RTN	43 32	141	+	40			
112	(f LBL 7)	42 21 7	142	STO I	44 25			
113	RCL 3	45 3	143	RCL 4	45 4			
114	RCL 6	45 6	144	X	20			
115	RCL 3	45 3	145	STO 0	44 0			
116	RCL 0	45 0	146	(f LBL 8)	42 21 8			
117	+	40	147*	f Π (gΠ)	42 16			
118	X	20	148	R/S	31			
119	+	40	149	RCL 6	45 6			
120	CHS	16	150	EEX	26			

Note: g MEM leads to P-00 r-6; therefore program memory is completely full, since registers 0 through 6 are used by the program.

*Indicates statements that must be changed for the HP-15C. Suggested or required change is shown in parentheses. Although the "code" for these statements will also change this is left for the 15C user to alter.

G. Formulas and background.

For convenience, the symbols defined in the "Purpose" section are repeated here:

- n = number of periods over which compounding takes place
- I = periodic interest rate expressed as a percent
- i = periodic interest rate expressed as a decimal
- PV = present value
- PMT = periodic payment made in each of the n periods
- FV = future value
- NAR = nominal annual rate of interest, as a percent
- APR = annual percentage rate; same as NAR
- EAR = effective annual rate, as a percent

Other symbols are defined below, where appropriate.

1. Compound interest in general
   a. basic formula when no payments are involved:

$$FV = PV\,(1+i)^n .$$

   b. basic formula when payments are made at the <u>end</u> of each period (sometimes called "ordinary annuity" problem):

$$FV = PMT\left[\frac{(1+i)^n - 1}{i}\right]$$

   c. basic formula when payments are made at the <u>beginning</u> of each period (sometimes called an "annuity due" or "terminating" annuity problem):

$$FV = PMT\left[\frac{(1+i)^n - 1}{i}\right](1+i)$$

   d. The general formula.

A significant computational breakthrough was made when Martin (1977) cited a formula, later modified by Dewey (See PPC ROM, 1981) which combines the above 3 formulas, enabling much more compact solution:

$$(PV)(1+i)^n + (PMT)(1+iX)\left[\frac{(1+i)^n - 1}{i}\right] + FV = 0$$

where $X = 0$ if payments are made at END of period
$X = 1$ if payments are made at BEG of period

and the algebraic signs used in the conventional cash flow diagram are employed.

As a simplification, the formula can be re-written as

$$(PV + C)A + PV + FV = 0$$

$$\left.\begin{aligned} \text{where } A &= (1+i)^n - 1 \\ B &= (1+iX)/i \\ \text{and } C &= (PMT)(B). \end{aligned}\right\} \quad \text{Label 9}$$

Now, the five components of the general formula can be solved for; these are the expressions evaluated in the program (Label indicated):

$$n = \ln \left[\frac{C - FV}{C + PV}\right] / \ln (1+i) \qquad \text{Label 3}$$

$$i = \left[\frac{FV}{PV}\right]^{1/n} - 1 \text{ , if } PMT = 0 \qquad \text{Label 4}$$

if $PMT \neq 0$, the solution is an iterative one, too complex for the current program.

$$PV = \frac{-[FV + (A)(C)]}{A + 1} \qquad \text{Label 5}$$

$$PMT = \frac{-[FV + (PV)(A+1)]}{(A)(B)} \qquad \text{Label 6}$$

$$FV = -[PV + A(PV+C)] \qquad \text{Label 7}$$

2. Effective annual rate calculations.

a. Discrete case.

The formula discussed in the examples section which relates EAR with NAR is

$$EAR = 100\left[\left(1 + \frac{NAR}{100C}\right)^{C} - 1\right]$$

where C = frequency of compounding per year.

This can be written more succintly as

$$i_e = (1+i')^n - 1$$

where i_e = effective annual rate as a decimal

i' = periodic nominal rate as a decimal

= i/C

and n = number of periods per year = C

Because of this simplification, the formula is seen to be equal to the factor A, routinely calculated in the program.

Therefore, EAR = 100 A   Label 8

b. Continuous case

The analagous two formulas under the assumption of continuous compounding are

$$EAR_c = 100[e^{NAR/100} - 1] \text{ , as percents}$$

and

$$i_e = e^i - 1 \text{ , as decimals}$$

c. Although the formulas presented in a. and b. were those used in the program, more general formulas have been published (PPC, 1981) and deserve documentation here:

(1) To convert nominal rates to effective rates

$$i_e = (1+i/CF)^{CF/PF} - 1 \quad \text{(for discrete compounding)}$$

and $$i_e = e^{i/PF} - 1 \quad \text{(for continuous compounding)}$$

where:

i = nominal annual interest rate as a decimal

i_e = effective interest rate (decimal) per payment period

CF = compounding frequency per year

PF = payment frequency per year

EXAMPLE:

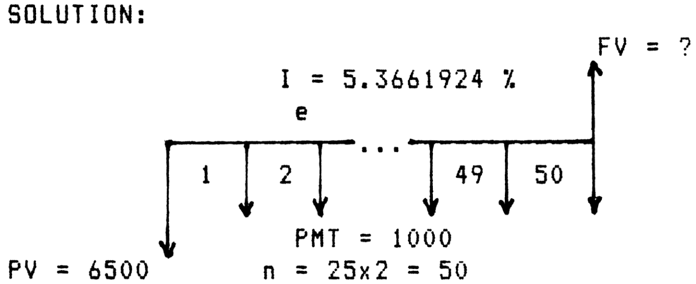
Consider an IRA program with a \$6500 initial deposit on Jan. 1, 10.5% nominal annual rate, monthly compounding, and semi-annual payments of \$1000 made on June 30 and Dec. 31 each year. What is the accumulation after 25 years?

In order to use the program FICALC-11/15, on the following cash flow diagram, the effective interest rate <u>per half-year</u> (I_e) must be calculated and used, since payments are made every half-year. Employing the discrete case of the above formula:

$$i_e = (1+.105/12)^{12/2} - 1 = .053661924$$

leading to $I_e = 100\, i_e = 5.3661924\ \%$,

SOLUTION:



Input	Key(s)	Display	Comment
-	GSB,2	0.00	END mode
50	A	50.00	n = 50
5.3661924	B	0.05	i_e = .053661924
6500	CHS,C	-6500.00	PV = 6500
1000	CHS,D	-1000.00	PMT = 1000
-	R/S	3.14	
-	E	324,406.14	FV = Total accumulation

Reference to the detailed example in Appendix B will show that the total accumulation here is between that obtained with annual \$2000 payments (\$318,247.45) and \$166.67 monthly payments (\$329,627.33), under the same interest rate and compounding frequency.

(2) To convert effective rates to nominal rates:

$$i = CF[(i+i_e)^{PF/CF} - 1] \qquad \text{(discrete case)}$$

and

$$i = \ln[(1+i_e)^{PF}] \qquad \text{(continuous case)}$$

3. Cash flow analysis

a. Net present value (NPV)

$$NPV = CF_o + \frac{CF_j}{(1+i)^j}$$

where CF_j = cash flow at period j

b. Modified internal rate of return (MIRR)

$$MIRR = 100\left[\left(\frac{NFV_P}{-NPV_N}\right)^{1/n} - 1\right]$$

where NFV_P = net future value of the positive cash flows

and NPV_N = net present value of the negative cash flows

4. Simple interest

AI = (P)(i)(N)

where AI = amount of interest (dollars)
P = principal (borrowed or invested)
i = annual interest rate as a decimal
N = number of years; may be less than one, such as 90 days implies N = 90/365

H. References

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Martin, R.E. 1977. Printing Financial Calculator Sets New Standards For Accuracy and Capability. Hewlett-Packard Journal, Oct. 1977. Hewlett-Packard Co., Corvallis, OR.

PPC. 1981. PPC ROM User's Manual. Personal Programming Center. Fountain Valley, CA.

Appendix A: The Concept of Time and Money [4/]

The following pages represent a fairly complete introduction to basic concepts of time and money. Most of the problems described can be solved from the HP-11C or HP-15C keyboard and/or use of the FICALC-11/15 program if one notes that:

1. Clearing the financial registers, CLFIN, is achieved on a FICALC-11/15 programmed HP-11C or HP-15C by GSB 0, GSB 1, or GSB 2, as appropriate.

2. The "BEG/END switch" mode is achieved by the choice of GSB 1 (= BEG) or GSB 2 (= END).

3. Using FICALC-11/15, you must indicate the end of the input phase by pressing R/S, then the appropriate label key for the desired unknown. In the following examples the financial calculator assumed has been "hard wired" to know when sufficient data have been supplied, therefore no signal (such as pressing R/S) to terminate input is needed.

4. Lower case i is used to indicate the interest rate as a percent, whereas in FICALC-11/15, capital I implies interest as a percent, and lower case i implies interest as a decimal.

5. The material which follows dealing with amortization schedules and discounted cash flow analyses is included for completeness; these topics are not directly handled by the FICALC-11/15 program.

[4/]Copied with permission of Hewlett-Packard Co. from "Your HP Financial Calculator", Feb. 1978 version. (see References, Section H).

Section 2

Concepts of Time and Money

Essentially, there are three things you can do with money: spend it, invest it, or sit on it. A savings account in a bank is considered an investment and we'all know what spending entails, but putting your money under a cushion...? Whether you spend or invest money, you want to receive something worthwhile in return.

This section looks at the nature of cash flows, or how time and money relate to one another when you borrow or invest. Your HP financial calculator has the common time-and-money formulas built-in and ready at your fingertips, so you are free to concentrate on the concepts themselves.

Percent: The Universal Yardstick

Percentage is the universal yardstick—the common standard of measurement—in the financial world. If your money increases or decreases, the gain or loss is measured in percent as well as in dollars. Taxes, interest rates, discounts, inflation, appreciation, depreciation, even the last raise you got, or the typewriter you bought last week for 40% off—all are expressed in terms of percent.

Percent, denoted by the symbol %, simply means "for each hundred." When you see 25%, it's the same as $^{25}/_{100}$ or 0.25 or ¼.

Percentage is a dynamic relationship, a comparison or ratio of two numbers that often signifies that a change has taken place. "Third-quarter earnings are down 27% from last year" may be cause for concern, while "a 12% raise effective today" may be cause for celebration.

Likewise, when you start with a given amount of money and receive money in return, the difference—whether it's a gain or loss—is viewed in relation to the original amount and expressed as a percentage. If you start out with one share of stock worth $100 and sell it for $125, you have earned $^{25}/_{100}$ or a 25% return.

When you superimpose that gain or loss against time, it's called the **rate of return.** The time period most commonly used in business is one year. So if you earned that $25 in one year, that's a 25% annual rate of return.

Percent and Your Financial Calculator

Hewlett-Packard financial calculators provide you with three separate functions for calculating percentage problems: [%], [Δ%], and [%T]. With the [%] key, you can key in a number and a percent and find that percent of the number. With the [Δ%] key, you can find the percent change (increase or decrease) between an old value and a new value. And with the [%T] key, you can find what percent one number is of another number or of the sum of several numbers. You are finding proportions when you use [%T].

When calculating percentage problems, you don't have to convert percents to their decimal equivalents: 10% need not be changed to .10. It can be keyed in the way you see it and say it, 10 [%].

Look at the following example, illustrating the use of percentage keys and concepts.

Example: Suppose you own 150 shares of stock in Coakley Laboratories, 52 shares of Idylwild Aircraft, and 200 shares of Burrell Industries.

If you sell 30% of your stock in Coakley Laboratories, how many shares would you have left?

Keystrokes	Display	
150 [ENTER↑] 30 [%]	*45.00*	30% of 150 is 45 (# shares to be sold).
[−]	*105.00*	Shares left.

Notice you didn't have to reenter 150 before subtracting; your calculator automatically retains the base number in percentage calculations.

Assume your Burrell Industries stock is now selling at $7.50 per share, down from $10.75 per share last year at this time. What is the percent decrease in cost per share over the last year? Notice that the keystroke sequence is "old value-enter-new value."

Keystrokes	Display	
10.75 [ENTER↑]	*10.75*	Cost per share last year.
7.50 [Δ%]	*-30.23*	% decrease.

Your calculator automatically displays a minus sign to show percentage decreases.

What percent of your portfolio does each corporation represent at this time?

Keystrokes	Display	
150 [ENTER↑] 52 [+]		
200 [+]	*402.00*	Sum of shares.
150 [%T]	*37.31*	% Coakley Laboratories.
[CLX]	*0.00*	Clear the display.
52 [%T]	*12.94*	% Idylwild Aircraft.
[CLX]	*0.00*	Clear the display.
200 [%T]	*49.75*	% Burrell Industries.

After you sell 45 shares of Coakley Laboratories?

Keystrokes	Display	
105 [ENTER↑] 52 [+]		
200 [+]	*357.00*	Sum of shares.
105 [%T]	*29.41*	% Coakley Laboratories.
[CLX]	*0.00*	
52 [%T]	*14.57*	% Idylwild Aircraft.
[CLX]	*0.00*	
200 [%T]	*56.02*	% Burrell Industries.

Interest

Percent is also used to calculate interest. Interest is a charge for the use of money. In a sense, you "rent" the money or someone "rents" it from you.

Interest is based on three things:

1. The amount of money borrowed or saved.
2. The length of time.
3. The interest rate (a percentage).

This makes sense because the longer you rent something, the more you pay for it. If you rent a car for a week, it will cost more than if you rent it one day.

You can charge for money by the day, the week, the month, etc., but usually money is loaned or borrowed at a yearly rate. This annual interest rate is expressed as a percent. If a certain investment pays 9% yearly, that means $9 per year for every $100 invested.

But there are other considerations, too, when you pay or receive interest—namely, what *type* of interest and how often it is paid.

Simple Interest

With **simple interest,** the principal—i.e., the original amount of money—earns interest for the entire life of the transaction. For example, suppose you borrow $1,000 at an 8% interest rate ($8 for each $100) for one year. The formula for calculating simple interest is:

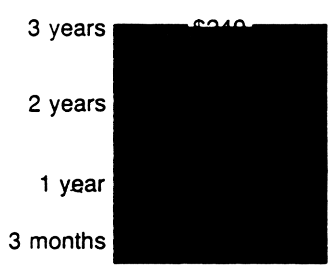
$$\text{Interest}_{(\text{simple})} = \text{Principal} \times \text{Interest rate} \times \text{Time}$$

The interest amount would be $80 ($1,000 × 8% × 1 year).

Borrowing the same amount of money for only 3 months would cost one-fourth as much, or $20 ($1,000 × 8% × ¼th of a year). And borrowing $1,000 for 3 years would cost 3 times as much, or $240 ($1,000 × 8% × 3 years).

Simple Interest

"Rent" on $1,000 at 8% simple interest:



On your calculator:

Keystrokes	Display	
1000 ENTER↑	*1,000.00*	Principal.
8 %	*80.00*	Interest amount (one year).
3 ENTER↑ 12 ÷	*0.25*	Portion of year (¼).
×	*20.00*	Interest for ¼th of a year.

Borrowing $1,000 for 3 years:

Keystrokes	Display	
1000 ENTER↑	*1,000.00*	Principal.
8 %	*80.00*	Interest amount (one year).
3 ×	*240.00*	Interest in 3 years.

Remember, percentages are like hundredths. Taking 8% of $1,000 is the same as multiplying $^8/_{100}$ (.08) times $1,000. Your calculator simplifies calculations by taking 8% of $1,000 automatically, as shown in the previous section.

Compound Interest

Although the concept of simple interest underlies most financial transactions, its use in the business world usually differs somewhat from the problem presented in the previous paragraph. For example, suppose you invested $1,000 for 2 years at 8%. How much interest would your investment earn? Using simple interest, the answer would be $160; you receive $80 at the end of the first year, and $80 at the end of the second year.

What would happen if at the end of the first year, the $80 of interest earned was invested for the second year along with the initial $1,000? At the end of the second year the $1,000 and $80 together would earn $86.40, $6.40 more than the initial $1,000 earned the first year. Each time the interest is paid, it is added to the balance. In effect, the interest is earning interest. Continuing this procedure, year after year, both the amount invested and the interest earned continue to grow.

This method of reinvesting earned interest, referred to as **compounding the interest,** is much more common in business transactions than earning interest on the principal alone (simple interest). **Compound interest** is usually stated as an annual rate, although it may be compounded (calculated) continuously, daily, monthly, quarterly, or semi-annually.

So if you have $1,000 at present, you can calculate how much money you will receive in the future, before you invest. This is called its **future value**[1]. Or perhaps you want $1,000 in the future—to take a trip to Acapulco next year—and want to know how much you have to invest to reach that goal. Since you have established the desired future value, you are solving for the money right now or its **present value.**[2]

What if you invested $1,000 at 8% *compounded annually* for 3 years? Let's use the calculator to compute the future value, or the amount of money you will have at the end of 3 years.

Keystrokes	Display	
[CL FIN]		Clear finanical entries.
1000 [CHS] [PV]	*-1,000.00*	Key in the initial investment and change the sign since you pay out money when you invest it. Press [PV] for present value.
8 [i]	*8.00*	Key in 8 and press [i] for 8% annual interest rate.
3 [n]	*3.00*	You want to find the value of your investment in 3 years. Press [n] for the number of compounding periods.
[FV]	*1,259.71*	Simply press [FV] to find the future value of your investment.

1. The formula for computing the future value of money with compound interest is:

$$FV = PV \times (1 + i)^n$$

2. The formula for computing the present value of money with compound interest is:

$$PV = \frac{FV}{(1 + i)^n}$$

The display shows that when you invest $1,000 at 8% compounded annually for 3 years, you will receive $1,259.71 at the end of those 3 years.

Is there a way to earn more money with that $1,000 at the same 8% interest rate? Yes, by compounding or adding the interest to the principal more than once a year. Suppose you put that money into an account where the 8% interest is *compounded quarterly*. How much do you have at the end of a year?

The $1,000 remains the same in our calculations, so you don't need to key the present value in again. But the number of compounding periods have changed from one per year to four per year.

Keystrokes	Display	
4 [n]	*4.00*	The number of compounding periods [n], has changed from 3 in 3 years to 4 in 1 year.
8 [ENTER↑] 4 [÷] [i]	*2.00*	Since the interest rate must always correspond with the compounding interval, you must divide the 8% annual interest rate by the number of compounding periods in a year, 4.
[FV]	*1,082.43*	When you press [FV], the calculator shows that you will have $1,082.43 at the end of one year.

And after 3 years?

Keystrokes	Display	
4 [ENTER↑] 3 [×] [n]	*12.00*	Simply change [n] to 12; 4 compounding periods per year × 3 years.
[FV]	*1,268.24*	The future value of your investment of $1,000 at 8% compounded quarterly at the end of 3 years.

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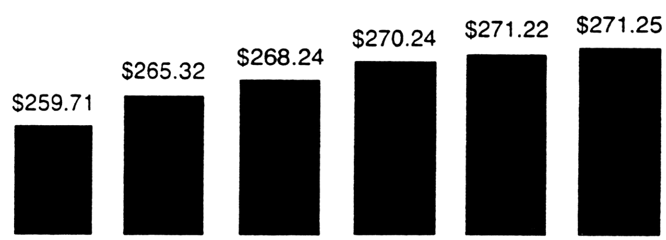
Now instead of $240.00 (no compounding) or $259.71 (compounding annually), you earned $268.24 in interest. It becomes apparent that the more often interest is compounded, the more money you receive in return.

Compound Interest

3 years "rent" on $1,000 at 8% compounded...



There is a limit, called *continuous compounding*, to the amount of money you can earn by increasing the frequency of compounding. If you compound continuously (more often than daily or hourly or every second), you reach the maximum mathematical limit. In other words, you reach the point where you just can't compound any more often.

Something interesting is emerging here. Even though 8% is stated as the annual rate, you actually receive more than 8% interest with compounding more than once a year.

$1,000 at 8% for 1 Year

COMPOUNDED	RETURN	% INTEREST
Quarterly	$1,082.43	8.243%
Monthly	$1,083.00	8.300%
Daily	$1,083.28	8.328%
Continuously	$1,083.29	8.329%

When interest is compounded more often than once a year, the stated annual rate (8% in the example above) is called the **nominal rate.** The

rate of interest actually earned in one year is called the **effective rate** (i.e., 8.328% compounding daily).

Many savings institutions quote both the nominal rate and the effective rate. And your calculator can quickly convert one to the other. (Refer to the applications manuals for interest rate conversions.) As the chart shows, the effective rate may differ considerably from the nominal rate, so it pays (literally!) to know what it is.

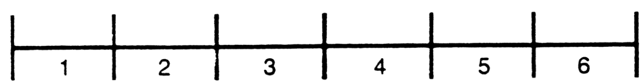
Clearing financial entries: Each time you begin a new problem press [CL FIN] (or CLEAR [FIN], depending on the model of your calculator) to erase previous financial values. When you press [CL FIN], all values for [n], [i], [PV], [PMT], and [FV] are replaced with 0.00 (zero). If you want to change some, but not all, of the values in a financial problem it is not necessary to press [CL FIN] and reenter all of the values again. Simply key in the new data and press the appropriate financial keys to change particular financial values, as we did in the last subsection.

You can view particular financial values held in the calculator at any time, by pressing the [RCL] *(recall)* key and then the desired financial key ([n], [i], [PV], [PMT], or [FV]). The designated financial value will then be displayed.

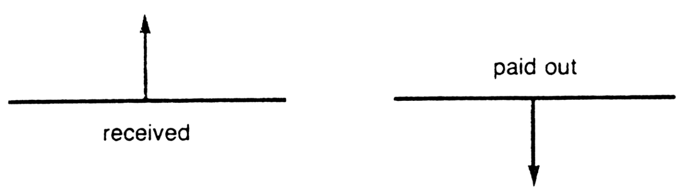
Compound Interest and the Cash Flow Diagram

The concept of compound interest is not difficult. The computations involved, however, can become exceedingly complex. Problems encountered often involve numerous payments and receipts before the transaction is concluded. Your financial calculator is designed to solve many of the most complicated calculations, but it requires a precise format for describing the problem. Such a format can be represented pictorially in the form of a cash flow diagram. The diagram is nothing more than a description of the timing and direction in which cash changes hands using terms that correspond to your calculator's financial keyboard. As long as you can picture your problem with a cash flow diagram and label it, your calculator can find the answers.

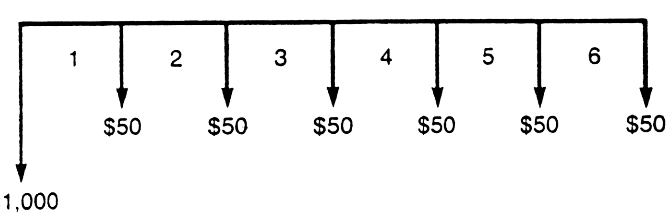
The diagram starts with a horizontal line called the time line. It represents the duration of a financial problem and is divided into compounding periods. For example a financial problem that transpires over 6 months with monthly compounding would look like this:



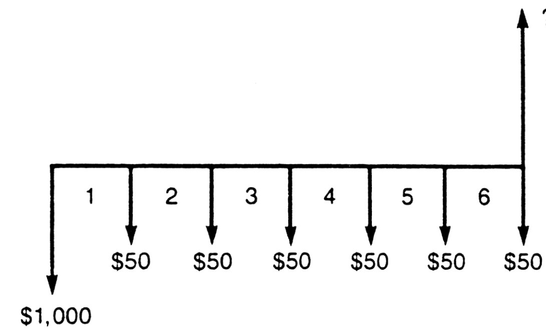
The exchange of money in a problem is pictured with vertical arrows; money received is represented with an arrow pointing up from the time line where the transaction occurred and money paid out is represented by an arrow pointing down.



For example, if you deposited (paid out) $1,000 at the beginning of the time period and then deposited an additional $50 at the end of each month for the remaining 6 months, you would label the diagram like this:



At the end of the period your account would have a balance that included the initial deposit, the subsequent payments, and any interest paid. This balance could be withdrawn (received), if necessary, and would represent a final cash exchange, completing the problem and the cash flow diagram.

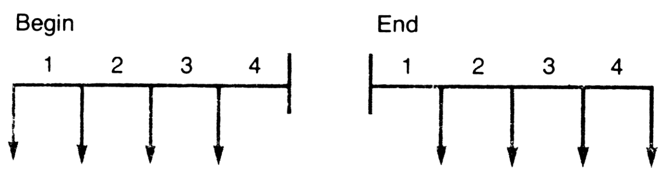


The following keys in the first row of your financial keyboard correspond exactly with our cash flow diagram: [n], [i], [PV], [PMT], and [FV]. The number of compounding periods* in a financial problem is represented by [n]: n would be 6 in our example. Interest rate *per compounding period* is denoted by [i] on the keyboard. (The interest rate must correspond with the compounding interval. Don't mix monthly interest with quarterly periods or daily interest with semiannual compounding periods.) The different cash flows are represented by PV, PMT, and FV. [PV] *(present value)* represents the cash flow at the start of the time line. In our example PV would be the $1,000 initial deposit. [FV] stands for future value and represents the cash flow at the end of the time line; the amount that could be withdrawn at the end of six months, indicated by the question mark in the above cash flow diagram. [PMT] *(payment)*

* Some financial problems involve a portion of a payment period as well as a series of whole payment periods. This occurs whenever a transaction begins on a date that does not correspond to the beginning of the usual payment period. Although there is no standard convention that applies to every problem of this kind, certain problems—such as purchasing a house in the middle of the month when regular payments are made at the beginning of each month—must be separated into two parts: one with the fractional portion of a payment period and one with the remaining whole number of payment periods. The payments made during the whole number of payment periods are calculated using *compound* interest, while the interest accumulated during the fractional portion of a payment period is often calculated using *simple* interest. Be sure that you partition problems when necessary, and calculate accordingly.

represents a series of cash exchanges of the same direction and amount, i.e., annuities. In our example $50 payments are deposited at the end of each month.

Payments can either start at the beginning of each period (BEGIN), or start at the end of each period (END). There are always the same number of payments as periods.



Whenever payments (PMT) are involved, it is necessary to specify which of the alternatives is applicable by setting the payment switch BEGIN ▯ END, found above the financial keys, to the proper position. BEGIN is for payments in advance and END is for payments in arrears. Or BEGIN is for annuities due and END is for ordinary annuities. The payment switch setting *does* make a difference in your calculated results. That's because interest accumulates on different amounts depending on whether payments are made at the beginning or the end of a compounding period. In our example, the payments occur at the end of each period, so the payment switch must be placed in the END position before starting calculations.

> **The Sign Convention:** Cash received (arrow pointing up) is represented by a positive value (+), and cash paid out (arrow pointing down) is represented by a negative value (−).

In our example, the $1,000 initial transaction (PV), and the periodic $50 payments would both be negative values. The amount received at the end of the time span would be positive.

The sign convention allows you to solve financial problems with 4 *or* 5 variables. (For instance, we shall soon solve for FV, given values for n, i, PV, and PMT.*) In fact, you can easily solve for any of the financial

* Some earlier handheld calculators could handle only two of the three kinds of cashflows at a time. The sign did not need to be specified because the cash flows were necessarily of opposite sign (e.g., PV positive, PMT negative; or PV negative, FV positive). Since your calculator can handle three kinds of cash exchanges (PV, PMT, and FV) at a time, their direction is no longer obvious. Thus, the sign needs to be specified using the described sign convention.

values above as long as you specify the values of at least three other financial keys (and include n or i as one of them).

Remember:

n = number of compounding periods
i = interest rate per compounding period
PV = present value
PMT = periodic payment
FV = future value

BEGIN = payments made at the beginning of the period
END = payments made at the end of the period

Now let's do the problem represented by the cash flow diagram and calculate the FV. Before beginning the calculation, one additional piece of information is necessary; the interest rate paid each compounding period. For this example let the interest rate be .75% per period (or 9% nominal interest). Remember, all cash that is paid out has a negative value.

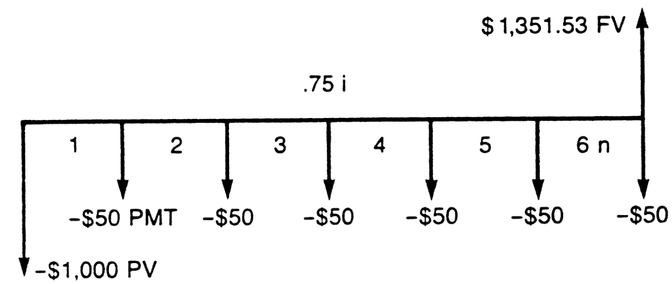
Since the $50 payments are made at the end of each period, set the payment switch BEGIN ▯ END to END.

Keystrokes	Display	
[CL FIN]		Clear previous financial values.
6 [n]	*6.00*	
.75 [i]	*0.75*	
1000 [CHS] [PV]	*-1,000.00*	Negative for cash paid out.
50 [CHS] [PMT]	*-50.00*	

The calculator now has all of the necessary information to solve for FV, which is the last key pressed.

Keystroke	Display	
[FV]	*1,351.53*	The calculated value is positive indicating we receive this amount.

As you can see, the keys on HP financial calculators and the signs of the values entered correspond precisely to the problem as represented by the cash flow diagram.

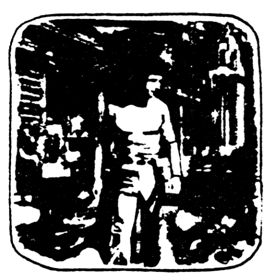


Suppose you wanted to increase your initial investment (PV) sufficiently to create an ending balance (FV) of $2,000 with the same interest rate, number of periods, and payments. What present value would be necessary?

There is no need to start the entire problem over again. The n, i, and PMT are unchanged and therefore do not have to be reentered. The only value that needs to be entered is the new desired FV. Enter the FV and solve for PV.

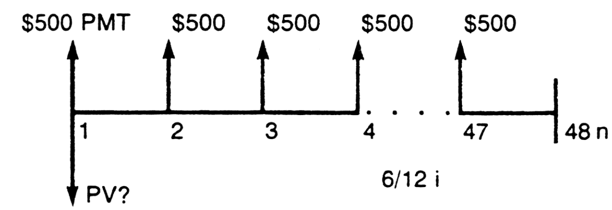
Keystrokes	Display	
2000 [FV]	*2,000.00*	Desired cash receipt.
[PV]	*-1,620.04*	Necessary cash paid out.

Looking over our example we find that with only a few easy keystrokes we have solved problems that would have required a great deal of time had we attempted to answer them by evaluating the complex mathematical formulas involved. The financial calculator's power allows you to consider numerous investment alternatives while concerning yourself only with the underlying concepts and the practicality of the values used.



Let's try another problem. Suppose you are concerned about providing for your daughter's college education 14 years from today. You expect that the cost will be about $6,000 a year or about $500 a month. If you withdrew the monthly payments for 4 years from a bank account paying 6% a year, compounded monthly, how much must you deposit in the bank at the start of the college years (PV) to make the monthly payments?

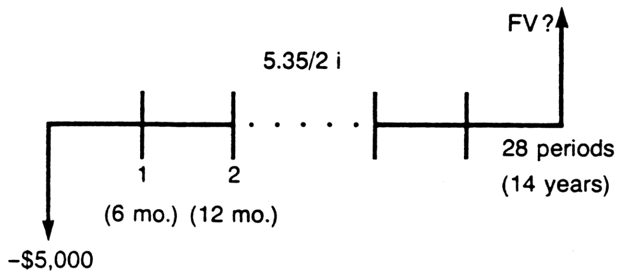
A cash flow diagram of the problem would look like the following:



The periodic interest rate must correspond to the time span between payments (compounding periods), so you must divide the yearly rate (6%) by 12 in order to produce a monthly rate [i]. As you can see from the diagram, the payments of $500 a month (PMT) start with the beginning of the time span; so you should set the payment switch BEGIN END to BEGIN. Since we are beginning a new problem, it is best to clear out any values remaining from the previous problem by pressing [CL FIN].

Keystrokes	Display	
[CL FIN]		Clears previous financial values.
6 [ENTER↑] 12 [÷] [i]	*0.50*	Calculate and enter interest rate per period.
4 [ENTER↑] 12 [×] [n]	*48.00*	Calculate and enter the number of compounding periods.
500 [PMT]	*500.00*	Amount received each period.
[PV]	*-21,396.61*	Total deposit required.

The next question we might ask is how do we accumulate such a sum by the time she enters college. We have several possibilities. Your daughter has a $5,000 paid up insurance policy that pays 5.35% (nominal) a year compounded semiannually. How much would it be worth by the time she enters college?

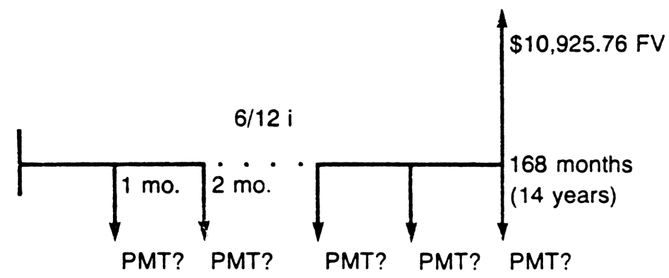


There are no payments so the payment switch BEGIN/END has no effect. In this problem our compounding periods occur semiannually so the yearly rate must be divided in half to obtain [i]. The value of [n] is 14 years times 2 periods per year. This is another new problem, so be sure to clear previous financial entries.

Keystrokes	Display	
[CL FIN]		
14 [ENTER↑] 2 [×] [n]	*28.00*	Total number of periods.
5.35 [ENTER↑] 2 [÷] [i]	*2.68*	Periodic interest rate.
5000 [CHS] [PV]	*-5,000.00*	Deposited (a negative value).
[FV]	*10,470.85*	Value of policy.

The insurance policy will supply about half of the needed amount. An additional amount must be set aside to make up the $10,925.76 deficit (21,396.61 − 10,470.85). Beginning next month, if we made monthly payments into a special college account, how large would the payments

have to be to accumulate the necessary future value of $10,925.76 in the 14 years remaining? Assume the account would pay 6% a year, compounded monthly.



Rather than multiplying 14 times 12 to get the proper number of compounding periods for [n] and dividing 6 by 12 for [i], we can use a short-cut provided on the financial keys for making quick conversions from years and yearly rates to months and monthly rates.

> **Remember:** [n] must always be the total number of compounding periods in the time span.
> [i] must always be the interest rate per compounding period.

Set the payment switch BEGIN/END to END. (Check your keyboard for correct prefix keys.)

Keystrokes	Display	
[CL FIN]		
14 [12×]	*168.00*	Automatically carries out the multiplication by 12 and stores the answer in [n].
6 [12÷]	*0.50*	Divides by 12 and stores in [i].
10925.76 [FV]	*10,925.76*	Future value desired.
[PMT]	*-41.65*	Necessary deposit each period (each month).

Note that we used the [12×] key to automatically compute and store the value of [n], and the [12÷] key to automatically compute and store the value of [i].

If we made the payment only $35 a month, how long (n) would it be before we reached the desired amount?

Keystrokes	Display	
35 [CHS] [PMT]	*-35.00*	
[n]	*188.54*	Number of periods.

In order to find the number of years, divide by 12.

Keystrokes	Display	
12 [÷]	*15.71*	Years.

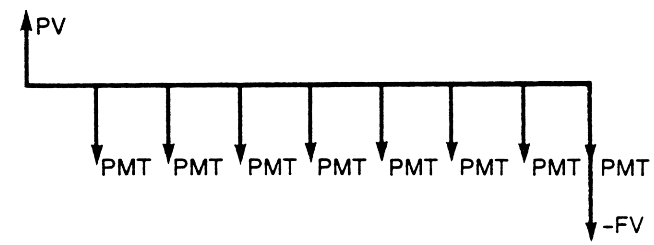
If, on the other hand, the monthly payment were increased to $45, with the 14-year term, the excess could be used as a contingency fund. For instance, with a $45 a month payment, what interest rate could the bank pay, while still enabling us to meet our goal?

Keystrokes	Display	
14 [12×]	*168.00*	Original term.
45 [CHS] [PMT]	*-45.00*	New monthly deposit.
[i]	*0.42*	Monthly interest rate.
12 [×]	*5.01*	Nominal yearly interest rate.

Note that it was necessary to reenter the length of the original term. Our previous computation of [n] (15.71) was stored in the calculator and would have otherwise been used for the term of this calculation.

In the preceding sample problems, we have seen how cash flow diagrams can be useful in representing a wide range of compound interest problems, and how the diagrams can be translated directly into solutions on an HP financial calculator. The diagrams are helpful tools that describe complex business and financial problems in a manner suitable for calculation. In addition, the cash flow diagram can be applied in other ways to become a valuable aid.

As we are all too often aware, each segment of the business community has its own special vocabulary. When considering compound interest problems of the kind we have been discussing, there are often numerous terms used throughout the business world describing the same problem, but which are not familiar outside a particular segment. For instance, this diagram:



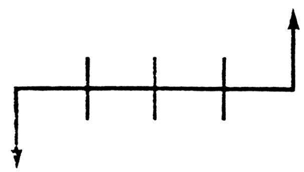
might represent a mortgage with a balloon payment in the terminology of the banking and real estate industries or a lease with a buy back (residual) in the leasing industry. There are probably many other terms in other industries as well as countries for describing this cash transaction. But regardless of the language, the essential problem is the same. In providing a means of describing business financial problems without using terminology specific to a particular segment, the cash flow diagram becomes, in a sense, a universal language.

The cash flow diagrams for four basic compound interest problems are presented in the following table along with some of the more common terminology.

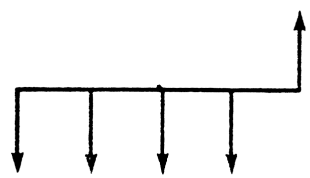
Some of the terms you see listed in the table may be common to your industry and some may not. There also may be diagrams represented that correspond to familiar transactions, but which do not bear familiar names. The important point to remember is that for financial calculations, it is the **magnitude** and **timing** of the cash exchanges represented by the cash flow diagram that are important, not the industry-dependent terminology.

Generalized Net Cash Flow Diagrams and Terminology

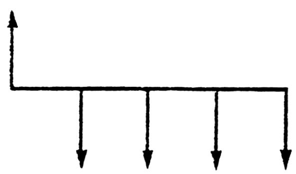
(Note that diagrams involving payments may be represented with payments at the beginning or end of the period.)



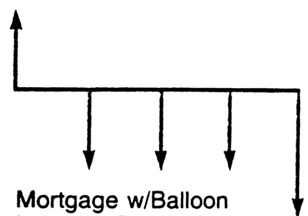
Compound Growth
Savings Account
Appreciation



Savings Plan
Sinking Fund
Pension Fund
Annuity (series of payments)



Mortgage
Lease
Direct Reduction Loan
Direct Reduction Installment
Amortization
Annuity



Mortgage w/Balloon
Lease w/Buy Back
Lease w/Residual
Annuity

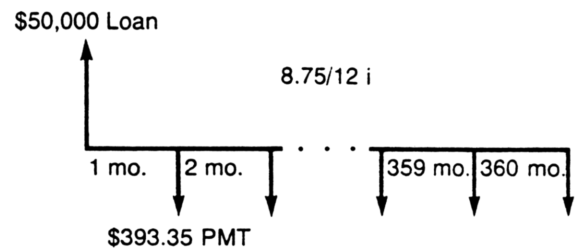
Amortization

If a loan or interest-bearing debt is paid off by (usually) equal payments, then it is said to be **amortized**. The word amortization comes from the French "a mort" meaning "at the point of death." Likewise, you are "killing" a loan by paying it off.

Most simple mortgages and installment loans are called **direct reduction loans**. The debt is discharged by equal periodic payments although varying portions of each payment are applied toward principal and interest.

The interest is paid first, then the remainder of the payment is used to reduce the debt. The time frame over which you make payments is called the **schedule of payments**. The breakdown of payments into interest portions and principal portions is called an **amortization schedule**.

Suppose you find your dream house. If you take out a \$50,000 mortgage for 30 years at 8¾% with monthly payments of \$393.35, your payment schedule would look like this:



At the end of the first month, interest is calculated on the entire \$50,000:

$$\frac{8.75\%}{12} \times 50{,}000 = \$364.58$$

and is added to the balance:

$$\$50{,}000 + \$364.58 = \$50{,}364.58$$

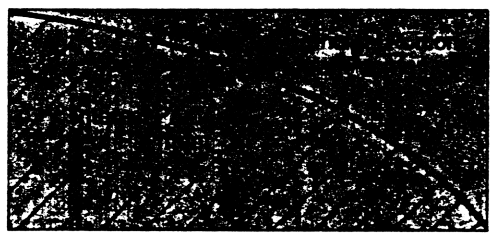
Then your first payment is deducted to obtain your new balance:

$$\$50{,}364.58 - \$393.35 = \$49{,}971.23$$

The next month and every month thereafter, the same procedure is followed, i.e., interest is calculated first and added to the balance before your payment is subtracted.

The amortization of your mortgage would look like this:

$393.35 Periodic
Payment
Amount



1 2 3. 360

Periodic Payment Number

As you reduce the size of the loan, the interest decreases. Gradually a higher percentage of each payment goes toward the debt itself or outstanding principal. By the time you reach your last payment, very little is deducted for interest.

With your HP financial calculator, you can easily compute the accumulated interest and remaining balance of your loan at any point in time. All you need to do is key in the principal [PV], periodic interest rate [i], and periodic payment [PMT]. Then key in the number of periods to be amortized and press [AMORT].

Your calculator will display the total interest portion of the payments. Simultaneously, your calculator generates the total principal portion of the payments (press [x⇄y]) and the remaining balance of the loan (press [RCL] [PV]).

When working amortization problems, remember to abide by the *sign convention* (cash received is positive, cash paid out is negative) and be sure to set the payment switch to the proper position.

When using the [AMORT] function, all calculated values are automatically rounded to match the display setting. The normal display shows numbers as dollars and cents. If your problem requires other rounding, refer to section 3 to set the display to the number of digits you wish carried.

Example: Generate an amortization schedule for the first 2 months of a 30-year mortgage for $50,000 at 8¾% annual interest with monthly payments of $393.35. Then find the balance on the loan after 1 year.

Set the payment switch BEGIN ▮▮ END to END.

Keystrokes	Display	
[CL FIN]		Clear old financial values.
50000 [PV]	*50,000.00*	Loan amount.
393.35 [CHS] [PMT]	*-393.35*	Periodic payment.
8.75 [12÷]	*0.73*	Periodic interest rate.

The [AMORT] function returns two values; the interest portion and the principal portion of the periodic payment. Since the display can only show one answer at a time, the second value is held in the automatic memory. It can be viewed by pressing the exchange key [x⇄y].

Keystrokes	Display	1st month of schedule.
1 [AMORT]	*-364.58*	Interest portion of payment.
[x⇄y]	*-28.77*	Principal portion of payment.
[RCL] [PV]	*49,971.23*	Remaining balance.
[RCL] [n]	*1.00*	Number of periods calculated (one month).

Notice that the [AMORT] function changes both the [n] and [PV] values: [PV] brings back the new balance, [n] provides the total number of periods amortized.

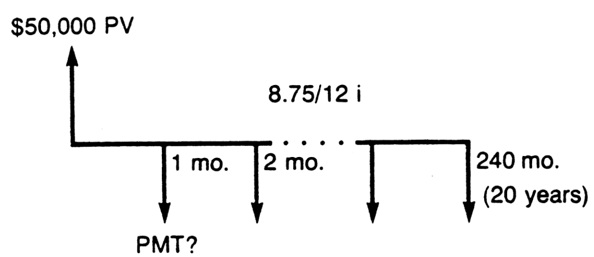
To generate the next period of the schedule simply press 1 [AMORT] again:

Keystrokes	Display	2nd month.
1 [AMORT]	*-364.37*	Interest portion of payment.
[x⇄y]	*-28.98*	Principal portion of payment.
[RCL] [PV]	*49,942.25*	Remaining balance.
[RCL] [n]	*2.00*	2nd payment period.

Now we want to find the balance on the loan after one year. We have already calculated the first 2 periods, so press 10 [AMORT] to compute the amount of interest and the amount of principal paid in the next 10 periods, leaving a balance on the loan after 1 year in [PV].

Keystrokes	Display	
10 [AMORT]	*-3,631.86*	Interest portion of payments made from month 3 through month 12.
[x⇄y]	*-301.64*	Principal portion of payments made from month 3 through month 12.
[RCL] [PV]	*49,640.61*	Balance after 12 months or 1 year.
[RCL] [n]	*12.00*	

What would the monthly payments be if we decide to pay off the mortgage in 20 years? The cash flow diagram would look like this:



Keystrokes	Display	
50000 [PV]	*50,000.00*	Reenter initial principal.
20 [12x]	*240.00*	# monthly payments in 20 years.
[PMT]	*-441.86*	Periodic payment.

It was not necessary to reenter the periodic interest rate as it remained the same in our last calculation.

How much interest and how much of the principal will be paid after 1 year with this 20-year mortgage? (Since we are starting the amortization schedule from the beginning we must set [n], the number of periods, to zero. We did this in our first amortization example by pressing [CL FIN], but this time we want to preserve the value of [i].)

Keystrokes	Display	
0 [n]	*0.00*	Set n to zero. (n was 240 from the last calculation.)
12 [AMORT]	*-4,336.87*	Interest portion of payments for 1 year.
[x⇄y]	*-965.45*	Principal portion of payments for 1 year.
[RCL] [PV]	*49,034.55*	Remaining balance.

How many payment periods have we calculated?

Keystrokes	Display
[RCL] [n]	*12.00*

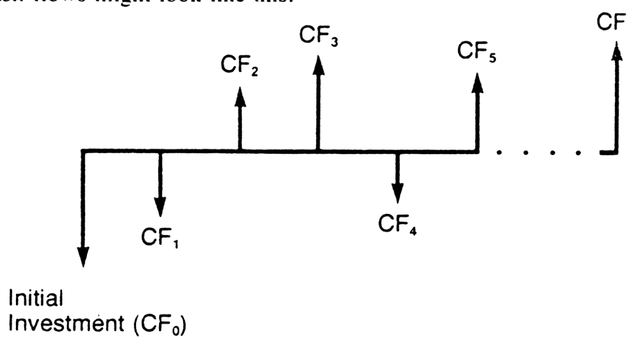
Remember: Set [n] to zero (by pressing 0 [n] or [CL FIN]) whenever you begin to calculate a new amortization schedule, to keep track of your payment number. When the payment switch is set to BEGIN, the whole first payment goes to pay off the principal (no interest has yet accumulated). In this case, it is essential to change [n] to zero so that interest is not calculated on the principal before the first payment.

Discounted Cash Flow Analysis

Except for PV and FV, the cash flow diagrams on page 36 contain *even* (equal) cash flows. Discounted cash flow analysis is a way of evaluating investments with *uneven* cash flows. Your calculator can help you evaluate future cash demands and returns to see which scheme or investment best meets your profit objectives. Two forms of discounted cash flow analysis are the net present value (NPV) approach and the internal rate of return (IRR) approach.

Net Present Value (NPV)

Suppose you invest a large amount of money into a scheme that generates a cash flow CF_1 (cash $flow_1$) the first year, CF_2 the second year, and so on, up to CF_n in the n^{th} year when the cash flow ends. A diagram of the cash flows might look like this:



Notice that the original investment (CF_0) is negative because it represents a cash outlay. Also note that the cash flows may not necessarily be positive. Maybe, in a new business, you have a loss the first year. Or perhaps after you've been in business a while, a recession causes you to have a bad year.

You also have to consider the *time* value of money, not just the dollar value. (Would you rather have someone give you $10,000 today or 5 years from today?) The cash flows (CF_1, CF_2, etc.) are mini-future values that you are going to receive. But realistically these future cash flows have to be translated back (discounted) to present value on the basis of a given rate of interest, for you to accurately assess the investment.

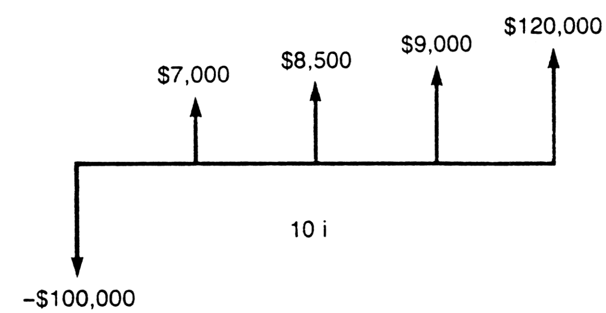
This method of analysis is called solving for **net present value*** because you are comparing the sum of the present values of all the future cash flows (CF_j) to the initial investment (CF_0). For the interest rate (i), use the rate of return that you want from the investment.

At the start, the net present value (NPV) is negative because you've put out a large amount of money in the original investment. As the cash returns flow in, NPV will increase. Eventually—hopefully—NPV will turn positive. When NPV=0, you have reached the break-even point in the investment.

It's a simple, clear-cut analysis: assuming a desired minimum yield, if NPV is *negative*, the investment DOES NOT meet your profit objectives; if NPV is *positive*, the investment DOES meet your profit objectives.

For example, you are thinking of buying an apartment building for $100,000. Based on the anticipated cash flows below, will this investment return 10% a year?

Year		Cash Flow
1		$7,000
2		$8,500
3		$9,000
4	(You intend to sell it.)	$120,000



* The formula for net present value is:

$$NPV = CF_0 + \frac{CF_1}{(1+i)^1} + \frac{CF_2}{(1+i)^2} + \ldots + \frac{CF_n}{(1+i)^n}$$

or net present value = initial investment + sum of present values of future cash flows. Notice that we are translating each cash flow from a predicted future value to a present value using the formula for finding present value:

$$PV = \frac{FV}{(1+i)^n}$$

Using the formula for NPV, we find that after the 4th year the net present value is positive ($2,111.88) so the investment does return 10% or greater per year.

Suppose you sell the building in the second year for $110,000. Would that be more profitable? No, the net present value is negative (−$2,727.27) so you won't even meet your desired 10% rate of return.

Internal Rate of Return (IRR)

Sometimes if you know your initial investment and can predict the periodic cash flows, you want to find the rate of return that you will earn. Internal rate of return (IRR) is the interest rate that equates the present value of all cash flows with an initial cash flow. IRR is also called the **yield** or **discounted rate of return**.

The formula (page 43) for finding NPV also applies for finding IRR, only we let NPV=0 and solve for i. The easiest way to find IRR is to choose a best-guess interest rate (i) and find the net present value. *When NPV is 0, your "guesstimate" is the actual IRR*. If NPV is negative, your estimated percentage is higher than the actual IRR—try a lower interest rate. If NPV is positive, the actual IRR is even better or higher than the rate you've chosen.

For example, what is the estimated rate of return for a restaurant costing $200,000 that produces the following cash flows?

Year		Cash Flow
1		-$4,000
2		$20,000
3		$27,000
4		$42,000
5		$56,500
6	(you sell it)	$230,000

If you try 12%, the NPV after the sixth year is $6,867.05 so the actual IRR is higher than 12%.

Next, try 13%. This time the NPV is negative (−$2,265.95) so the IRR must be less than 13%.

As a result of these two iterations, the IRR must be between 12% and 13%. Since the NPV for 13% is closer to zero than the NPV for 12%, the IRR must be closer to 13%. The actual yield or IRR on this restaurant investment is approximately 12.75%.

Specific keystrokes for NPV and IRR are described in your owner's handbook. If you have understood these concepts, the mechanics of solving a problem—be it in real estate, banking, leasing, insurance or investments—will not be difficult. You can use your calculator to evaluate time and money relationships before you invest and to explore several financial alternatives.

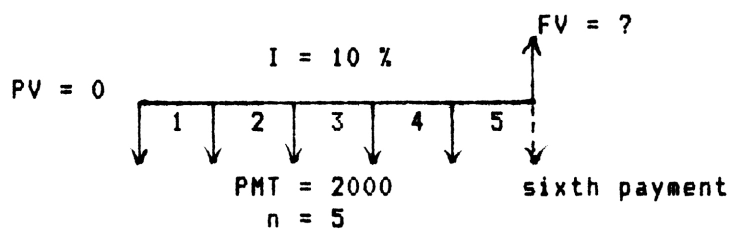
Appendix B: An Elaboration of a Compound Interest Problem and Solution Using the HP-11C/15C

A more complete understanding of the compound interest process can be obtained by considering an example such as that provided by a typical IRA investment problem:

1. Payments at the Beginning of Periods

Assume we start an IRA today with a $2000 payment and plan to continue depositing $2000 each year from now on. At 10% annual compounding, how much will be the accumulation (FV) of principal and interest at the end of five years?

The appropriate cash flow diagram is



Notice that the way the problem is posed, the payments occur at the beginning of each period and we want the future value at the end of the five years, to make the number of years coincide with the number of payments -- a normal constraint in such problems.

Another way of considering the problem is to seek the future value just before the sixth payment is made, which of course, is the same as "at the end of five years".

a. Solution using the stack

Since the basic formula relating present value (PV), future value (FV), interest expressed as a decimal (i), and the number of compounding periods (n) is

$$FV = PV(1+i)^n ,$$

the solution to the problem is to find the sum of the FVs when each $2000 is considered a PV for 1, 2, 3, 4, and 5 years.

The stack on all modern HP calculators provides an easy calculation mechanism:

(1) load the stack with the constant multiplier, (1+i), in the example 1.10;

(2) key the amount to be "futured", in the example $2000;

(3) press the times key once for each year, and record the results (the FVs).

Thus, the keystroke sequence:

(1) 1.10, ↑, ↑, ↑

(2) 2000

(3) X, read 2200.00 = 2000 @ 10% for 1 year
X, read 2420.00 = 2000 @ 10% for 2 years
X, read 2662.00 = 2000 @ 10% for 3 years
X, read 2928.20 = 2000 @ 10% for 4 years
X, read 3221.02 = 2000 @ 10% for 5 years

(4) The sum of the above 5 FVs is $13,431.22, the solution to the posed problem.

b. Solution by formula.

The appropriate formula, algebraically derived from the above procedure, is

$$FV = PMT\left[\frac{(1+i)^n - 1}{i}\right](1+i).$$

Upon substitution of the data given in the problem this becomes

$$FV = 2000\left[\frac{(1.10)^5 - 1}{.10}\right](1.10),$$

and using the following keystrokes leads to the answer 13,431.22, as before:

2000, ↑, 1.1, ↑, 5, Y^x, 1, -, .1, ÷, 1.1, X, X

c. Solution using the program FICALC-11/15

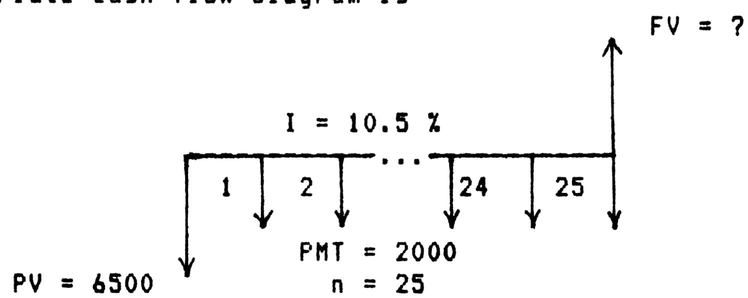
Input	Key(s)	Display	Comment
-	GSB,1	0.00	BEG mode (observe GRAD)
5	A	5.00	n = 5
10	B	0.10	i = 0.10
2000	CHS,D	-2000.00	PMT = 2000
-	R/S	3.14	
-	E	13,431.22	FV, as before

2. Payments at the End of Periods

The above problem exemplifies an investment scenario sometimes called an "annuity due" variation for the calculation of the future value of a terminating annual annuity. The other common variation (called an "ordinary annuity") occurs if payments are made at the end of each period, as the following problem suggests.

Suppose we change IRA vendors and transfer $6500 already accumulated, into an IRA account with the new vendor and plan to deposit $2000 annually for the next 25 years. At 10.5% annual compounding how much will accumulate after 25 years?

The appropriate cash flow diagram is



Notice that in this case the payments are at the end of each period, and because of this the 25th payment will accrue no interest.

a. Solution using the stack

It should be obvious that this procedure is impractical for such long periods, however we still might want to see how the initial $6500 (or each $2000 payment, for that matter) would grow year-by-year. Using the $6500:

(1) 1.105, ↑, ↑, ↑ to fill the stack with (1+i)

(2) 6500

(3) X, read 7182.50 after 1 year
X, read 7936.66 after 2 years
. .
. .
. .
X, read 78,880.62 after 25 years

Of course, for the complete solution we would have to add the future value of each of the $2000 payments. This can be done far more efficiently by either of the following methods.

b. Solution by formula

The appropriate formula for the sum of the future values of the payments made at the end of each period is

$$FV_1 = PMT\left[\frac{(1+i)^n - 1}{i}\right] \quad ^{5/},$$

but to this must be added the FV of the initial deposit (PV), found by

$$FV_2 = PV(1+i)^n$$

Substitution and the appropriate solution leads to:

Total accumulation = $FV_1 + FV_2$

$$= 2000\left[\frac{1.105^{25} - 1}{.105}\right] + 6500(1.105)^{25}$$

$$= 212{,}104.38 + 78{,}880.62$$

$$= 290{,}985.00$$

c. Solution using the program FICALC-11/15

Input	Key(s)	Display	Comment
-	GSB,2	0.00	END mode
25	A	25.00	n = 25
10.5	B	0.11	i = 0.105
6500	CHS,C	-6500.00	PV = 6500
2000	CHS,D	-2000.00	PMT = 2000
-	R/S	3.14	
-	E	290,985.00	Total accumulation, as before

5/Observe that this differs from the BEG mode of payment formula only by the absence of the (1+i) multiplier.

3. Additional Use of the Program

Advantages of the programmed solution should be very obvious to this point, however the ease with which repeated, but slightly different solutions can be made, really emphasizes this point. For example, if the time period changes, say to n = 30, simply do this:

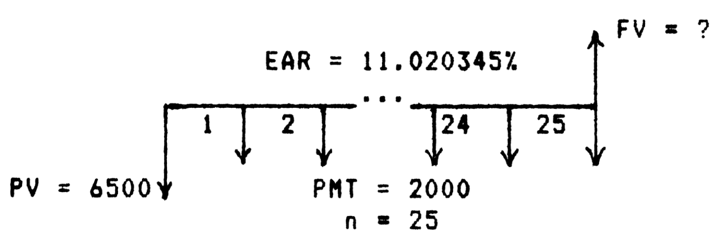
30	A	30.00	n = 30
-	R/S	3.14	
-	E	491,714.61	Total accumulation after 30 years

If the frequency of either payments or compounding is not annual, the formula procedure can become confusing and very prone to error, whereas the program approach is perhaps easier to implement.

a. Consider an IRA plan starting with the same $6500 initial investment and 10.5% nominal annual rate but with <u>monthly</u> compounding and <u>annual</u> payments of $2000. This situation apparently violates one of the assumed requirements for the use of the program -- that the total number of payments must be the same as the total number of compounding periods. However if we can calculate the effective <u>annual</u> rate (EAR) corresponding to the 10.5% nominal annual rate the problem satisfies the cited constraint. Using the program (from scratch) to calculate the EAR:

Input	Key(s)	Display	Comment
-	GSB,0	0.00	BEG/END mode immaterial
12	A	12.00	n = C = 12, compounding frequency
10.5 ↑12 ÷	B	0.01	actual i = 0.00875
-	R/S	3.14	
-	R/S	11.020	actual EAR = 11.020345

Now, the advantage of <u>monthly</u> compounding (at the 10.5% nominal rate), equivalent to 11.020345% annual compounding, can be shown using the program. NOTE, the appropriate cash flow diagram is

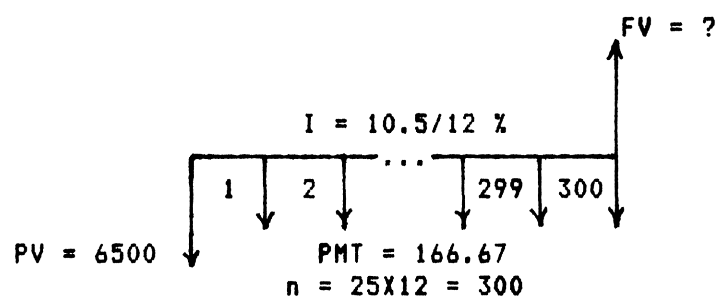


Input	Key(s)	Display	Comment
-	GSB,2	0.00	END mode
25	A	25.00	n = 25
11.020345	B	0.11	actual i = 0.11020345
6500	CHS,C	-6500.00	PV = 6500
2000	CHS,D	-2000.00	PMT = 2000
-	R/S	3.14	
-	E	318,247.45	Total accumulation

Note that monthly compounding resulted in considerably more money (318,247.45 - 290,985.00 = $27,262.45) than annual compounding, illustrating the powerful effect of compounding at shorter intervals.

b. Another point needs to be considered, which can be assessed easily by the program. Consider basically the same IRA plan; $6500 initial investment, 10.5% nominal annual rate, monthly compounding, but monthly payments of 2000 / 12 = $166.67, perhaps made by monthly payroll withholding. Now, what is the total accumulation after 25 years?

SOLUTION:

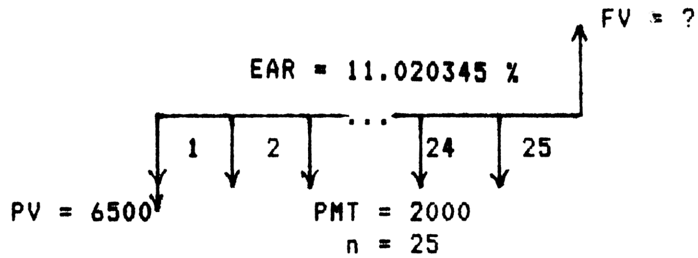


Input	Key(s)	Display	Comment
-	GSB,2	0.00	END mode
25 ↑ 12 X	A	300.00	n = 300
10.5 ↑ 12 ÷	B	0.01	i = 0.00875
6500	CHS,C	-6500.00	PV = 6500
166.67	CHS,D	-166.67	PMT = 166.67
-	R/S	3.14	
-	E	329,627.33	Total accumulation = FV after 300 months

Thus, the effect of depositing the money in monthly installments rather than waiting until the end of each year is considerable (329,627.33 - 318,247.45 = $11,379.88).

c. The thoughtful reader who has followed this example through all the details might ask, why not make the annual payments at the beginning of each year (if he can afford it) rather than monthly partial payments or end-of-year full payments? If possible, this is the best way, as the following solution shows. Note that this now becomes a beginning-of-period payment situation rather the end-of-period done previously.

SOLUTION:



Input	Key(s)	Display	Comment
-	GSB,1	0.00	BEG mode
25	A	25.00	n = 25
11.020345	B	0.11	i = 0.11020345
6500	CHS,C	-6500.00	PV = 6500
2000	CHS,D	-2000.00	PMT = 2000
-	R/S	3.14	
-	E	343,543.16	Total accumulation

In summary, the results of the four fairly typical alternatives are:

Initial investment	Compounding frequency at 10.5% NAR	Payments frequency	Payments amount	Payments number	Payments BEG/END	Total accumulation
$			$			$
6500	annual	annual	2000	25	END	290,985.00
6500	monthly	annual	2000	25	END	318,247.45
6500	monthly	monthly	166.67	300	END	329,627.33
6500	monthly	annual	2000	25	BEG	343,543.16

Appendix C: FICALC, A version of FICALC-11/15
Appropriate for the HP-41C, CV, or CX

The program found on the following pages, as a listing and in barcode form, can be used with an HP-41C, HP-41CV, or HP-41CX.

It was written so that the user could follow the instructions found in the main section of this paper, appropriate for the HP-11C or HP-15C, with the reminder that the XEQ key on the HP-41 is equivalent to the GSB key on the HP-11C or HP-15C.

In addition to the obvious improvements regarding the identification of input data and answers, two other refinements were incorporated into the FICALC program:

1. A "12 multiplier" is provided in LBL a; by keying the number of years and pressing the "label a" key (shift Σ+) the number of months is calculated and becomes N, stored appropriately.

2. A "12 divisor" is provided in LBL b; by keying the annual interest rate (%) and pressing the "label b" key (shift 1/X) the monthly interest is calculated and becomes i, stored appropriately.

Thus, these two refinements replace the keystroke sequences otherwise needed:

number of years, ↑, 12, X, A

and

annual interest percent, ↑, 12, ÷ , B.

Program Listing for FICALC, an HP-41 Analogy of the Program FICALC-11/15.

```
01♦LBL "FICALC"
02 SF 27
03 GTO 00
04♦LBL 01
05 SF 01
06♦LBL 00
07 ADV
08 FIX 2
09 CLRG
10 CF 00
11 1
12 STO 01
13 CLX
14 "0.00 END MODE"
15 FS? 01
16 GRAD
17 FS? 01
18 "0.00 BEG MODE"
19 AVIEW
20 ADV
21 STOP
22♦LBL 02
23 CF 01
24 DEG
25 GTO 00
26♦LBL a
27 12
28 *
29♦LBL A
30 FS? 00
31 GTO 03
32 STO 01
33 "N= "
34 ARCL X
35 AVIEW
36 STOP
37 GTO 09
38♦LBL b
39 12
40 /
41♦LBL B
42 1 E2
43 /
44 FS? 00
45 GTO 04
46 STO 02
47 1 E2
48 *
49 "I= "
50 ARCL X
51 "⊦ %"
52 AVIEW
53 STOP
54 GTO 09
55♦LBL C
56 FS? 00
57 GTO 05
58 STO 03
59 "PV= "
60 ARCL X
61 AVIEW
62 STOP
63 GTO 09
64♦LBL D
65 FS? 00
66 GTO 06
67 STO 04
68 "PMT= "
69 ARCL X
70 AVIEW
71 STOP
72 GTO 09
73♦LBL E
74 FS? 00
75 GTO 07
76 STO 05
77 "FV= "
78 ARCL X
79 AVIEW
80 STOP
81 GTO 09
82♦LBL 03
83 RCL 00
84 RCL 05
85 -
86 RCL 00
87 RCL 03
88 +
89 /
90 LN
91 RCL 02
92 1
93 +
94 LN
95 /
96 STO 01
97 CF 00
98 GTO A
99♦LBL 04
100 RCL 04
101 X≠0?
102 GTO 10
103 RCL 05
104 RCL 03
105 /
106 ABS
107 RCL 01
108 1/X
109 Y↑X
110 1
111 -
112 STO 02
113 1 E2
114 *
115 CF 00
116 GTO B
117♦LBL 05
118 RCL 05
119 RCL 06
120 RCL 00
121 *
122 +
123 CHS
124 RCL 06
125 1
126 +
127 /
128 STO 03
129 CF 00
130 GTO C
131♦LBL 06
132 RCL 05
133 RCL 03
134 RCL 06
135 1
136 +
137 *
138 +
139 CHS
140 RCL 06
141 RCL 07
142 *
143 /
144 STO 04
145 CF 00
146 GTO D
147♦LBL 07
148 RCL 03
149 RCL 06
150 RCL 03
151 RCL 00
152 +
153 *
154 +
155 CHS
156 STO 05
157 CF 00
158 GTO E
159♦LBL 09
160 SF 00
161 RCL 02
162 X=0?
163 GTO 08
164 1
165 +
166 RCL 01
167 Y↑X
168 1
169 -
170 STO 06
171 0
172 RCL 02
173 1/X
174 FS? 01
175 1
176 +
177 STO 07
178 RCL 04
179 *
180 STO 00
181♦LBL 08
182 "3.14 : READY"
183 ADV
184 PROMPT
185 RCL 06
186 1 E2
187 *
188 CF 00
189 FIX 3
190 "EAR= "
191 ARCL X
192 "⊦ %"
193 AVIEW
194 STOP
195 GTO 08
196♦LBL 10
197 "ERROR 3 "
198 AVIEW
199 END
```

HP-41 Program Barcode: FICALC

Program Registers Required: 51

ROW 1: LINES 1 - 3

ROW 2: LINES 3 - 12

ROW 3: LINES 13 - 14

ROW 4: LINES 14 - 18

ROW 5: LINES 18 - 23

ROW 6: LINES 23 - 30

ROW 7: LINES 31 - 37

ROW 8: LINES 38 - 44

ROW 9: LINES 45 - 50

ROW 10: LINES 51 - 57

ROW 11: LINES 57 - 63

ROW 12: LINES 64 - 68

ROW 13: LINES 69 - 76

ROW 14: LINES 77 - 83

ROW 15: LINES 84 - 96

FICALC (continued)

ROW 16: LINES 97 - 105

ROW 17: LINES 106 - 115

ROW 18: LINES 116 - 126

ROW 19: LINES 127 - 136

ROW 20: LINES 137 - 146

ROW 21: LINES 147 - 158

ROW 22: LINES 158 - 167

ROW 23: LINES 168 - 179

ROW 24: LINES 180 - 182

ROW 25: LINES 182 - 189

ROW 26: LINES 190 - 193

ROW 27: LINES 194 - 197

ROW 28: LINES 198 - 199

Program No. 11F007

Calculator: HP-11C (and HP-15C with minor changes)
HP-41C (see Appendix A and B)

Program Name: AREA-11/15 (AREA calculations on the HP-11C or HP-15C)

Author: Thomas W. Beers

Date: June 1985

Purpose: To calculate the area of a closed traverse, given input in the form of straight-side distances and directions either as azimuths or as bearings and quadrants. Area is calculated in square units corresponding to the distance units and optionally in acres or hectares. The linear error of closure is calculated and displayed if desired. The program can also be used as a bearing-to-azimuth converter. Note that there is no provision to "balance" the traverse; if this is necessary it must be done separately.

A. Storage Assignments

Register	15C	Use
0-5	(2-7)	under the control of the summation key
0	(2)	number of sides
1	(3)	sum of latitudes
3	(5)	sum of departures
6	(0)	bearing, then azimuth, in degree and decimal form
7	(1)	double meridian distance of the previous side (DMD_{i-1})
8		departure of the previous side (DEP_{i-1})
9		cumulative area = half the sum of DMD_i
.0		quadrant number: 1 = NE, 2 = SE, 3 = SW, 4 = NW
.1		the constant 43,560 or 10,000

B. Labels

Name	Use
[A]*	converts keyed-in Azimuth angle from D.MS (degrees, minutes, seconds) format to D.dd (degrees, decimal degrees) format, pauses, then transfers to label 1

B. Labels (continued)

Name	Use
[B]	stores keyed-in quadrant number and converts keyed-in Bearing in D.MS format to an azimuth angle in D.dd format
[C]	Calculates (actually just recalls and displays) area in the square of input units, and optionally in acres or hectares, and linear error of closure
[D]	uses the keyed-in Distance and stored azimuth angle to calculate latitudes, departures, DMDs, and cumulative area
[E]	clears the registers for a new data set and stores and displays the appropriate metric or U.S. units divisor, depending on the status of flag 0
1	stores azimuth angle in D.dd format then displays it in D.MS format
[9] [6] [5] [8]	generates the quadrant number implied by the key's position (9 = NE: 1; 6 = SE: 2; 5 = SW: 3; 8 = NW: 4) and using the keyed-in bearing passes control to label B

*brackets indicate meaningful external (i.e., in RUN mode) accessibility.

C. Flags

Number	Use
0	clear assumes the input distance units are in feet, therefore output area is in sq. ft. and acres set assumes the input distance units are in meters, therefore output area is in sq. meters and hectares[1/]

D. Program Procedure

1. In program mode, key in the program.

2. In run mode, activate USER mode and clear flag 0 (distances in feet) or set flag 0 (distances in meters or chains).

3. Press E to initialize the calculator; observe either 43,560. or 10,000. depending upon the status of flag 0.

[1/] If distance input units are chains, choose this option, then area in acres = 1000 x "no. of hectares".

D. Program Procedure (continued)

4. Provide the angle of the first side of the traverse, using any one of the options cited:[2/]

   a. key the azimuth angle as DD.MMSS, press A, observe the angle as DD.dddd then as DD.MMSS to verify the input, go to step 5;

   b. key the bearing, press ENTER, key the quadrant number (NE=1, SE=2, SW=3, NW=4), press B, observe the angle as an azimuth in DD.MMSS format, go to step 5;

   c. key the bearing, press GSB followed by either 9, 6, 5, or 8 (thus implying quadrants 1, 2, 3, or 4, respectively), observe the angle as an azimuth in D.MS format, go to step 5.

5. Key the distance, press either D or R/S;[3/] observe 1., indicating the first side has been processed.

6. Repeat steps 4 and 5 for all sides of the traverse.

7. Press C to display the area in sq. input units, press R/S to display the area in acres or hectares (or acres/1000 if chains are the input units), press R/S to display the linear error of closure. (step 7 can be repeated.)

   Special note: The area answers may be displayed with negative signs, which simply means that the direction of the traverse was clockwise, rather than counterclockwise.

[2/] If a mistake is detected just after the current bearing or angle has been processed, it can be corrected by repeating any of the three options using the correct angle or bearing. Mistakes detected after the distance is processed (step 5) mean "start over".

[3/] In the HP-41 version of the program, R/S must be pressed, since label D has another use.

E. Examples

EXAMPLE 1

Given the following closed traverse with distance expressed in feet, calculate the area in sq. ft., in acres, and the linear error of closure.

Side	Bearing as DD.MMSS	Quadrant	Distance
1	S 51.0656 W	3	199.123
2	S 13.2944 E	2	128.550
3	N 63.2606 E	1	111.803
4	N 7.0730 E	1	201.556

SOLUTION:

Note that each of the three angle input options are used below to demonstrate the interchangeability of the procedures. USER mode is presumed.

Step	Input	Key(s)	Display	Comment
2.	-	g CF 0	-	clear flag 0
3.	-	E	43,560.	implies distances in feet
4b.	51.0656↑3	B	231.0656	azimuth displayed as DD.MMSS
5.	199.123	D (or R/S)	1.	one side done
4c.	13.2944	GSB 6	166.3016	azimuth displayed
5.	128.55	D (or R/S)	2.	two sides done
4c.	63.2606	GSB 9	63.2606	azimuth displayed
5.	111.803	D (or R/S)	3.	three sides done
4a.	7.073	A	7.1250 (pause) 7.0730	azimuth as DD.dddd then in D.MS format
5.	201.556	D (or R/S)	4.	four sides done
7.	-	C	20,937.438	area in sq. ft.
	-	R/S	0.481	area in acres
	-	R/S	0.001	error of closure in feet

EXAMPLE 2

Assuming the same traverse data as in EX. 1, except that distances are expressed in meters, calculate the area in sq. meters, in hectares, and the linear error of closure.

SOLUTION:

Step	Input	Key(s)	Display	Comment
2.	-	g SF 0	-	set flag 0
3.	-	E	10,000.	implies distances in meters
4b. and 5.	51.0656	GSB 5	231.0656	azimuth
(repeated)	199.123	R/S	1.	
	13.2944	GSB 6	166.3016	azimuth
	128.55	R/S	2.	
	63.2606	GSB 9	63.2606	azimuth
	111.803	R/S	3.	
	7.073	GSB 9	7.0730	azimuth
	201.556	R/S	4.	
7.	-	C	20937.438	area in sq. meters
	-	R/S	2.094	area in hectares
	-	R/S	0.001	error of closure in meters

EXAMPLE 3

Assuming the same data except that distances are in chains, calculate the area in sq. chains, in acres, and the linear error of closure.

SOLUTION:

The procedure is identical in all respects to EX. 2 down to step 7. which is

Step	Input	Key(s)	Display	Comment
7.	-	C	20937.438	area in sq. chains
	-	R/S	2.094	area in acres/1000
	1000	X	2093.744	area in acres
	-	R/S	0.001	error of closure in chains

F. Program Statements and Code

Step	Statement	Code	Step	Statement	Code	Step	Statement	Code
01	[f LBL E][4/]	42 21 15	31	1	1	61	X	20
02	f CLEAR REG	42 34	32	8	8	62	2	2
03	4	4	33	0	0	63	÷	10
04	3	3	34	X	20	64	STO + 9	44 40 9
05	5	5	35	X≷Y	34	65*	RCL 0 (2)	45 0
06	6	6	36	g LSTx	43 36	66	f FIX 0	42 7 0
07	0	0	37	RCL .0	45 .0	67	R/S	31
08	STO .1	44 .1	38	X	20	68	[f LBL C]	42 21 13
09	EEX	26	39	COS	24	69	f FIX 3	42 7 3
10	4	4	40	X	20	70	RCL 9	45 9
11	g F? 0	43 6 0	41	-	30	71	R/S	31
12	STO .1	44 .1	42	(f LBL 1)	42 21 1	72	RCL .1	45 .1
13	RCL .1	45 .1	43*	STO 6 (0)	44 6	73	÷	10
14	f FIX 0	42 7 0	44	f -> H.MS	42 2	74	R/S	31
15	g RTN	43 32	45	R/S	31	75*	RCL 3 (5)	45 3
16	[f LBL A]	42 21 11	46	[f LBL D]	42 21 14	76*	RCL 1 (3)	45 1
17	f FIX 4	42 7 4	47*	RCL 6 (0)	45 6	77	g -> P	43 26
18	g -> H	43 2	48	X≷Y	34	78	R/S	31
19	f PSE	42 31	49	f -> R	42 26	79	[f LBL 5]	42 21 5
20	GTO 1	22 1	50	Σ+	49	80	3	3
21	[f LBL B]	42 21 12	51	R↓	33	81	GTO B	22 12
22	f FIX 4	42 7 4	52	g LSTx	43 36	82	[f LBL 6]	42 21 6
23	STO .0	44 .0	53	X≷Y	34	83	2	2
24	X≷Y	34	54	RCL 8	45 8	84	GTO B	22 12
25	g -> H	43 2	55	X≷Y	34	85	[f LBL 8]	42 21 8
26*	STO 6 (0)	44 6	56	STO 8	44 8	86	4	4
27	X≷Y	34	57	+	40	87	GTO B	22 12
28	2	2	58*	RCL 7 (1)	45 7	88	[f LBL 9]	42 21 9
29	÷	10	59	+	40	89	1	1
30	g INT	43 44	60*	STO 7 (1)	44 7	90	GTO B	22 12

Note: g MEM leads to
P - 01 r - .5

[4/] Brackets and parentheses are not on the keyboard; they are used to indicate externally meaningful or internal use labels, respectively.

* Indicates statements that must be changed for the HP-15C. Suggested or required change is shown in parentheses. Although the "code" for these statements will also change this is left for the 15C user to alter.

G. Formulas and Unique Features

1. Formulas

a. Traverse area (A) was calculated using the double meridian distance (DMD) method which can be found in most elementary surveying textbooks. The DMD procedure makes use of latitudes (LAT) and departures (DEP) calculated from the traverse data. The pertinent formulas are:

$$A = \frac{1}{2} \sum_{i=1}^{n} (DMD_i)(LAT_i)$$

in which

$$DMD_i = DMD_{i-1} + DEP_{i-1} + DEP_i \ ,$$

$$LAT_i = (D_i)(\cos Z_i) \ ,$$

where

$$DEP_i = (D_i)(\sin Z_i) \ ;$$

D_i = distance of the i^{th} side,

and Z_i = bearing of the i^{th} side expressed as an azimuth clockwise from north.

b. Area in acres or hectares was calculated simply by dividing the area (A) by 43,560, or 10,000 respectively.

c. The linear area of closure (C), the distance by which the traverse fails to close, is defined by the formula

$$C = \sqrt{(\text{sum of DEP})^2 + (\text{sum of LAT})^2}$$

2. Unique Features

The calculation of area of a closed traverse on a programmable calculator has been a popular one since the HP-25 came on the market in the early 1970's. As a result, many efficient computational short-cuts have been employed. Some of these were used in this program; they warrant the attention of the serious programmer and therefore are discussed below.

a. Use of azimuth rather than bearing angle

Those of us who learned the DMD method before computers came along recall calculating latitudes and departures from the formulas LAT = (distance)(cos bearing), DEP = (distance)(sin bearing), then placing the results in a table, according to the convention LAT = + for north bearing, - for south and DEP = + for east bearing, - for west, and proceeding with the DMD calculation algorithm. This process can be automated by converting the bearings to azimuths (Z) and using the formulas LAT = (D)(cos Z) and DEP = (D)(sin Z), which assures that the proper sign is employed for the LATs and DEPs. This fortuitous happening does not seem immediately logical since azimuth angles are measured clockwise from the north axis, while angles in the trigonometric sense are turned counterclockwise from the "east axis"; but it works!

b. Conversion of bearings to azimuths

The procedure for the conversion of bearings (which of course represent the angle from north or south measured toward east or west) to azimuths is commonly done by visualizing the N-S, E-W axes and applying some mental arithmetic. Obviously unsuited for programming! A number of formulations have been proposed, such as:

To convert a bearing in degrees (B) to an azimuth in degrees (Z):

If Quadrant, Q, =	then
1 (NE)	Z = B
2 (SE)	Z = 180 - B
3 (SW)	Z = 180 + B
4 (NW)	Z = 360 - B

But such relations are still somewhat awkward to use, therefore either of the following are to be preferred:

$$Z = (180)(\text{integer portion of } \tfrac{Q}{2}) + (-1)^{Q+1}(B)$$

or $$Z = (180)(\text{integer portion of } \tfrac{Q}{2}) - (B)\cos(180\ Q)$$

The latter formula was used in the AREA-11/15 program.

c. Angle format conversion

The relations described above (in section b.) require that bearings and azimuths are expressed in degree format (D.dd). It is convenient, however, to input and refer to bearings in degrees, minutes and seconds (D.MS) format. Conversion from one format to the other is accomplished on most HP calculators by either of the following functions:[5/]

-> H ("to hours")--to convert D.MS to D.dd

or -> H.MS ("to hours, minutes, seconds")--to convert D.dd to D.MS

d. Calculation of LATs and DEPs

If the distance (D) and azimuth (Z) are located in the X and Y registers respectively, the formulas for LATs and DEPs (LAT = D cos Z and DEP = D sin Z) can be solved in one program step using the function ->R ("to rectangular coordinates"), leaving the latitude in the X register and the departure in Y. This efficient process works because in the usual polar and rectangular coordinate representation, X = r cos θ and Y = r sin θ, and in our application r = D, and θ = Z.

e. Simultaneous summation of LATs and DEPs

In order to calculate the error of closure, the sum of latitudes and the sum of departures must be obtained. This is easily accomplished in one program step on most HP calculators by the Σ+ ("sum plus") function. So, if LAT_i is in X and DEP_i is in Y, executing Σ+ on the HP-11C adds these to register 1 and register 3, respectively (on the HP-11C).

f. Calculation of error of closure

The error of closure (C) defined by the formula

$$C = \sqrt{(\Sigma LAT)^2 + (\Sigma DEP)^2}$$

can be found in one program step using the function ->P ("to polar coordinates"). The rationale behind this calculation is that C is actually the hypotenuse of the right triangle whose sides are X = r cos θ and Y = r sin θ, and since the magnitude, r, in polar coordinates is calculated from the X and Y rectangular coordinates by

$$r = \sqrt{X^2 + Y^2},$$

the ->P function solves the Pythagorean Theorem for the hypotenuse! This meets our needs since we can position ΣLAT in the X register and ΣDEP in the Y register, use the ->P function, and the result in the X register will be r = C.

[5/] On the HP-41, the corresponding functions are HR (from D.MS to D.dd) and HMS (from D.dd to D.MS).

Appendix A: AREA41 (program number 41F039), Appropriate for the HP-41C, CV, or CX.

The program found on the following pages, as a listing and in barcode form, was written so that the user could in general follow the instructions found in the main section of this paper. However, because of the prompting available on the HP-41 and other design differences from the HP-11, the following should be noted before the user tries AREA41 initially:

1. Initialization is first achieved by XEQ AREA41; re-initialization for a new data set by shift E.

2. The three bearing options, providing cryptic prompts, are available on keys A, B, C, implying Azimuths, Bearings and quadrant number, and bearing Combined with quadrant key, respectively. The prompts for options A and B must be followed by proper input then R/S depression, while the prompt for option C is followed by the bearing input and keys D, E, I, or J as needed to indicate quadrants 4, 1, 3, or 2 respectively.

3. The distance prompt is automatically obtained, and after proper input, R/S must be pressed (not D!).

4. When all sides have been processed and the prompt for the next direction is in the display, the area (and other output) is obtained by pressing R/S or by shift C (which can be repeated). Press R/S between answers, unless a printer is attached, in which case the output is continuous.

5. Storage register assignments in the HP-41 program are slightly different from the HP-11 since the summation register configuration is not the same. The AREA41 program designates R00 through R05 as the statistical registers (step 06), but because "n" is positioned differently from the HP-11, the following are the pertinent register assignments in AREA41:

Register		Use
00		ΣX (= sum of latitudes)
02		ΣY (= sum of departures)
05		n (= number of sides)
06	same as in AREA-11/15	bearing or azimuth, in D.dd format
07		DMD_{i-1}
08		DEP_{i-1}
09		cumulative area
10		quadrant number keyed in
11		43,560 or 10,000
12		azimuth in D.MS format
13		quadrant number generated in labels D, E, I, or J
14		sum of distances

CAUTION: after AREA41 has been executed, the user should note that the statistical register assignment remains R00 through R05; if the "normal" assignment of R11-R16 is desired, ΣREG 11 must be executed.

6. An additional calculation was made in AREA41: the precision of the survey, defined by the formula

   PREC. = 1: X

   where $X = \frac{\text{sum of the distances}}{\text{error of closure}}$.

7. The audio part of the bearing and distance prompts employs tones which were assembled using synthetic programming. Unfortunately, the program listing does not identify the tones precisely, but for those who care about such details, "TONE 0" is actually TONE 100 and "TONE 7" is actually TONE 87. An advantage of loading the program from the barcode is that these synthetic tones will be compiled in the program.

8. Printed output can be obtained if an appropriate printer is attached to the HP-41 when AREA41 is used. The samples shown at the end of the program listing are from the HP 82143A printer and demonstrate options A, B, and C for bearing entries and the U.S.-Metric option.

Program Listing for AREA41, an HP-41 Analogy of the Program AREA-11/15

```
01♦LBL "AREA41"
02♦LBL e
03 CLRG
04 SF 27
05 CF 29
06 ΣREG 00
07 1
08 ST+ 05
09 43560
10 FS? 00
11 1 E4
12 STO 11
13 FIX 0
14 "U.S.: "
15 FS? 00
16 "METRIC: "
17 ARCL 11
18 AVIEW
19 PSE
20 ADV
21 FS? 55
22 CF 21
23 "BRG. OPTIONS:"
24 AVIEW
25 SIN
26 SIN
27 "KEY A,B, OR C"
28 AVIEW
29 SIN
30 SIN
31 FS? 55
32 SF 21
33♦LBL C
34♦LBL 10
35 FIX 0
36 CF 05
37 CF 06
38 CF 22
39 CLA
40 ARCL 05
41 "⊦ BRG.,Q. KEY"
42 TONE 7
43 TONE 7
44 PROMPT
45 FC?C 22
46 GTO c
47 "THINK"
48 AVIEW
49 SIN
50 SIN
51 "PRESS A,B, OR C"
52 PROMPT
53♦LBL A
54 FIX 0
55 CF 22
56 SF 05
57 CF 06
58 CLA
59 ARCL 05
60 "⊦ AZIMUTH?"
61 TONE 0
62 PROMPT
63 FC?C 22
64 GTO c
65 STO 12
66 CLA
67 XEQ 16
68 AVIEW
69 SIN
70 RCL 12
71 FIX 4
72 HR
73 "= "
74 ARCL X
75 "⊦ DEG."
76 AVIEW
77 PSE
78 GTO 01
79♦LBL B
80 FIX 0
81 CF 22
82 SF 06
83 CF 05
84 CLA
85 ARCL 05
86 "⊦ BRG.↑Q. NO. ?"
87 TONE 0
88 TONE 0
89 PROMPT
90 FC?C 22
91 GTO c
92 FIX 4
93 CLA
94 ARCL Y
95 "⊦ ↑ Q"
96 FIX 0
97 ARCL X
98 AVIEW
99♦LBL 11
100 STO 10
101 X<>Y
102 HR
103 STO 06
104 X<>Y
105 2
106 /
107 INT
108 180
109 *
110 X<>Y
111 LASTX
112 RCL 10
113 *
114 COS
115 *
116 -
117♦LBL 01
118 STO 06
119♦LBL d
120 FIX 0
121 CLA
122 TONE 7
123 ARCL 05
124 "⊦ DISTANCE?"
125 PROMPT
126 FIX 3
127 "  DIST: "
128 ARCL X
129 AVIEW
130 ADV
131 ST+ 14
132 RCL 06
133 X<>Y
134 P-R
135 Σ+
136 RDN
137 LASTX
138 X<>Y
139 RCL 08
140 X<>Y
141 STO 08
142 +
143 RCL 07
144 +
145 STO 07
146 *
147 2
148 /
149 ST+ 09
150 FS?C 05
151 GTO A
152 FS?C 06
153 GTO B
154 GTO 10
155♦LBL D
156 4
157 "N "
158 XEQ 12
159 "⊦ W"
160 GTO 13
161♦LBL E
162 1
163 "N "
164 XEQ 12
165 "⊦ E"
166 GTO 13
167♦LBL I
168 3
169 "S "
170 XEQ 12
171 "⊦ W"
172 GTO 13
173♦LBL J
174 2
175 "S "
176 XEQ 12
177 "⊦ E"
178♦LBL 13
179 AVIEW
180 GTO 11
181♦LBL 12
182 STO 12
183 X<>Y
184 STO 13
185♦LBL 16
186 FIX 0
187 INT
188 ARCL X
189 LASTX
190 FRC
191 1 E2
192 *
193 INT
194 "⊦-"
195 10
196 X>Y?
197 "⊦0"
198 RDN
199 ARCL X
200 LASTX
```

Program Listing for AREA41 (continued) and Sample Printed Output Obtained in MAN Printer Mode

```
201 FRC
202 1 E2
203 *
204 INT
205 "⊢-"
206 10
207 X>Y?
208 "⊢0"
209 RDN
210 ARCL X
211 RCL 13
212 RCL 12
213 RTN
214♦LBL c
215 FIX 2
216 "AREA= "
217 ARCL 09
218 FC? 00
219 "⊢ FT.↑2"
220 FS? 00
221 "⊢ M.↑2"
222 AVIEW
223 XEQ 15
224 RCL 09
225 RCL 11
226 /
227 FIX 3
228 "= "
229 ARCL X
230 FC? 00
231 "⊢ ACRES"
232 FS? 00
233 "⊢ HECT."
234 AVIEW
235 ADV
236 XEQ 15
237 RCL 02
238 RCL 00
239 R-P
240 "EOC= "
241 ARCL X
242 FC? 00
243 "⊢ FT."
244 FS? 00
245 "⊢ M."
246 AVIEW
247 XEQ 15
248 RCL 14
249 X<>Y
250 /
251 INT
252 FIX 0
253 "PREC.= 1:"
254 ARCL X
255 AVIEW
256 RTN
257♦LBL 15
258 FS? 21
259 PSE
260 FC? 21
261 STOP
262 END
```

```
U.S.: 43560

51.0656 ↑ Q3
 DIST: 199.123

13.2944 ↑ Q2
 DIST: 128.550

63.2606 ↑ Q1
 DIST: 111.803

7.0730 ↑ Q1
 DIST: 201.556

AREA= 20937.44 FT.↑2
= 0.481 ACRES

EOC= 0.001 FT.
PREC.= 1:509937
```

```
U.S.: 43560

231-06-56
= 231.1156 DEG.
 DIST: 199.123

166-30-16
= 166.5044 DEG.
 DIST: 128.550

63-26-06
= 63.4350 DEG.
 DIST: 111.803

7-07-30
= 7.1250 DEG.
 DIST: 201.556

AREA= 20937.44 FT.↑2
= 0.481 ACRES

EOC= 0.001 FT.
PREC.= 1:509937
```

```
U.S.: 43560

S 51-06-56 W
 DIST: 199.123

S 13-29-44 E
 DIST: 128.550

N 63-26-06 E
 DIST: 111.803

N 7-07-30 E
 DIST: 201.556

AREA= 20937.44 FT.↑2
= 0.481 ACRES

EOC= 0.001 FT.
PREC.= 1:509937
```

```
                SF 00
METRIC: 10000

S 51-06-56 W
 DIST: 199.123

S 13-29-44 E
 DIST: 128.550

N 63-26-06 E
 DIST: 111.803

N 7-07-30 E
 DIST: 201.556

AREA= 20937.44 M.↑2
= 2.094 HECT.

EOC= 0.001 M.
PREC.= 1:509937
```

HP-41 Program Barcode: AREA41

Program Registers Required: 88

ROW 1: LINES 1 - 3

ROW 2: LINES 4 - 9

ROW 3: LINES 9 - 14

ROW 4: LINES 14 - 16

ROW 5: LINES 16 - 23

ROW 6: LINES 23 - 25

ROW 7: LINES 26 - 27

ROW 8: LINES 27 - 35

ROW 9: LINES 35 - 41

ROW 10: LINES 41 - 42

ROW 11: LINES 43 - 47

ROW 12: LINES 47 - 51

ROW 13: LINES 51 - 55

ROW 14: LINES 55 - 60

ROW 15: LINES 60 - 64

AREA41 (continued)

ROW 16: LINES 64 - 73

ROW 17: LINES 73 - 77

ROW 18: LINES 78 - 84

ROW 19: LINES 85 - 86

ROW 20: LINES 86 - 91

ROW 21: LINES 91 - 95

ROW 22: LINES 96 - 106

ROW 23: LINES 107 - 117

ROW 24: LINES 118 - 124

ROW 25: LINES 124 - 127

ROW 26: LINES 127 - 131

ROW 27: LINES 132 - 144

ROW 28: LINES 145 - 152

ROW 29: LINES 153 - 158

ROW 30: LINES 158 - 163

AREA41 (continued)

ROW 31: LINES 164 - 169

ROW 32: LINES 169 - 173

ROW 33: LINES 174 - 179

ROW 34: LINES 180 - 188

ROW 35: LINES 189 - 196

ROW 36: LINES 197 - 204

ROW 37: LINES 205 - 211

ROW 38: LINES 212 - 216

ROW 39: LINES 217 - 220

ROW 40: LINES 220 - 224

ROW 41: LINES 225 - 231

ROW 42: LINES 231 - 233

ROW 43: LINES 233 - 240

ROW 44: LINES 240 - 243

ROW 45: LINES 243 - 248

AREA41 (continued)

ROW 46: LINES 249 - 253

ROW 47: LINES 253 - 260

ROW 48: LINES 261 - 262

Appendix B: AREA-S (program number 41F040), A Short Version of AREA41.

While the AREA41 program is flexible and demonstrates how various options can be incorporated into one program, it was felt that a shorter version incorporating the most likely options might be equally desirable. Thus, AREA-S is included on the following pages in listing and barcode form. It assumes distances are measured in feet and bearings in D.MS format are input using option C -- key the bearing, and press D, E, I, or J as appropriate for NW, NE, SW, and SE quadrants, respectively.

Program Listing and Sample Printed Output for AREA-S, a Short Version of AREA41

```
01♦LBL "AREA-S"
02 ADV
03 CLRG
04 SF 27
05 CF 29
06 ΣREG 00
07 1
08 ST+ 05
09♦LBL C
10♦LBL 10
11 FIX 0
12 CF 22
13 CLA
14 ARCL 05
15 "├ BRG.,Q. KEY"
16 TONE 7
17 TONE 7
18 PROMPT
19 FC?C 22
20 GTO c
21 "THINK"
22 AVIEW
23 SIN
24 SIN
25 GTO 10
26♦LBL 11
27 STO 10
28 X<>Y
29 HR
30 X<>Y
31 2
32 /
33 INT
34 180
35 *
36 X<>Y
37 LASTX
38 RCL 10
39 *
40 COS
41 *
42 -
43 STO 06
44 FIX 0
45 CLA
46 TONE 7
47 ARCL 05
48 "├ DISTANCE?"
49 PROMPT
50 FIX 3
51 " DIST: "
52 ARCL X
53 AVIEW
54 ADV
55 ST+ 14
56 RCL 06
57 X<>Y
58 P-R
59 Σ+
60 RDN
61 LASTX
62 X<>Y
63 RCL 08
64 X<>Y
65 STO 08
66 +
67 RCL 07
68 +
69 STO 07
70 *
71 2
72 /
73 ST+ 09
74 GTO 10
75♦LBL D
76 4
77 "N "
78 XEQ 12
79 "├ W"
80 GTO 13
81♦LBL E
82 1
83 "N "
84 XEQ 12
85 "├ E"
86 GTO 13
87♦LBL I
88 3
89 "S "
90 XEQ 12
91 "├ W"
92 GTO 13
93♦LBL J
94 2
95 "S "
96 XEQ 12
97 "├ E"
98♦LBL 13
99 AVIEW
100 GTO 11
101♦LBL 12
102 STO 12
103 X<>Y
104 STO 13
105 FIX 0
106 INT
107 ARCL X
108 LASTX
109 FRC
110 1 E2
111 *
112 INT
113 "├-"
114 10
115 X>Y?
116 "├0"
117 RDN
118 ARCL X
119 LASTX
120 FRC
121 1 E2
122 *
123 INT
124 "├-"
125 10
126 X>Y?
127 "├0"
128 RDN
129 ARCL X
130 RCL 13
131 RCL 12
132 RTN
133♦LBL c
134 FIX 2
135 "AREA= "
136 ARCL 09
137 "├ FT.↑2"
138 AVIEW
139 XEQ 15
140 RCL 09
141 43560
142 /
143 FIX 3
144 "= "
145 ARCL X
146 "├ ACRES"
147 AVIEW
148 ADV
149 XEQ 15
150 RCL 02
151 RCL 00
152 R-P
153 "EOC= "
154 ARCL X
155 "├ FT."
156 AVIEW
157 XEQ 15
158 RCL 14
159 X<>Y
160 /
161 INT
162 FIX 0
163 "PREC.= 1:"
164 ARCL X
165 AVIEW
166 RTN
167♦LBL 15
168 FS? 21
169 PSE
170 FC? 21
171 STOP
172 END
```

```
S 51-06-56 W
 DIST: 199.123

S 13-29-44 E
 DIST: 128.550

N 63-26-06 E
 DIST: 111.803

N 7-07-30 E
 DIST: 201.556

AREA= 20937.44 FT.↑2
= 0.481 ACRES

EOC= 0.001 FT.
PREC.= 1:509937
```

HP-41 Program Barcode: AREA-S

Program Registers Required: 52

ROW 1: LINES 1 - 4

ROW 2: LINES 4 - 11

ROW 3: LINES 12 - 15

ROW 4: LINES 15 - 19

ROW 5: LINES 20 - 25

ROW 6: LINES 25 - 35

ROW 7: LINES 36 - 46

ROW 8: LINES 47 - 48

ROW 9: LINES 48 - 52

ROW 10: LINES 52 - 63

ROW 11: LINES 64 - 74

ROW 12: LINES 75 - 79

ROW 13: LINES 80 - 85

ROW 14: LINES 85 - 90

ROW 15: LINES 91 - 96

AREA-S (continued)

ROW 16: LINES 96 - 103

ROW 17: LINES 104 - 112

ROW 18: LINES 113 - 119

ROW 19: LINES 120 - 127

ROW 20: LINES 127 - 135

ROW 21: LINES 135 - 137

ROW 22: LINES 137 - 141

ROW 23: LINES 142 - 146

ROW 24: LINES 146 - 153

ROW 25: LINES 153 - 156

ROW 26: LINES 157 - 163

ROW 27: LINES 163 - 168

ROW 28: LINES 168 - 172

Program No. 11F008

Calculator: HP-11C
HP-41C (see Appendix A)

Program Name: MUGO-11 (Means, with Ungrouped and Grouped Options on the HP-11C)

Author: Thomas W. Beers

Date: February 1986

Purpose: To calculate, for a set of univariate data, the arithmetic, quadratic, geometric, and harmonic means, and the standard deviation, coefficient of variation, and standard error. Data can be ungrouped, or grouped using either true frequencies or weights other than frequencies.

A. Storage assignments

Register	15C	Use
0	(2)	number of observations, n, or sum of frequencies (Σf_i), or sum of weights (Σw_i)
1	(3)	ΣX_i, $\Sigma f_i X_i$ or $\Sigma w_i X_i$
2	(4)	ΣX_i^2, $\Sigma f_i X_i^2$, or $\Sigma w_i X_i^2$
3	(5)	$\Sigma \log X_i$, $\Sigma f_i \log X_i$, or $\Sigma w_i \log X_i$
4	(6)	forced to be $(\Sigma \log X_i)^2$ to prevent an "ERROR" stop, or f_i or w_i
5	(7)	not used, or X_i
6	(0)	$\Sigma 1/X_i$, $\Sigma f_i/X_i$, or $\Sigma w_i/X_i$
7	(1)	arithmetic mean ($\bar{X}$ or $\bar{X}_{wtd.}$)
8		standard deviation (s or $s_{wtd.}$)
I		number of classes, c, in the case of grouped data

B. Labels

Name	Use
[0]*	program start if data are <u>ungrouped;</u> all storage registers are cleared, flag 1 is cleared and the X register is set to zero as a signal
[1]	program start if data are <u>grouped</u> (either frequencies or weights); all storage registers are cleared, flag 1 is cleared, and Π is returned to the X register as a signal
[A]	calculates and displays the arithmetic mean, standard deviation, coefficient of variation, and the standard error
[B]	calculates and displays the quadratic mean
[C]	calculates and displays the geometric mean
[D]	calculates and displays the harmonic mean
[E]	calculates and displays all four means separated by pauses
2	data summary loop for the ungrouped case
3	data summary loop for the grouped case; also tests for negative "frequencies" which indicates the weighted case, thus setting flag 1
4	if flag 1 is set, a correction is made to the standard deviation so that the effective denominator under the radical is $(c-1)\Sigma w_i$ rather than $(\Sigma f_i)(\Sigma f_i-1)$ which is appropriate for the frequency case

*brackets indicate meaningful "RUN mode" accessability.

C. Flags

Number	Use
1	Cleared in labels 0 and 1 but then set in label 3 if keyed-in "frequencies" are negative, thus indicating that they are actually weights.

D. Program procedure and example

I. In PRGM mode, load the program.

II. In RUN mode, proceed as given below by way of example. The three cases described are ungrouped, grouped assuming frequencies, and grouped assuming weights. Assume FIX 3, and in USER mode.

1. Calculate the four means and standard deviation(s), coefficient of variation (CV), and standard error ($s_{\bar{X}}$) for the following data set:

   62, 58, 62, 73, 84, 68

Step	Input	Keys	Output	Comment
a.	-	GSB 0	0.000	ungrouped signal
b.	62	R/S	1.000	first observation
	58	R/S	2.000	2nd observation
	62	R/S	3.000	3rd observation
	73	R/S	4.000	4th observation
	84	R/S	5.000	5th observation
	68	R/S	6.000	6th observation
c.	-	A	67.833	$\bar{X}$ = A.M.
	-	R/S	9.517	s
	-	R/S	14.029	SV as a percent
	-	R/S	3.885	$s_{\bar{X}}$
d.	-	B	68.387	Q.M.
	-	C	67.306	G.M.
	-	D	66.809	H.M.
e.	-	E	67.833 (pause)	A.M.
			68.387 (pause)	Q.M.
			67.306 (pause)	G.M.
			66.809	H.M.

   Note that the user can select among steps c, d, and e, and the order of execution is immaterial.

   f. for a new set of ungrouped data go to step a.

2. Calculate the same statistics (as in example 1) for the following data:

X_i	f_i
58	4
62	2
68	10
73	6
84	8

Step	Input	Keys	Output	Comment
a.	-	GSB 1	3.142	grouped signal
b.	58↑4	R/S	1.000	first class
	62↑2	R/S	2.000	2nd class
	68↑10	R/S	3.000	3rd class
	73↑6	R/S	4.000	4th class
	84↑8	R/S	5.000	5th class
c.	-	A	71.533	$\overline{x}$ (weighted)
	-	R/S	8.897	s (weighted)
	-	R/S	12.437	CV as a percent
	-	R/S	1.624	$s_{\overline{x}}$ (weighted)
d.		E	71.533 (pause)	A.M. (weighted)
			72.066 (pause)	Q.M. (weighted)
			71.000 (pause)	G.M. (weighted)
			70.470	H.M. (weighted)

e. for a new set of grouped data go to step a.

3. Calculate the same statistics (as in example 1) for the following data:

X_i	w_i
58	.4
62	.2
68	1.0
73	.6
84	.8

Step	Input	Keys	Output	Comment
a.	-	GSB 1	3.142	grouped signal
b.	58↑.4 CHS	R/S	1.000	negative weight
	62↑.2	R/S	2.000	indicates weighted
	68↑1	R/S	3.000	data -- this need
	73↑.6	R/S	4.000	be done only for
	84↑.8	R/S	5.000	one of the
				keyed-in weights.
c.	-	A	71.533	$\overline{X}$ (weighted)
	-	R/S	7.575	s (weighted)
	-	R/S	10.590	CV as a percent
	-	R/S	4.374	$s_{\overline{x}}$ (weighted)

d. pressing A through E will give the same values as in example 2, since the weights were developed by scaling the frequencies by a factor of ten.

E. Program statements and code.

Step	Statement	Code	Step	Statement	Code	Step	Statement	Code
01	[f LBL 0]1/	42 21 0	31*	RCL 4 (6)	45 4	61	EEX	26
02	f CLEAR REG	42 34	32	X	20	62	2	2
03	g CLX	43 35	33*	STO + 2 (4)	44 40 2	63	X	20
04	g CF 1	43 5 1	34*	RCL 5 (7)	45 5	64	R/S	31
05	[f LBL 2]	42 21 2	35	g LOG	43 13	65	RCL 8	45 8
06	R/S	31	36*	RCL 4 (6)	45 4	66*	RCL 0 (2)	45 0
07	1/X	15	37	X	20	67	$\sqrt{X}$	11
08*	STO + 6 (0)	44 40 6	38*	STO + 3 (5)	44 40 3	68	÷	10
09	g LST X	43 36	39*	RCL 4 (6)	45 4	69	g RTN	43 32
10	g LOG	43 13	40*	RCL 5 (7)	45 5	70	[f LBL B]	42 21 12
11	g LST X	43 36	41	÷	10	71*	RCL 2 (4)	45 2
12	Σ+	49	42*	STO + 6 (0)	44 40 6	72*	RCL 0 (2)	45 0
13	GTO 2	22 2	43*	f ISG (f ISG I)	42 6	73	÷	10
14	[f LBL 1]	42 21 1	44	·	48	74	$\sqrt{X}$	11
15	g CF 1	43 5 1	45	RCL I	45 25	75	g RTN	43 32
16	f CLEAR REG	42 34	46	GTO 3	22 3	76	[f LBL C]	42 21 13
17*	f Π (g Π)	42 16	47	[f LBL A]	42 21 11	77	g $\bar{x}$	43 0
18	(f LBL 3)	42 21 3	48*	RCL 3 (5)	45 3	78	X⇄Y	34
19	R/S	31	49	g X^2	43 11	79	10^X	13
20*	g X<0 (g TEST 2)	43 10	50*	STO 4 (6)	44 4	80	g RTN	43 32
21	g SF 1	43 4 1	51	g $\overline{X}$	43 0	81	[f LBL D]	42 21 14
22	g ABS	43 16	52*	STO 7 (1)	44 7	82*	RCL 0 (2)	45 0
23*	STO 4 (6)	44 4	53	R/S	31	83*	RCL 6 (0)	45 6
24*	STO + 0 (2)	44 40 0	54	g s	43 48	84	÷	10
25	X⇄Y	34	55	g F? 1	43 6 1	85	g RTN	43 32
26*	STO 5 (7)	44 5	56	GSB 4	32 4	86	[f LBL E]	42 21 15
27	X	20	57	STO 8	44 8	87	g $\bar{x}$	43 0
28*	STO + 1 (3)	44 40 1	58	R/S	31	88	f PSE	42 31
29*	RCL 5 (7)	45 5	59*	RCL 7 (1)	45 7	89	GSB B	32 12
30	g X^2	43 11	60	÷	10	90	f PSE	42 31

1/ Brackets and parentheses are not programmable; they are used to indicate externally meaningful or internal use labels, respectively.

E. Program statements and code. (continued)

Step	Statement	Code
91	GSB C	32 13
92	f PSE	42 31
93	GTO D	22 14
94	(f LBL 4)	42 21 4
95*	RCL 0 (2)	45 0
96	1	1
97	-	30
98	RCL I	45 25
99	1	1
100	-	30
101	÷	10
102	$\sqrt{x}$	11
103	X	20
104	g RTN	43 32

NOTE: g MEM leads to P-01 r-.3

*Indicates statements that must be changed for the HP-15C. Suggested or required change is shown in parentheses. Although the "code" for these statements will also change this is left for the 15C user to alter.

F. Formulas

1. Ungrouped case

Given n observations, and summations over the range indicated by $\sum_{i=1}^{n}$,

$$\text{arithmetic mean} = \bar{X} = \text{A.M.} = \frac{\Sigma X_i}{n}$$

$$\text{quadratic mean} = \text{Q.M.} = \sqrt{\frac{\Sigma X_i^2}{n}}$$

$$\text{geometric mean} = \text{G.M.} = \text{antilog}\left(\frac{\Sigma \log X_i}{n}\right)$$

$$\text{harmonic mean} = \text{H.M.} = \frac{n}{\Sigma \frac{1}{X_i}}$$

$$\text{standard deviation} = s = \sqrt{\frac{\Sigma (X_i - \bar{X})^2}{n-1}}$$

$$= \sqrt{\frac{n\Sigma X_i^2 - (\Sigma X_i)^2}{n(n-1)}}$$

$$\text{coefficient of variation} = CV = \frac{s}{\bar{x}}(100)$$

$$\text{standard error} = s_{\bar{X}} = \frac{s}{\sqrt{n}}$$

2. Grouped case using frequencies.

Given c groups (or classes) of observations, each having a frequency indicated by f_i, the summations are over the c classes, i.e., $\sum_{i=1}^{c}$, therefore the sum of the frequencies, $\Sigma f_i = n$, the total number of observations.

$$\overline{X}_{wtd} = A.M. = \frac{\Sigma f_i X_i}{\Sigma f_i} = \frac{\Sigma f_i X_i}{n}$$

$$Q.M._{wtd.} = \sqrt{\frac{\Sigma f_i X_i^2}{n}}$$

$$G.M._{wtd.} = \text{antilog} \ \left(\frac{\Sigma f_i \log X_i}{n}\right)$$

$$H.M._{wtd.} = \frac{n}{\Sigma \frac{f_i}{X_i}}$$

$$s_{wtd.} = \sqrt{\frac{\Sigma f_i (X_i - \overline{X})^2}{n-1}}$$

$$= \sqrt{\frac{(\Sigma f_i)(\Sigma f_i X_i^2) - (\Sigma f_i X_i)^2}{n(n-1)}}$$

$$CV = \frac{s}{\bar{x}}(100)$$

$$s_{\overline{X}} = \frac{s}{\sqrt{n}}$$

3. Grouped case using weights.

Given c groups of observations, each having a weight indicated by w_i, the summations are again over the c classes, $\sum_{i=1}^{c}$, but the sum of the weights is <u>not</u> the total number of observations. We now have c observations, and $\Sigma w_i \neq c$. This not so subtle change, however, affects primarily the standard deviation calculation. The casual user is cautioned that only the weighted arithmetic mean, s, CV, and $s_{\overline{X}}$ are used to any extent. The other weighted means as defined below have questionable utility and indeed are difficult to justify. Their formulas are given for completeness only.

$$A.M._{wtd.} = \frac{\Sigma w_i X_i}{\Sigma w_i}$$

$$Q.M._{wtd.} = \sqrt{\frac{\Sigma w_i X_i^2}{\Sigma w_i}}$$

$$G.M._{wtd.} = \text{antilog}\left(\frac{\Sigma w_i \log X_i}{\Sigma w_i}\right)$$

$$H.M._{wtd.} = \frac{\Sigma w_i}{\Sigma \frac{w_i}{X_i}}$$

$$s_{wtd.} = \sqrt{\frac{\Sigma w_i (X_i - \bar{X})^2}{c-1}}$$

$$s_{wtd.} = \sqrt{\frac{(\Sigma w_i)(\Sigma w_i X_i^2) - (\Sigma w_i X_i)^2}{(\Sigma w_i)(c-1)}}$$

$$CV = \frac{s}{\bar{X}}(100)$$

$$s_{\bar{X}} = \frac{s}{\sqrt{\Sigma w_i}}$$

Appendix A: MUG041 (program number 41F042), Appropriate for the HP-41C, CV, or CX.

The program found on the following pages, as a listing and in barcode form, can be used with any model of the HP-41. It was written so that the user could in general follow the instructions found in the HP-11C section of this documentation, assuming he knows that the XEQ key on the HP-41 is equivalent to the GSB key on the HP-11C.

Briefly, the steps in using MUG041 are as follows, assuming the program has been loaded into the calculator:

1. Program start and initialization is achieved by XEQ MUG041 (which can be assigned to a key, say shift LN, i.e., -15).

2. Select ungrouped or grouped option.

   a. The ungrouped option is initially assumed, as evident in the first prompt: KEY X(1), R/S, meaning of course, "key in the first value of X and press R/S". The prompt for X(2) will follow, after keying X1 and pressing R/S.

   b. Grouped option -- using frequencies (F) or other weights (W).

      (1) At the initial prompt described above, pressing XEQ 01 will lead to the prompt:

      (C1) X↑F, R/S, meaning "for class 1 key in the value of X, press ENTER , key in the frequency for the class, and press R/S." The prompt for the next class X and frequency will follow after R/S has been pressed.

      (2) If in response to the (C1), X↑F, R/S prompt the user keys in a negative value for "F", this and all subsequent "frequencies" will be assumed to be positive weights; the prompt for the second class then will be (C2), X↑W, R/S.

3. After all data have been keyed in, the means and other statistics are obtained using the same labels and keys as in the HP-11C program, for example

Key Pressed	Output
A	AM= XX.XXX
R/S	SDEV= XX.XXX
R/S	CV= XX.XXX %
R/S	SXBAR= XX.XXX
B	QM= XX.XXX
C	GM= XX.XXX
D	HM= XX.XXX
E	All four means, separated by pauses.

4. For a new set of data:
   ungrouped: XEQ 00 (or XEQ MUGO41)
   grouped: XEQ 01 followed by input of X values and either positive frequencies or an initial negative "frequency" (i.e., weights), as appropriate.

5. Storage register assignments in the HP-41 program ar slightly different from the HP-11C since the summation register configuration is not the same. The MUGO41 program designates R00 through R05 as the statistical registers (step 03), but because "n" is positioned differently from the HP-11C, the following are the pertinent register assignments in MUGO41:

Register		Use
00		ΣX_i, $\Sigma f_i X_i$, or $\Sigma w_i X_i$
01		ΣX_i^2, $\Sigma f_i X_i^2$, or $\Sigma w_i X_i^2$
02		$\Sigma \log X_i$, $\Sigma f_i \log X_i$, or $\Sigma w_i \log X_i$
03		forced to be $(\Sigma \log X_i)^2$ to prevent an "ERROR" stop, or f_i or w_i
04		not used, or X_i
05		number of observations, n, or sum of frequencies (Σf_i), or sum of weights (Σw_i)
06	same as in MUGO-11	$\Sigma 1/x$, $\Sigma f_i/X_i$, or $\Sigma w_i/X_i$
07	same as in MUGO-11	arithmetic mean ($\overline{X}$ or $\overline{X}_{wtd.}$)
08	same as in MUGO-11	standard deviation (s or $s_{wtd.}$)
09		number of classes, c, in the case of grouped data

CAUTION: After MUGO41 has been executed, the user should note that the statistical register assignment remains R00 through R05; if the "normal" assignment of R11-R16 is desired, ΣREG 11 must be executed.

6. Careful attention to the display will show that parentheses have been used rather than the < and > symbols available on the standard HP-41 keyboard. The user who loads the program from the keyboard therefore can substitute these "brackets" for parentheses. Alternatively, the program can be loaded using the barcode provided, wherein parentheses are made part of the program.

7. Printed output can be obtained if an appropriate printer is attached to the HP-41 when MUGO41 is used. The samples shown at the end of the program listing are from the HP 82143A printer and make use of the same examples cited in the HP-11C part of this documentation.

Program Listing for MUGO41, an HP-41 Analogy of the Program MUGO-11

```
01♦LBL "MUGO41"
02 CF 29
03 ΣREG 00
04♦LBL 00
05 CLΣ
06 0
07 STO 06
08 CF 01
09♦LBL 02
10 FIX 0
11 RCL 05
12 1
13 +
14 "KEY X("
15 ARCL X
16 "⊦), R/S"
17 PROMPT
18 1/X
19 ST+ 06
20 LASTX
21 LOG
22 LASTX
23 Σ+
24 "X("
25 ARCL 05
26 "⊦)= "
27 FIX 3
28 ARCL L
29 AVIEW
30 GTO 02
31♦LBL 01
32 CF 01
33 CLΣ
34 0
35 STO 06
36 STO 09
37♦LBL 03
38 FIX 0
39 RCL 09
40 1
41 +
42 "(C"
43 ARCL X
44 "⊦) X↑"
45 FS? 01
46 "⊦W"
47 FC? 01
48 "⊦F"
49 "⊦, R/S"
50 PROMPT
51 X<0?
52 SF 01
53 ABS
54 STO 03
55 ST+ 05
56 X<>Y
57 STO 04
58 *
59 ST+ 00
60 RCL 04
61 X↑2
62 RCL 03
63 *
64 ST+ 01
65 RCL 04
66 LOG
67 RCL 03
68 *
69 ST+ 02
70 RCL 03
71 RCL 04
72 /
73 ST+ 06
74 ISG 09
75 .
76 "(C"
77 ARCL 09
78 "⊦):"
79 FIX 3
80 AVIEW
81 " X= "
82 ARCL 04
83 AVIEW
84 FS? 01
85 " W"
86 FC? 01
87 " F"
88 "⊦= "
89 FC? 01
90 FIX 0
91 ARCL 03
92 AVIEW
93 ADV
94 GTO 03
95♦LBL A
96 ADV
97 XEQ 05
98 FIX 3
99 RCL 02
100 X↑2
101 STO 03
102 MEAN
103 STO 07
104 "AM= "
105 ARCL 07
106 AVIEW
107 STOP
108 SDEV
109 FS? 01
110 XEQ 04
111 STO 08
112 " SDEV= "
113 ARCL 08
114 AVIEW
115 STOP
116 RCL 07
117 /
118 1 E2
119 *
120 " CV= "
121 ARCL X
122 "⊦ %"
123 AVIEW
124 STOP
125 RCL 08
126 RCL 05
127 SQRT
128 /
129 " SXBAR= "
130 ARCL X
131 AVIEW
132 STOP
133♦LBL B
134 RCL 01
135 RCL 05
136 /
137 SQRT
138 "QM= "
139 ARCL X
140 AVIEW
141 RTN
142♦LBL C
143 MEAN
144 X<>Y
145 10↑X
146 "GM= "
147 ARCL X
148 AVIEW
149 RTN
150♦LBL D
```

Program Listing for MUG041 (continued) and Sample Printed Output
Obtained in MAN Printer Mode

```
151 RCL 05
152 RCL 06
153 /
154 "HM= "
155 ARCL X
156 AVIEW
157 RTN
158♦LBL E
159 MEAN
160 "AM= "
161 ARCL X
162 ADV
163 AVIEW
164 PSE
165 XEQ B
166 PSE
167 XEQ C
168 PSE
169 XEQ D
170 PSE
171♦LBL 05
172 SF 12
173 "******"
174 AVIEW
175 CF 12
176 RTN
177♦LBL 04
178 RCL 05
179 1
180 -
181 RCL 09
182 1
183 -
184 /
185 SQRT
186 *
187 END
```

```
X(1)= 62.000
X(2)= 58.000
X(3)= 62.000
X(4)= 73.000
X(5)= 84.000
X(6)= 68.000

******
AM= 67.833
 SDEV= 9.517
 CV= 14.029 %
 SXBAR= 3.885
QM= 68.387
GM= 67.306
HM= 66.809

AM= 67.833
QM= 68.387
GM= 67.306
HM= 66.809
******
```

```
(C1):
 X= 58.000
 F= 4

(C2):
 X= 62.000
 F= 2

(C3):
 X= 68.000
 F= 10

(C4):
 X= 73.000
 F= 6

(C5):
 X= 84.000
 F= 8

******
AM= 71.533
 SDEV= 8.897
 CV= 12.437 %
 SXBAR= 1.624

AM= 71.533
QM= 72.066
GM= 71.000
HM= 70.470
******
```

```
(C1):
 X= 58.000
 W= 0.400

(C2):
 X= 62.000
 W= 0.200

(C3):
 X= 68.000
 W= 1.000

(C4):
 X= 73.000
 W= 0.600

(C5):
 X= 84.000
 W= 0.800

******
AM= 71.533
 SDEV= 7.575
 CV= 10.590 %
 SXBAR= 4.374

AM= 71.533
QM= 72.066
GM= 71.000
HM= 70.470
******
```

HP-41 Program Barcode: MUG041

Program Registers Required: 52

ROW 1: LINES 1 - 2

ROW 2: LINES 2 - 10

ROW 3: LINES 10 - 14

ROW 4: LINES 15 - 17

ROW 5: LINES 18 - 25

ROW 6: LINES 25 - 29

ROW 7: LINES 30 - 38

ROW 8: LINES 38 - 44

ROW 9: LINES 44 - 47

ROW 10: LINES 48 - 50

ROW 11: LINES 51 - 59

ROW 12: LINES 59 - 68

ROW 13: LINES 69 - 76

ROW 14: LINES 76 - 80

ROW 15: LINES 81 - 85

ROW 16: LINES 85 - 88

ROW 17: LINES 89 - 95

ROW 18: LINES 95 - 102

HP-41 Program Barcode: MUG041 (continued)

ROW 19: LINES 103 - 108

ROW 20: LINES 109 - 112

ROW 21: LINES 112 - 118

ROW 22: LINES 118 - 122

ROW 23: LINES 122 - 129

ROW 24: LINES 129 - 132

ROW 25: LINES 133 - 138

ROW 26: LINES 139 - 146

ROW 27: LINES 146 - 152

ROW 28: LINES 153 - 158

ROW 29: LINES 158 - 163

ROW 30: LINES 164 - 169

ROW 31: LINES 169 - 173

ROW 32: LINES 173 - 182

ROW 33: LINES 183 - 187

Some Programs Dealing with Amortization Schedules

Thomas W. Beers*

The programs presented and described in this package provide the means by which various entries of a common home mortgage amortization schedule can be obtained. The usual scenario assumed is that the monthly payments are to be made at the end of each month over the life of the loan and interest is charged based on the outstanding balance just after the previous payment. Part of the payment, then, pays this interest and the remainder (reduction in principal) reduces the outstanding balance for the next interest calculation. Thus, the proportion of each month's payment attributed to interest starts out high and S-L-O-W-L-Y drops. Conversely the part of each payment applied to reduce the principal starts out L-O-W and increases gradually.

Also included in some of the programs is the capability of calculating entries in a schedule which is developed by a financial scheme known as "the rule of 78s". In this approach the borrower makes monthly payments as above, but the interest component of each payment starts out very high (compared to

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the normal amortization schedule) and drops off precipitously in the late stages of the loan life. The net effect of this way of opportioning the finance charge is to discourage the borrower from making an early payoff of the loan, since the principal is reduced so slowly during the first part of the loan. On the positive side, apportioning the interest this way may be beneficial to the borrower when claiming income tax deductions.

The programs found herein are

1. AMORT78-11. AMORTization schedules and similar financial calculations using the rule of 78s with the HP-11C. (Program number 11F009).

2. FINC78-11. FINancial Calculations assuming the rule of 78s with the HP-11C. (Program number 11F010).

3. AMORT-12. AMORTization schedules with the HP-12C. (Program number 12F001).

4. AMORT. Normal AMORTization schedules with the HP-41. (Program number 41F043).

5. AM7841. AMortization schedules and similar financial calculations using the rule of 78s with the HP-41. (Program number 41F044).

Several tables found on the following pages will be referred to repeatedly in examples used in the programs. Table 1 is a portion of a normal amortization schedule generated using an HP-71 computer equipped with a finance module and connected to an HP Thinkjet Printer. The loan is a 25-year home mortgage with initial principal (PV) of $35,000, an annual percentage rate (APR) of 11% (monthly rate = 11/12 = .916666667%), leading to a constant monthly payment of $343.04. This payment value can be found easily by a number of options:

Table 1. Portion of a Normal Amortization Schedule for a Home Mortage Loan. Loan amount is $35,000, term is 25 years with 11 percent annual interest applied monthly, leading to equal monthly payments of $343.04.

Loan balance is 35000
Payment amount is -343.04
made at the End of each period
Periodic interest rate % is 0.91666666667

Period #	Amt. of Int.	Change in Balance	Ending Balance
0	-0.00	0.00	35000.00
1	-320.83	-22.21	34977.79
2	-320.63	-22.41	34955.38
3	-320.42	-22.62	34932.76
4	-320.22	-22.82	34909.94
5	-320.01	-23.03	34886.91
6	-319.80	-23.24	34863.67
7	-319.58	-23.46	34840.21
8	-319.37	-23.67	34816.54
9	-319.15	-23.89	34792.65
10	-318.93	-24.11	34768.54
11	-318.71	-24.33	34744.21
12	-318.49	-24.55	34719.66
13	-318.26	-24.78	34694.88
14	-318.04	-25.00	34669.88
15	-317.81	-25.23	34644.65
16	-317.58	-25.46	34619.19
17	-317.34	-25.70	34593.49
18	-317.11	-25.93	34567.56
19	-316.87	-26.17	34541.39
20	-316.63	-26.41	34514.98
21	-316.39	-26.65	34488.33
22	-316.14	-26.90	34461.43
23	-315.90	-27.14	34434.29
24	-315.65	-27.39	34406.90
25	-315.40	-27.64	34379.26
26	-315.14	-27.90	34351.36
27	-314.89	-28.15	34323.21
28	-314.63	-28.41	34294.80
29	-314.37	-28.67	34266.13
30	-314.11	-28.93	34237.20
31	-313.84	-29.20	34208.00
32	-313.57	-29.47	34178.53
33	-313.30	-29.74	34148.79
34	-313.03	-30.01	34118.78
35	-312.76	-30.28	34088.50
36	-312.48	-30.56	34057.94
298	-9.26	-333.78	676.07
299	-6.20	-336.84	339.23
300	-3.11	-339.93	-.70

(1) from the keyboard of an HP-12C;

(2) using either of the programs FICALC-11/15 (HP-11C or HP-15C) or FICALC (HP-41);

(3) using the TVM program in an HP-41 equipped with the Advantage module;

(4) using the FI program and an HP-41 equipped with a PPC ROM module.

Table 2 exemplifies a 12-month loan where the interest allocation procedure is by the rule of 78s and month-by-month values are shown. Table 3 also demonstrates the application of the rule of 78s, but a 3-year loan is assumed and totals are obtained for each year. It is interesting to note that the annual percentage rate of interest corresponding to the cited monthly payments does not appear in either Tables 2 or 3, but can be shown to be 35.07% in Table 2 and 22.60% in Table 3. Fortunately for the unsuspecting borrower the use of the rule of 78s is seldom seen in home loans or other long-term loans and is gradually being phased out by most lending institutions. Tables 2 and 3 were extracted from an article titled "The Rule of 78s" by E.T. Clare which appeared in <u>Real Estate Today</u>, July/August, 1984.

Table 2. Monthly summary of a Twelve-Month Loan with Total Finance Charge Allocated Using the Rule of 78s. Loan amount is $5000, total finance charge is $1000, thus the $6000 is paid back in 12 equal monthly payments of $500.

Exhibit I

Interest Allocation Per Payment Period

Loan amount: $5,000
Loan term: 12 months
Loan monthly payments: $500
Finance charge (interest): $1,000

$$\text{SOD} = \frac{n(n+1)}{2} = \frac{12(12+1)}{2}$$

SOD = 6(13)
SOD = 78 (fraction denominator)

Payment Period	Applicable Fraction		Total Interest		Interest Allocation	Principal Reduction	Remaining Balance
1	12/78	×	$1,000	=	$153.85	$346.15	$4,653.85
2	11/78	×	1,000	=	141.03	358.97	4,294.87
3	10/78	×	1,000	=	128.21	371.79	3,923.08
4	9/78	×	1,000	=	115.38	384.62	3,538.46
5	8/78	×	1,000	=	102.56	397 44	3,141.03
6	7/78	×	1,000	=	89.74	410.26	2,730.77
7	6/78	×	1,000	=	76.92	423.08	2,307.77
8	5/78	×	1,000	=	64.10	435.90	1,871.80
9	4/78	×	1,000	=	51.28	448.72	1,423.08
10	3/78	×	1,000	=	38.46	461.54	961.54
11	2/78	×	1,000	=	25.64	474.36	487.17
12	1/78	×	1,000	=	12.83	487.17	0
					$1,000.00	$5,000.00	

Table 3. Monthly Detail and Annual Summary of A Three-Year Loan with Interest Allocation Using the Rule of 78s. Loan amount is $20,000, total finance charge is $7720, thus the $27,720 is paid back in 36 equal monthly payments of $770.

Exhibit II

Monthly Payment Change over Life of Loan

Loan amount: $20,000
Loan term: 36 months
Loan monthly payment: $770
Loan finance charge: $7,720

$$SOD = \frac{n(n+1)}{2}$$

$$SOD = \frac{36(36+1)}{2} = 18(37)$$

$$SOD = 666$$

Payment Period	Applicable Fraction		Total Interest		Interest Allocation	Principal Reduction	Loan Balance
1	36/666	×	$7,720	=	$417.30	$352.70	$19,647.30
2	35/666	×	7,720	=	405.71	364.29	19,283.00
3	34/666	×	7,720	=	394.11	375.89	18,907.12
4	33/666	×	7,720	=	382.52	387.48	18,519.64
5	32/666	×	7,720	=	370.93	399.07	18,120.57
6	31/666	×	7,720	=	359.34	410.66	17,709.91
7	30/666	×	7,720	=	347.75	422.25	17,287.66
8	29/666	×	7,720	=	336.16	433.84	16,853.81
9	28/666	×	7,720	=	324.56	445.44	16,408.38
10	27/666	×	7,720	=	312.97	457.03	15,951.35
11	26/666	×	7,720	=	301.38	468.62	15,482.73
12	25/666	×	7,720	=	289.79	480.21	15,002.52
					$4,242.52	**$4,997.48**	
13	24/666	×	$7,720	=	$278.20	$491.80	$14,510.72
14	23/666	×	7,720	=	266.61	503.39	14,007.32
15	22/666	×	7,720	=	255.02	514.98	13,492.34
16	21/666	×	7,720	=	243.42	526.58	12,965.76
17	20/666	×	7,720	=	231.83	538.17	12,427.60
18	19/666	×	7,720	=	220.24	549.76	11,877.84
19	18/666	×	7,720	=	208.65	561.35	11,316.48
20	17/666	×	7,720	=	197.06	572.94	10,743.54
21	16/666	×	7,720	=	185.47	584.53	10,159.01
22	15/666	×	7,720	=	173.87	596.13	9,562.88
23	14/666	×	7,720	=	162.28	607.72	8,955.16
24	13/666	×	7,720	=	150.69	619.31	8,335.85
					$2,573.34	**$6,666.66**	
25	12/666	×	$7,720	=	$139.10	$630.90	$ 7,704.95
26	11/666	×	7,720	=	127.51	642.49	7,062.46
27	10/666	×	7,720	=	115.92	654.08	6,408.38
28	9/666	×	7,720	=	104.32	665.68	5,742.70
29	8/666	×	7,720	=	92.73	677.27	5,065.43
30	7/666	×	7,720	=	81.14	688.86	4,376.57
31	6/666	×	7,720	=	69.55	700.45	3,676.12
32	5/666	×	7,720	=	57.96	712.04	2,964.08
33	4/666	×	7,720	=	46.37	723.63	2,240.45
34	3/666	×	7,720	=	34.77	735.23	1,505.22
35	2/666	×	7,720	=	23.18	746.82	758.41
36	1/666	×	7,720	=	11.59	758.41	0
					$904.14	**$8,335.86**	
Totals					**$7,720.00**	**$20,000.00**	

Program No. 11F009

Calculator: HP-11C

Program Name: AMORT78-11 (**AMORT**ization schedules and similar financial calculations using the "rule of 78s" with the HP-11C)

Author: Thomas W. Beers

Date: March 1986

Purpose: To provide the usual entries in an amortization schedule (payment number, interest paid, reduction in principal, and outstanding balance) assuming the input values are

k = the number of periods over which the amount of interest and the reduction in principal are to be summed (can be 1 for the complete schedule, month by month or year by year)

I = periodic interest rate as a percent

PV = initial value of the loan, the amount borrowed

PMT = periodic payment (input as a negative)

The program provides the option of calculating similar entries for a schedule which assumes the rule of 78s interest allocation procedure, in which case the required input values are PV, PMT, and k as defined above, and

n = the total number of periods (usually months) in the loan

and

INT = the total finance charge for the loan, i.e., the sum of all the periodic interest payments (input as a negative)

A. Storage assignments

Register	Use
0	k = no. of periods per group
1	ΣINT = the sum of the interest paid through the current payment (within the group)
2	ΣPRIN = the sum of the reduction in principal paid through the current payment (within the group)

A. Storage assignments (continued)

Register	Use
3	PV = initially the amount borrowed, then the current remaining balance
4	I = periodic interest rate (a percent), or INT = the total finance charge for the loan (for the 78s rule option)
5	PMT = the periodic payment
6	the current number of periods amortized or processed; 1, 2, ..., n
7	not used, or a "count-down" index of the stage of processing; n, n-1, n-2, ..., 0 (for the 78s rule option)
8	not used, or the sum of digits (SOD) from 1 through n (for the 78s rule option)
I	an index for each group, starting each time with k; then k-1, k-2,...,0

B. Labels

Name	
[E]*	program start; all storage registers are cleared, FIX 2 is set, and the X register is cleared as a signal
[A]	the access to the part of the program under the calculation procedure of the rule of 78s; sets flag 0, then calculates the sum of digits from 1 to n; input assumption is n ENTER k
[B]	stores the input value, I, or INT
[C]	stores the input value, PV
[D]	stores the input value, PMT, and after input of k, clears flag 0 and proceeds to label 3
1	the calculation loop for an amortization schedule; current allocation for interest is calculated then control passes to label 6
2	establishes the within group index (in register I) then clears R1 and R2
3	stores the new value for k if just input
4	calculation loop for the rule of 78s; calculates ΣINT, ΣPRIN, and PV, making use of label 6 as a subroutine

B. Labels (continued)

Name	Use
5	displays the number of periods amortized or processed, ΣINT, ΣPRIN, and PV; control is transferred (upon R/S depression) to label 2 if no number is keyed (i.e., k remains the same) or to label 3 if a new k (group size) is keyed
6	completes the calculation loop for an amortization schedule, calculating the current allocation for reduction in principal, ΣINT and ΣPRIN, and incrementing the number of periods currently amortized or processed

*brackets indicate meaningful "RUN mode" accessability.

C. Flags

Number	Use
0	cleared in label D if R/S is pressed after a value for k is keyed, indicating the selected option is the common amortization schedule; set in label A if A is pressed after a value for n and k have been keyed at the "label D" stop, indicating the selected option is a table summarizing the rule of 78s results

D. Program procedure and example

I. In PRGM mode, load the program (first refer to comment 5 found after the examples).

II. In RUN mode. proceed as given below by way of example.

1. Example 11-1. Duplicate the amortization schedule (for the first three months) shown in Table 1. That is, for a home mortgage loan of $35,000 (PV) where annual percentage rate = 11% (periodic rate, I, therefore is 11/12 %), the monthly payment, PMT, is $343.04, and the number of periods to be grouped, k, is 1. USER mode is assumed.

Step	Input	Key	Output	Comment
a. initialize	-	E	0.00	"clear" signal
b. input	11 ENTER 12 ÷	B	0.92	I=.91666...7
	35000	C	35,000.00	PV=35,000
	343.04	CHS,D	-343.04	PMT=343.04

D. Program procedure and example (continued)

Step	Input	Key	Output	Comment
c. solution	1	R/S	1. (pause)	k=1, month 1
			-320.83(pause)	interest
			-22.21(pause)	principal reduction
			34,977.79	after-payment balance
	-	R/S	2.(pause)	month 2
			-320.63(")	interest
			-22.41(")	prin. red.
			34,955.38	balance
	-	R/S	3.(pause)	month 3
			-320.42(")	interest
			-22.62(")	prin. red.
			34,932.76	balance

etc., for all months, if more are desired.

Note: if the pause is too fast, current interest, principal reduction, and balance can be recalled from R1, R2, and R3 respectively.[1/]

d. for a new value of k (group size) key this value then press R/S; for an entirely new problem, press E (step a.).

2. Example 11-2. Given the same basic loan data as in the previous example except that the first payment is due on Dec. 15, 1985 and all subsequent payments are due the 15th of the month. For tax reports we need the sums of interest paid and principal reduction paid by calendar years for 1985 (k=1 payment), 1986 (k=12), 1987 (k=12), and 1988 (k=11). We also want to know the loan balance remaining after the 36 payments have been made, i.e., the "balloon" or pay-off amount.

Step	Input	Key	Output	Comment
a. initialize	-	E	0.00	clear signal
b. input	11 ENTER 12 ÷	B	0.92	I=.91666...7
	35000	C	35,000.00	PV=35,000
	343.04	CHS,D	-343.04	PMT=-343.04
c. solution	1	R/S	1.(pause)	month 1
			-320.83(")	interest
			-22.21(")	prin. red.
			34,977.79	balance

[1/] If this is done, however, the group size (1 in this case) must be re-keyed prior to pressing R/S to continue.

D. Program procedure and example (continued)

Step	Input	Key	Output	Comment
	12	R/S	13.(pause)	months 2-13
			-3,833.57(")	12-month interest
			-282.91(")	12-month prin. red.
			34,694.88	balance
	-	R/S	25.(pause)	months 14-25
			-3,800.86(")	12-month interest
			-315.62(")	12-month prin. red.
			34,379.26	balance
	11	R/S	36.(pause)	months 26-36
			-3,452.12(")	11-month interest
			-321.32(")	11-month prin. red.
			34,057.94	balance

The sums can be verified by adding up the pertinent values from Table 1. Note that the balance after the 36th payment (the balloon) also checks.

CAUTION: because of the iterative nature of the program, some 18 seconds are required to determine the 12-month sums, so don't panic; think how long it would take you to make the calculations "by hand"! More speed is possible if the corresponding HP-12C or HP-41 programs are used.

3. Example 11-3. Duplicate the interest and principal reductions for the first three months of Table 2, which assumes use of the rule of 78s. Also find the pay-off value (remaining balance) immediately after the 9th payment. Note that

PV = \$5000, amount borrowed
n = 12, term of the loan in months
PMT = \$500, monthly payment
INT = \$1000, total finance charge = n(PMT) - PV

and k = 1, number of periods to be grouped

Step	Input	Key	Output	Comment
a. initialize	-	E	0.00	clear signal
b. input	1000	CHS,B	-1,000.00	INT=-1000
	5000	C	5,000.00	PV= 5000
	500	CHS,D	-500.00	PMT=-500.00

D. Program procedure and example (continued)

Step	Input	Key	Output	Comment
c. solution	12 ENTER 1	A	1. (pause) -153.85(") -346.15(") 4,653.85	month 1 (n=12) interest prin. red. balance
	-	R/S	2. (pause) -141.03(") -358.97(") 4,294.88	month 2 interest prin.red. balance
	-	R/S	3. (pause) -128.21(") -371.79(") 3,923.09	month 3 interest prin. red. balance
	6	R/S	9. (pause) -499.98(") -2,500.02(") 1,423.07	months 4-9 interest sum prin. red. sum balance

4. Example 11-4. Duplicate the 12-month totals for interest and principal paid and remaining balances for each of the three terminal years shown in Table 3. Note that the rule of 78s prevails and

PV = \$20,000, amount borrowed
n = 36, term of the loan in months
PMT = \$770, monthly payment
INT = \$7720, total finance charge
and k = 12, number of periods to be grouped

Step	Input	Key	Output	Comment
a. initialize	-	E	0.00	clear signal
b. input	7720 20000 770	CHS,B C CHS,D	-7,720.00 20,000.00 -770.00	INT=-7720 PV= 20,000 PMT=-770
c. solution	36 ENTER 12	A	12.(pause) -4,242.52(") -4,997.48(") 15,002.52	months 1-12 interest sum prin. red. sum balance
	-	R/S	24.(pause) -2,573.34(") -6,666.66(") 8,335.86	months 13-24 interest sum prin. red. sum balance

D. Program procedure and example (continued)

Step	Input	Key	Output	Comment
	-	R/S	36.(pause) -904.14(") -8,335.86(") 0.00	months 25-36 interest sum prin. red. sum balance

III. Comments

1. If the output answers appear too fast for recording, they can be recalled from the assigned registers:[2/]

   R1 = current grouped interest
   R2 = current grouped principal reduction
   R3 = current balance

   Alternatively, the R/S key can be pressed during any display, thus "freezing" the answer; pressing R/S again will re-start the program at the point it was interrupted.

2. As should be evident from the examples, input and output data make use of the conventional signs from the viewpoint of the borrower, that is, money received (the loan amount and the current balance) is positive, whereas money paid (amount of interest, payments, and principal reduction) is negative.

3. CAUTION: The program does not check to assure that the proper input signs are observed, therefore care should be taken to conform to the sign convention; otherwise, erroneous results ensue.

4. As evidenced by the examples dealing with amortization schedules, only the straight-forward cases can be easily treated with the programmable calculator. In cases, for example, where the compounding frequency (CF) and payment frequency (PF) do not match, the development of an amortization schedule can become abstruse and in general a suitable computer program should be sought.

   It is always advisable to ask the lending agency to specify in detail how the interest charges are obtained and how and when the reduction in principal takes place.

In situations where interest charges are compounded (like in a savings framework) assuming one frequency (CF) while payments are made at another frequency (PF), an effective periodic (period corresponding to the payment period) interest rate in percent (I_e) can be calculated from

$$I_e = 100[(1+\frac{i}{CF})^{CF/PF}-1]$$

[2/] If this is done, however, the group size must be re-keyed prior to pressing R/S to continue.

D. Program procedure and example (continued)

where i = APR/100 = annual percentage rate expressed as a decimal.
and CF, PF are 365 for daily, 12 for monthly, 2 for semi-annually, etc.

The use of this relation is more applicable to determine periodic payments or to determine future values of regular servings rather than to develop amortization schedules.

Thus, as an HP-11C program it is more appropriate to consider it a corollary to the progrram FICALC-11/15, or to keep permanently stored in an HP-12C. For the HP-11C the sequence of steps to solve the equation for I_e [assuming input of APR, ENTER, PF, ENTER, CF] would be

LBL 0	1
STO 4	+
X⇆Y	X⇆Y
÷	Y^X
X⇆Y	1
EEX	-
2	EEX
÷	2
RCL 4	X
÷	R/S or RTN

5. When keying the AMORT78-11 program initially, the user interested only in developing normal amortization schedules, i.e., not by the rule of 78s, can shorten the program considerably by deleting those statements marked with a preceding asterisk, numbers 15, 24, 25, 31, 41, 42, 45, and 64 through 89 (see next section). Remember, deleting statements from the end of the program toward the beginning preserves the original statement numbers and minimizes deletion blunders.

E. Program statements and code.

Step	Statement	Code
01	[f LBL E][3]	42 21 15
02	f FIX 2	42 7 2
03	f CLEAR REG	42 34
04	g CLX	43 35
05	R/S	31
06	[f LBL B]	42 21 12
07	STO 4	44 4
08	R/S	31
09	[f LBL C]	42 21 13
10	STO 3	44 3
11	R/S	31
12	[f LBL D]	42 21 14
13	STO 5	44 5
14	R/S	31
*15	g CF 0	43 5 0
16	(f LBL 3)	42 21 3
17	STO 0	44 0
18	(f LBL 2)	42 21 2
19	RCL 0	45 0
20	STO I	44 25
21	0	0
22	STO 1	44 1
23	STO 2	44 2
*24	g F? 0	43 6 0
*25	GTO 4	22 4
26	(f LBL 1)	42 21 1
27	RCL 3	45 3
28	CHS	16
29	RCL 4	45 4
30	g %	43 14
*31	(f LBL 6)	42 21 6
32	g RND	43 34
33	STO + 1	44 40 1
34	CHS	16
35	RCL 5	45 5
36	+	40
37	STO + 3	44 40 3
38	STO + 2	44 40 2
39	1	1
40	STO + 6	44 40 6
*41	g F? 0	43 6 0
*42	g RTN	43 32
43[4]	f DSE (f DSE I)	42 5
44	GTO 1	22 1
*45	(f LBL 5)	42 21 5
46	RCL 6	45 6
47	f FIX 0	42 7 0
48	f PSE	42 31
49	f FIX 2	42 7 2
50	RCL 1	45 1
51	f PSE	42 31
52	f PSE	42 31
53	RCL 2	45 2
54	f PSE	42 31
55	f PSE	42 31
56	RCL 3	45 3
57	ENTER	36
58	R/S	31
59	-	30
60	g X=0	43 40
61	GTO 2	22 2
62	g LSTX	43 36
63	GTO 3	22 3
*64	[f LBL A]	42 21 11
*65	g SF 0	43 4 0
*66	STO 0	44 0
*67	STO I	44 25
*68	X≶Y	34
*69	STO 7	44 7
*70	1	1
*71	+	40
*72	RCL 7	45 7
*73	X	20
*74	2	2
*75	÷	10
*76	STO 8	44 8
*77	(f LBL 4)	42 21 4
*78	RCL 7	45 7
*79	g X=0	43 40
*80	GTO 5	22 5
*81	RCL 8	45 8
*82	÷	10
*83	RCL 4	45 4
*84	X	20
*85	GSB 6	32 6
*86	STO - 7	44 30 7
*87[4]	f DSE (f DSE I)	42 5
*88	GTO 4	22 4
*89	GTO 5	22 5

NOTE: g MEM leads to P-02 r-.5

[3]Brackets imply meaningful "RUN mode" accessibility.

*Indicates those statements which are needed only by the rule of 78s application of the program; they can be deleted for normal amorization schedule development.

[4]The 15C user should key f DSE I for statements 43 and 87.

F. Formulas

1. Regarding <u>conventional mortgage calculations</u>:

The algorithm used to determine the part of each periodic payment (PMT) attributed to interest (INT_j) and reduction in principal ($PRIN_j$) starts with the initial loan amount (PV_o) and proceeds as follows:

$INT_1 = i(PV_o)$, i = periodic interest rate expressed as a <u>decimal</u>

$PRIN_1 = PMT - INT_1$

$PV_1 = PV_o - PRIN_1$ = balance immediately after the first period payment.

Then:

$INT_2 = i(PV_1)$

$PRIN_2 = PMT - INT_2$

$PV_2 = PV_1 - PRIN_2$

etc., thus

for the jth payment: $INT_j = i\ (PV_{j-1})$

$PRIN_j = PMT - INT_j$,

and just after the jth payment $PV_j = PV_{j-1} - PRIN_j$.

2. Regarding loans processed using the <u>rule of 78s</u>:

The algorithm used to determine the part of each periodic (usually monthly) payment (PMT) attributed to interest (INT_j) differs from the conventional loan in that the total finance charge (INT) is first determined by

$INT = n(PMT) - PV_o$

where n = total number of periods (months)

and PV_o = initial loan amount.

F. Formulas (continued)

Then:

$$INT_1 = \frac{n}{SOD}(INT),$$

where

$$SOD = \text{sum of digits} = 1+2+3+...+n$$

$$= \frac{n(n+1)}{2} ;$$

$$INT_2 = \frac{n-1}{SOD}(INT) ,$$

etc., up to INT_n .

The amount of each payment attributed to reduction in principal ($PRIN_j$), and the remaining balance (PV_j) are calculated the same as described in F.1 for conventional loans:

$$PRIN_1 = PMT - INT_1$$

and $PV_1 = PV_o - PRIN_1$

etc., thus

for the jth payment: $INT_j = \frac{n-j+1}{SOD}(INT)$

$$PRIN_j = PMT - INT_j$$

and, just after the jth payment $PV_j = PV_{j-1} - PRIN_j$.

3. <u>CAUTION</u>. For simplicity and readability in both cases covered by 1. and 2., the negative signs appropriate for input and output were deleted in the variables INT_j, $PRIN_j$, PMT, and INT. Alternatively, one could place "absolute value" signs about each of these variables in the equations cited.

Program No. 11F010

Calculator: HP-11C

Program Name: FINC78-11 (**FIN**ancial **C**alculations assuming the rule of **78s** with the HP-**11**C)

Author: Thomas W. Beers

Date: April 1986

Purpose: This program serves as a companion to the rule-of-78's part of the previous program (AMORT78-11), providing direct solution for

INT_j = the interest part of the jth payment

$PRIN_j$ = the reduction in principal part of the jth payment

and PV_j = the remaining balance after the jth payment.

The results are obtained much faster than in AMORT78-11 since sequential calculations are not necessary. Additionally, values can be obtained by direct calculation for

ΣINT_j = sum of the interest paid through the jth payment

$\Sigma PRIN_j$ = sum of reduction in principal through the jth payment

and a new value,

REB_j = the "rebate" (a misleading term) or the amount of unpaid interest (better phraseology) immediately after the jth payment has been made.

A. Storage assignments

Register	Use
0	j = the number of the current payment
1	ΣINT_j = sum of the interest paid through the jth payment, stored as a negative value
2	$\Sigma PRIN_j$ = sum of the principal reduction through the jth payment, stored as a negative value

A. Storage assignments (continued)

Register	Use
3	PV = the initial amount of the loan
4	INT = total finance charge for the loan, stored as a negative
5	PMT = periodic payment, stored as a negative
6,7,8	not used
9	n = total number of payments in the loan
.0	n-j
.1	PV_j = remaining balance after the jth payment
.2	$PRIN_j$ = the reduction in principal part of the jth payment, stored as a negative.
I	INT_j = interest part of the jth payment, stored as a negative.

B. Labels

Name	
[E]*	program start; all storage registers are cleared, FIX 2 is set, and the X register is cleared as a signal
[B]	stores the input value, INT (= total finance charge) as a negative number
[C]	stores the input value, PV (= loan amount) as a positive number
[D]	stores the input value, PMT (= periodic payment) as a negative number
[7]	achieves the calculation, storage and display of the interest and principal reduction part of the jth payment (INT_j and $PRIN_j$) and proceeds to label 9 for the display of the balance after the jth payment (PV_j); the input assumption is n ENTER j

B. Labels (continued)

Name	Use
[8]	recalls and displays the sums of the interest paid and prinipal reductions through the jth payment (ΣINT_j and $\Sigma PRIN_j$) and the remaining balance (PV_j); optionally, by R/S depression, the amount of the unpaid interest after the jth payment (REB_j) is calculated and displayed. As for label 7, the input assumed is n ENTER j
9	calculates and stores ΣINT_j; $\Sigma PRIN_j$, and PV_j, then returns control to label 8 (flag 1 set) or calculates and displays PV_j if not called as a subroutine (flag 1 clear)

* brackets indicate meaningful "RUN mode" accessability.

C. Flags

Number	Use
1	cleared in label 7 so that execution will stop in label 9 to display PV_j set in label 8 so label 9 will be executed as a subroutine

D. Program procedure and example.

I. In PRGM mode, load the program.

II. IN RUN mode, proceed as given below by way of example.

1. Example 11-5. Referring to the rule of 78s schedule and loan summarized in Table 3, verify the interest, principal reduction and balance for the 8th payment (GSB 7 is appropriate), then obtain the sums of interest, principal reduction, and balances through the 12th, 24th and 36th payments (GSB 8 is appropriate).

Note that

PV = \$20,000, amount borrowed
n = 36, term of loan in months
PMT = \$770, monthly payment
INT = \$7720, total finance charge

and j = 8, 12, 24, or 36, payment number pertinent to the problem

D. Program procedure and example (continued).

Step	Input	Key	Output	Comment
a. initialize	-	E	0.00	clear signal
b. input	7720	CHS,B	-7,720.00	INT=-7720
	20000	C	20,000.00	PV= 20,000
	770	CHS,D	-770.00	PMT=-770
c. solution	36 ENTER 8	GSB 7	8.(pause)	for n=36 and j=8
			-336.16(")	INT_8
			-433.84(")	$PRIN_8$
			16,853.81	PV_8

- -

Step	Input	Key	Output	Comment
c. solution	36 ENTER 12	GSB 8	12.(pause)	through payment 12
			-4,242.52(")	ΣINT_{12}
			-4,997.48(")	$\Sigma PRIN_{12}$
			15,002.52	PV_{12}

- -

Step	Input	Key	Output	Comment
c. solution	36 ENTER 24	GSB 8	24.(pause)	through payment 24
			-6,815.86(")	ΣINT_{24}
			-11,664.14(")	$\Sigma PRIN_{24}$
			8,335.86	PV_{24} (off .01 due to rounding)

Note in Table 3: 4242.52 + 2573.34 = 6815.86
and 4997.48 + 6666.66 = 11,664.14

- -

Step	Input	Key	Output	Comment
c. solution	36 ENTER 36	GSB 8	36.(pause)	through payment 36
			-7,720.00	ΣINT_{36}=INT
			-20,000.00	$\Sigma PRIN_{36}$=-PV
			0.00	PV_{36}

2. Example 11-6. Using the same loan as in the previous example, find the sums of interest, principal reduction, and the balance after the 33rd payment, and find the "rebate"-- the amount of interest (displayed as a positive number) left to be paid immediately after the 33rd payment.

Step	Input	Key	Output	Comment
a. initialize	-	E	0.00	clear signal
b. input	7720	CHS, B	-7,720.00	INT=-7720
	20000	C	20,000.00	PV=20,000
	770	CHS, D	-770.00	PMT=-770

D. Program procedure and example (continued).

c. solution	36 ENTER 33	GSB 8	33.(pause)	through payment 33
			-7,650.45	ΣINT_{33}
			-17,759.55	$\Sigma PRIN_{33}$
			2,240.45	PV_{33}
		R/S	69.55	REB_{33}=rebate after 33rd payment

Note in Table 3:

$$REB_{33} = INT_{34} + INT_{35} + INT_{36}$$

$$= 34.77 + 23.18 + 11.59$$

$$= 69.54 \text{ (checks, except for rounding)}$$

$$\text{or} \quad REB_{33} = INT - \sum_{j=1}^{33} INT_j$$

$$= INT - \Sigma INT_{33}$$

$$= 7720.00 - 7650.45$$

$$= 69.55$$

III. Comments

1. If the output answers appear too fast for recording, they can be recalled from the assigned registers:[1/]

When using GSB 7 (input = n ENTER j)

RI = amount of interest in payment j
R.2 = amount of reduction in principal in payment j
R.1 = loan balance just after payment j,

or, when using GSB 8 (input = n ENTER j)

R1 = sum of the interest through payment j
R2 = sum of the principal reduction through payment j
R.1 = loan balance just after payment j

R1-R4 = rebate after the jth payment.

Alternatively, the R/S key can be pressed during any display, thus "freezing" the answer; pressing R/S again will re-start the program at the point it was interrupted.

[1/] If this is done, however, both n and j must be re-keyed prior to pressing R/S to continue.

D. Program procedure and example (continued).

2. The program makes use of conventional signs for the values input and displayed, from the viewpoint of the borrower; money received or credited (the loan amount, current balance, and rebate) is positive, whereas money paid (amount of interest, payments, and principal reduction) is negative.

   This program, however, differs from AMORT78-11 in that here the input INT, PV, and PMT are checked for the proper sign and if wrong, forced to be correct.

3. Because of the similarity of this program (FINC78-11) with the previous one (AMORT78-11) it would seem desirable that both should be combined into one. Unfortunately, the HP-11C lacks sufficient storage to do this. Instead, the user is referred to the HP-41 program AM7841 for this convenience.

E. Program statements and code.

Step	Statement	Code	Step	Statement	Code	Step	Statement	Code
01	[f LBL E][2/]	42 21 15	31	STO.0	44 .0	61	RCL 9	45 9
02	f FIX 2	42 7 2	32	1	1	62	+	40
03	f CLEAR REG	42 34	33	+	40	63	÷	10
04	g CLX	43 5	34	RCL 9	45 9	64	CHS	16
05	R/X	31	35	g X^2	43 11	65	1	1
06	[f LBL B]	42 21 12	36	RCL 9	45 9	66	+	40
07*	g X>0 (g TEST 1)	43 20	37	+	40	67	X	20
08	CHS	16	38	÷	10	68	STO 1	44 1
09	STO 4	44 4	39	2	2	69	CHS	16
10	R/S	31	40	X	20	70	RCL 0	45 0
11	[f LBL C]	42 21 13	41	RCL 4	45 4	71	RCL 5	45 5
12*	g X<0 (g TEST 2)	43 10	42	X	20	72	X	20
13	CHS	16	43	STO I	44 25	73	+	40
14	STO 3	44 3	44	f PSE	42 31	74	STO 2	44 2
15	R/S	31	45	f PSE	42 31	75	RCL 3	45 3
16	[f LBL D]	42 21 14	46	CHS	16	76	+	40
17*	g X>0 (g TEST 1)	43 20	47	RCL 5	45 5	77	STO .1	44 .1
18	CHS	16	48	+	40	78	g F? 1	43 6 1
19	STO 5	44 5	49	STO .2	44 .2	79	g RTN	43 32
20	R/S	31	50	f PSE	42 31	80	R/S	31
21	[f LBL 7]	42 21 7	51	f PSE	42 31	81	[f LBL 8]	42 21 8
22	g CF 1	43 5 1	52	(f LBL 9)	42 21 9	82	g SF 1	43 4 1
23	f FIX 0	42 7 0	53	RCL 4	45 4	83	STO 0	44 0
24	f PSE	42 31	54	RCL .0	45 .0	84	X≶Y	34
25	f FIX 2	42 7 2	55	1	1	85	STO 9	44 9
26	STO 0	44 0	56	+	40	86	X≶Y	34
27	X≶Y	34	57	RCL .0	45 .0	87	-	30
28	STO 9	44 9	58	X	20	88	STO .0	44 .0
29	X≶Y	34	59	RCL 9	45 9	89	GSB 9	32 9
30	-	30	60	g X^2	43 11	90	f FIX 0	42 7 0

[2/] Brackets and parentheses are not programmable; they are used to indicate externally meaningful or internal use labels, respectively.

E. Program statements and code (continued).

Step	Statement	Code
91	RCL 0	45 0
92	f PSE	42 31
93	f PSE	42 31
94	f FIX 2	42 7 2
95	RCL 1	45 1
96	f PSE	42 31
97	f PSE	42 31
98	RCL 2	45 2
99	f PSE	42 31
100	f PSE	42 31
101	RCL .1	45 .1
102	R/S	31
103	RCL 4	45 4
104	RCL 1	45 1
105	-	30
106	CHS	16

Note: g MEM leads to P-06 r-.2

*Indicates statements that must be changed for the HP-15C. Suggested or required change is shown in parentheses. Although the "code" for these statements will also change this is left for the 15C user to alter.

F. Formulas

Because of the nature of the interest allocation scheme using the rule of 78s, the formulas used are directly solved in FINC78-11 rather than the iterative solution used and described in AMORT78-11. The results obtained in label 7 and label 8 are described separately.

Label 7

Assuming a final input of n and j,

where

n = total number of payments in the loan

and j = the number of the current payment,

the part of the jth payment, PMT, allocated to interest, INT_j, and to reduction in principal, $PRIN_j$,

as well as the after-payment remaining balance, PV_j, are obtained by[1/]

$$INT_j = \frac{2(n-j+1)}{n(n+1)}(INT),$$

$$PRIN_j = PMT - INT_j\ ,$$

and

$$PV_j = PV - \Sigma PRIN_j\ ,$$

where

$$\Sigma PRIN_j = j(PMT) - \Sigma INT_j\ ,$$

$$\Sigma INT_j = INT - REB_j\ ,$$

and

$$REB_j = \frac{(n-j)(n-j+1)}{n(n+1)}(INT)\ .$$

Reminder:

PV = the original amount of the loan
PMT = the constant periodic payment
INT = the total finance charge

[1/] The formulas are derived from formulas found in "HP-12C Solutions Handbook" Hewlett-Packard. March, 1984.

F. Formulas (continued)

Label 8

Assuming the same input as for label 7, n and j, the sum of the interest paid, ΣINT_j, and sum of principal paid through payment j, $\Sigma PRIN_j$, and the remaining balance, PV_j, are calculated as shown above using label 9 as a subroutine. The amount of the unpaid interest, or rebate, REB_j, is calculated upon R/S depression using the formula

$$REB_j = INT - \Sigma INT_j$$

CAUTION. For simplicity and readability in the formulas given the negative signs appropriate for input and output were deleted in the variables INT_j, $PRIN_j$, PMT, and INT. Alternatively one could place "absolute value" signs about each of these variables in the equations cited.

Program No. 12F001

Calculator: HP-12C

Program Name: AMORT-12 (**AMORT**ization schedules with the HP-**12**C)

Author: Thomas W. Beers

Date: April, 1986

Purpose: To provide the usual entries in an amortization schedule (payment number, interest paid, reduction in principal, and outstanding balance) assuming the input values are

k = the number of periods over which the amount of interest and the reduction in principal are to be summed (can be 1 for the complete schedule, month by month or year by year)

I = periodic interest rate as a percent

PV = initial value of the loan, the amount borrowed

and PMT = periodic payment (input as a negative)

A. Storage assignments

Register	Use
0	k = number of periods to be grouped
1	ΣINT = the sum of the interest paid through the current payment (within the group)
2	ΣPRIN = the sum of the reduction in principal paid through the current payment (within the group)
n	represents the total number of payments which have been amortized (by the AMORT function)
PV	initially used to store the original amount of the loan, part of the input; after the program is run, the register contains the loan balance, PV_n, immediately after the nth payment has been made
i	I = the periodic interest rate as a percent
PMT	PMT = the periodic payment (as a negative)

B. Labels -- there are no labels used in HP-12C programming.

C. Flags -- no flags are available in the HP-12C.

D. Program procedure and program

I. Put calculator in PRGM mode (press f P/R), then

a. f CLEAR PRGM to clear other programs

b. key in the following program:

Step	Statement	Code	Step	Statement	Code
01	STO 0	44 0	13	g PSE	43 31
02	RCL 0	45 0	14	RCL 2	45 2
03	f AMORT	42 11	15	g PSE	43 31
04	STO 1	44 1	16	g PSE	43 31
05	X≶Y	34	17	RCL PV	45 13
06	STO 2	44 2	18	ENTER	36
07	f 0	42 0	19	R/S	31
08	RCL n	45 11	20	-	30
09	g PSE	43 31	21	g X=0	43 35
10	f 2	42 2	22	g GTO 02	43 33 02
11	RCL 1	45 1	23	g LSTX	43 36
12	g PSE	43 31	24	g GTO 01	43 33 01

II. Put calculator in RUN mode (press f P/R), then

a. f CLEAR FIN (or f CLEAR REG) to clear the financial (or all) the storage registers

b. load the financial registers with the pertinent loan data:

key periodic interest rate, I, press i
" loan value, PV, press PV
" periodic payment, press CHS then PMT

D. Program procedure and program (continued)

c.[1] ensure that the program pointer is at step 00 by <u>any</u> of the following:

(1) turn calculator off, then on (press ON twice)

(2) go into and out of PRGM mode (press keys f P/R twice)

(3) press g GTO .00

d. key in k, the number of periods for which totals are to be grouped, press R/S and observe the results:

	n.	(the number of periods amortized)
(pause)		
	ΣINT	(the sum of interest paid in the group of payments, displayed as a negative)
(pause)		
(pause)		
	ΣPRIN	(the sum of the principal reduction paid in the group of payments, displayed as a negative)
	PV_n	(the remaining loan balance after the nth payment)

e. for the next group of periods <u>of the same size</u>, simply press R/S and see the results indicated in d.; for a different group size, key the new "k", then press R/S.

If the results are displayed too rapidly, they can be recalled by[2]

RCL	n	to see n
RCL	1	to see INT
RCL	2	to see PRIN
RCL	PV	to see PV_n

Alternatively, the results, when displayed, can be "frozen" by pressing R/S; pressing R/S again will continue the program where it was interrupted.

---

[1] This step is not always necessary but it is a good idea to routinely do it.
[2] If this is done, however, the group size must be re-keyed prior to pressing R/S to continue.

D. Program procedure and program (continued)

III. Examples (assuming the program has been properly keyed in, and the calculator is in RUN mode).

1. Example 12-1. (same as Example 11-1).
Duplicate the amortization schedule (for the first three months) shown in Table 1. The pertinent loan data being \$35,000 to be amortized over 25 years at 11 percent annual percentage rate, leading to monthly payments of \$343.04. Therefore, as input data:

I = 11/12%, the periodic (i.e., monthly) rate
PV = 35,000
PMT = -343.04
and k = 1 ("group" size = 1 month)

Step	Input	Key	Output	Comment
a. clear	-	f CLEAR FIN	?	display is not cleared
	or	f CLEAR REG	0.00	display is cleared
b. input	11	g i	0.92	I=.91666...7
	35000	PV	35,000.00	PV=35,000
	343.04	CHS,PMT	-343.04	PMT=-343.04
c. orient pointer	-	ON,ON	-343.04	not always needed
d. solution	1	R/S	1. (pause)	k=1, month 1
			-320.83(")	interest
			-22.21(")	principal reduction
			34,977.79	balance
e. next month	-	R/S	2. (pause)	month 2
			-320.63(")	interest
			-22.41(")	prin. red.
			34,955.38	balance
e. next month	-	R/S	3. (pause)	month 3
			-320.42(")	interest
			-22.62(")	prin.red.
			34,932.76	balance

e. etc., for additional months simply press R/S; if say the next 12 payments are to be grouped one would key 12, then press R/S (see next example).

D. Program procedure and program (continued)

2. Example 12-2. (same as Example 11-2.)
Given the same basic loan data as in the previous example except that only one payment is to be made in the first year of the loan (1985). For tax reports we need the sums of interest paid and principal reduction paid by calendar years for 1985 (k=1 payment), 1986 (k=12), 1987 (k-12), and 1988 (k=11). We also want to know the loan balance remaining after the 36 payments have been made, i.e., the "balloon" or pay-off amount.

Step	Input	Key	Output	Comment
a. clear	-	f CLEAR FIN	?	display not clear
b. input	11	g i	0.92	I=.91666...7
	35000	PV	35,000.00	PV=35,000
	343.04	CHS,PMT	-343.04	PMT=-343.04
c. orient pointer	-	ON,ON	-343.04	optional step
d. solution	1	R/S	1. (pause)	month 1
(Dec. 1985)			-320.83(")	interest
			-22.21(")	principal reduction
			34,977.79	balance
e. next 12 months	12	R/S	13.(pause)	months 2-13
(1986)			-3,833.57(")	12-month interest
			-282.91(")	12-month prin.red.
			34,694.88	balance
e. next 12 months	-	R/S	25.(pause)	months 14-25
			-3,800.86(")	12-month interest
			-315.62(")	12-month prin.red.
			34,379.26	balance
e. next 11 months	11	R/S	36. (pause)	months 26-36
(Jan.-Nov., 1988)			-3,452.12(")	11-month interest
			-321.32(")	11-month prin.red.
			34,057.94	balance

The sums can be verified by adding up the pertinent values from Table 1. Note that the balance after the 36th payment (the balloon) also checks.

IV. Comments

1. As noted in the instructions (II.e.), if the results display too quickly they can be recalled from the indicated registers or R/S can be used to interrupt the output.

D. Program procedure and program (continued)

2. The input and output data make use of conventional signs from the viewpoint of the borrower, that is, money received (the loan amount and the current balance) is positive, whereas money paid (amount of interest, payments, and principal reduction) is negative.

3. CAUTION: The program does not check to assure that the proper input signs are observed, therefore care should be taken to conform to the sign convention; otherwise, erroneous results ensue.

4. Although the concept has limited application to loans, the facility with which the HP-12C handles savings type problems suggests that a short program reside permanently in the calculator which could calculate an effective percent periodic interest rate (I_e) in situations where the compounding frequency (CF) and the payment frequency (PF) do not coincide. (See Comment 4 in AMORT78-11).

   The formula[3/] to be solved is

$$I_e = 100[(1+\frac{i}{CF})^{CF/PF}-1]$$

   where $i = \frac{APR}{100}$ = annual percentage rate expressed as a decimal.

   Example 12-3. Consider an IRA investment program with the following characteristics:

   PV = -\$6500.00, the initial investment

   PMT = -\$2000.00, annual payments made at the end of each year.

   APR = 10.5%, the annual percentage rate

   What is the effective periodic (annual in this case) interest rate if compounding takes place monthly (CF=12) even though payments are made annually (PF=1)?

   What is the total accumulation at the end of 25 years?

3/ The formula shown is adapted from that given in "PPC ROM Users Manual" PPC, Box 9599, Fountain Valley CA. 1981.

D. Program procedure and program (continued)

To solve such a problem using the HP-12C where the AMORT-12 program is already in memory (taking up 24 lines of program) one would proceed as described below.

a. In RUN mode, move the program pointer to line 24 by: g GTO 24

b. Go into PRGM mode by f P/R and observe 24-43, 33 01

c. Key in the following program:

Step	Statement	Code	Step	Statement	Code
25	STO 0	44 0	35	+	40
26	X⪌Y	34	36	X⪌Y	34
27	÷	10	37	Y^X	21
28	X⪌Y	34	38	1	1
29	EEX	26	39	-	30
30	2	2	40	EEX	26
31	÷	10	41	2	2
32	RCL 0	45 0	42	X	20
33	÷	10	43	g GTO 00	43 33 00
34	1	1			

d. Go into RUN mode by f P/R.

e. Ensure that END (of period) mode is active by g END.

f. Proceed as follows, using the given data.

Step	Input	Key	Output	Comment
(1) locate pointer	-	g GTO 25	?	moves the program pointer to the start of the program (step 25)

D. Program procedure and program (continued).

Step	Input	Key	Output	Comment
(2) input	10.5 ENTER 1 ENTER 12	R/S	11.02	for APR=10.5%, PF=1, and CF=12, I_e=11.02=effective periodic (annual) percentage rate

Note, the exact effective rate is 11.020345%, as can be seen by pressing f 9 (then return to 2 places by f 2).

(3) clear	-	f CLEAR FIN	11.02	clear financial registers
(4) input	-	i	11.02	loads the <u>exact</u> I_e into the i register
	25	n	25.00	n=25 years
	6500	CHS,PV	-6500.00	PV=6500
	2000	CHS,PMT	-2000.00	PMT=2000
(5) solution	-	FV	318,247.45	FV=total accumulation just after the 25th payment h been made

Note, this result checks with the result found by another procedure in Appendex B, part 3a. of the FICALC-11/15 program.

E. Formulas

The program makes use of the AMORT function, therefore no formulas need be solved. When executed from the keyboard, AMORT sums the interest and reduction in principal for the number of payments (periods) indicated in the X register (the display), using the amount of the loan in the PV register, and the periodic payment in the PMT register. The result returned to the X register is the sum of interest paid (ΣINT); the sum of the principal reduction (ΣPRIN) is in the Y register and can be displayed by pressing X≶Y. The remaining balance after n payments, PV_n, is found in the PV register (RCL PV to display it), and the total number of payments amortized is in the n register (RCL n to display it).

Program No. 41F043

Calculator: HP-41

Program Name: AMORT (normal **AMORT**ization schedules with the HP-41)

Author: Thomas W. Beers

Date: June 1986

Purpose: To provide the usual entries in an amortization schedule (payment number, interest paid, reduction in principal, and outstanding balance) assuming the input values are

k = the number of periods over which the amount of interest and the reduction in principal are to be summed (can be 1 for the complete schedule, month by month or year by year)

I = periodic interest rate as a percent

PV = initial value of the loan, the amount borrowed

PMT = periodic payment (input as a negative)

A printed schedule can be obtained when a printer (HP-82143A or HP-82162A) is connected, and the user has the option (by setting flag 02) of having a continuous printout if the size of the group (k) is uniform.

A. Storage assignments

Register	Use
00	k = no. of periods per group
01	ΣINT = the sum of the interest paid through the current payment (within the group)
02	ΣPRIN = the sum of the reduction in principal paid through the current payment (within the group)
03	PV = initially the amount borrowed, then the current remaining balance (BAL)
04	I = periodic interest rate (a percent)
05	PMT = the periodic payment

A. Storage assignments (continued)

Register	Use
06	the current number of periods amortized; 1, 2, ..., n
07-08	not used
09	N = the final period to be amortized and printed (only used in the "auto" option, i.e., flag 02 set)
10-12	not used
13	an index for each group, starting each time with k; then k-1, k-2, ..., 0

B. Labels

Global	Use
AMORT	program start, clears all registers and flag 02, provides flag 02 reminder, then prompts for next step (label E)
Local	
B	stores the input value for interest rate (I), displays and prints it, then prompts for the loan amount
C	stores the input value for the loan amount (PV), displays and prints it, then prompts for the payment amount. PV is checked to ensure a positive value.
D	stores the input value for the payment amount (PMT), displays and prints it, then prompts for the number of periods to be grouped (k); PMT is checked to ensure a negative value
E	program re-start if most of the input parameters have not changed; prints heading for the amortization schedule, then prompts for the periodic interest rate
b	provides a convenient divide-by-12 operation to easily convert an annual interest rate to a monthly one; control passes automatically, then, to label B for the PV input prompt
01	the calculation loop for the amortization schedule; current allocation for interest is calculated then control passes to label 06
03	stores the new value for k, if just input, clears registers 01 and 02, and proceeds to label 01

B. Labels (continued)

Local	Use
05	displays and prints the number of payments grouped (can be 1) then proceeds to labels 11, 12, and 13 for the grouped results
06	completes the calculation loop for the amortization schedule, accumulating the amounts for interest and reduction in principal, ΣINT and ΣPRIN, and incrementing the number of periods currently amortized
11	displays and prints the grouped amount of interest (Σ INT)
12	displays and prints the grouped reduction in principal (Σ PRIN)
13	displays and prints the remaining balance (BAL)
14	tests for "end of schedule" condition and/or automatic printout mode, directs control accordingly or prompts for a new group size (k)
95	sets flag 04 and stores the terminal non-uniform group size to be used as "k" for the final group summary; called as a subroutine in label 97
96	prompts for and stores the final N if flag 02 is set, indicating the "auto-display and print mode"; called as a subroutine in label E
97	tests for end-of-schedule condition, and either recycles to label 03 or passes control to label 99
98	used to print dashed-line in schedule heading; called as a subroutine in labels D and E
99	used to print the initial, and if flag 02 is set, the final asterisk-line

C. Flags

Type and Number	Use
User:	
02	set externally by the user to skip the prompt for the input of group size, k, after the first such prompt. This achieves a continuous display or printing of results and is appropriate when k remains constant

C. Flags (continued)

Type and Number	Use
User:	
03	set in label 05 to indicate group size is 1 and to select the proper heading (PAYMENT: if set; PMTS:, if clear)
04	set in label 95 to indicate the final-group condition
12	print double wide flag; set in labels 98 and 99 to achieve double-wide printing
22	numeric data input flag; tested throughout to detect a keyed-in number
29	digit grouping flag; cleared (in presence of FIX 0) to suppress decimal point
System:	
55	printer existence flag; used throughout to test for the presence of a printer in the system and therefore print results and skip the pauses necessary for displayed results

D. Size and key assignments

SIZE: $\geq$ 014

Suggested key assignments:

"AMORT" on shift, Σ+ (i.e., -11)

"B", "C", "D", "E", and "b" all done internally

E. Program procedure and examples

I. Load the program "AMORT" into the calculator.

II. In RUN mode, proceed as given below by way of example. USER mode is assumed, and "AMORT" is assigned to shift, Σ+.

1. Example 41-1 (same as Example 11-1).

Duplicate the amortization schedule (for the first three months) shown in Table 1. That is, for a home mortgage loan of $35,000 (PV) where annual percentage rate = 11% (periodic rate, I, therefore is 11/12%), the monthly payment, PMT, is $343.04, and the number of periods to be grouped, k, is 1.

E. Program procedure and examples (continued)

Therefore the input data are:

I = 11/12%, the periodic (i.e., monthly) rate
PV = 35,000
PMT = -343.04
and k = 1 ("group" size = 1 month)

Step	Prompt	Input	Key(s)	Comment or Output
a. prelim.	-	-	USER	activates USER mode
b. initialize	-	-	shift, Σ +	starts program
c. auto, or not	-	-	-	AUTO: SF 02,
	THEN PRESS E	-	E	AM. SCHEDULE
d. data input	I(%)= 0.00?	11	shift, B	I= 0.92 %
	PV= 0.00?	35000	C(or R/S)	PV= 35000.00
	PMT= 0.00?	343.04	D(or R/S)	PMT= -343.04
e. results	KEY K, R/S	1	R/S	PAYMENT: 1 INT= -320.83 PRIN= -22.21 BAL= 34977.79
	R/S OR K, R/S	-	R/S	PAYMENT: 2 INT= -320.63 PRIN= -22.41 BAL= 34955.38
	R/S OR K, R/S	-	R/S	PAYMENT: 3 INT= -320.42 PRIN= -22.62 BAL= 34932.76
	R/S OR K, R/S			problem finished

Note 1. If flag 02 had been set before pressing E in step c., and a "FINAL N" of 3 had been provided at the subsequent prompt, the pressing of R/S each time in step e. would not be necessary. This feature is perhaps most useful when a printer is attached.

Note 2. If the display of results is too fast, the ΣINT (or INT), ΣPRIN (or PRIN), and BAL can be recalled from R 01, R 02, and R 03, respectively (RCL Σ+, RCL 1/X, or RCL $\sqrt{X}$). If this is done at the "R/S OR K, R/S" prompt, the schedule can be continued by following the prompt directions.

E. Program procedure and examples (continued)

Note 3. The following printed output shows the results obtained with the printer in NORM mode to detail the keys pressed, then in MAN mode to demonstrate the "cleaner" output.

```
              XEQ "AMORT"
AUTO: SF 02,THEN PRESS E
                    SF 02
                    XEQ E
FINAL N?
              3.00    RUN
*******
AM. SCHEDULE
------
I(%)= 0.00?
             11.00  XEQ E
I= 0.92 %
PV= 0.00?
          35000.00    RUN
PV= 35000.00
PMT= 0.00?
            343.04    RUN
PMT= -343.04
------

KEY K, R/S
              1.00    RUN
PAYMENT: 1
INT= -320.83
PRIN= -22.21
BAL= 34977.79

PAYMENT: 2
INT= -320.63
PRIN= -22.41
BAL= 34955.38

PAYMENT: 3
INT= -320.42
PRIN= -22.62
BAL= 34932.76

*******
```

```
*******
AM. SCHEDULE
------
I= 0.92 %
PV= 35000.00
PMT= -343.04
------

PAYMENT: 1
INT= -320.83
PRIN= -22.21
BAL= 34977.79

PAYMENT: 2
INT= -320.63
PRIN= -22.41
BAL= 34955.38

PAYMENT: 3
INT= -320.42
PRIN= -22.62
BAL= 34932.76

*******
```

E. Program procedure and examples (continued)

2. Example 41-2 (same as Example 11-2).

Given the same basic loan data as in the previous example except that the first payment is due on Dec. 15, 1985 and all subsequent payments are due the 15th of the month. For tax reports we need the sums of interest paid and principal reduction paid by calendar years for 1985 (k=1 payment), 1986 (k=12), 1987 (k=12), and 1988 (k=11). We also want to know the loan balance remaining after the 36 payments have been made, i.e., the "balloon" or pay-off amount.

Flag 02 is assumed clear.

Step	Prompt	Input	Key(s)	Comment or Output
a. prelim.	-	-	USER	activates USER mode
b. initialize	-	-	E	since the basic input data are the same as previous example, E is pressed for re-start

c. since E was used to re-start the program, the "AUTO: SF 02" prompt is skipped.

Step	Prompt	Input	Key(s)	Comment or Output
d. data input	I(%)= 0.92?	-	R/S	I= 0.92 %
	PV= 34932.76?	35000	C(orR/S)	PV= 35000.00
	PMT= -343.04?	-	R/S	PMT= -343.04
e. results	KEY K, R/S	1	R/S	PAYMENT: 1 INT= -320.83 PRIN= -22.21 BAL= 34977.79
	R/S OR K, R/S	12	R/S	PMTS. 2--13 INT= -3833.57 PRIN= -282.91 BAL= 34694.88
	R/S OR K, R/S	-	R/S	PMTS. 14--25 INT= -3800.86 PRIN= -315.62 BAL= 34379.26
	R/S OR K, R/S	11	R/S	PMTS. 26--36 INT = -3452.12 PRIN= -321.32 BAL = 34057.94
	R/S OR K, R/S			problem finished

E. Program procedure and examples (continued)

Note 4. The following printed output shows the results obtained with the printer in NORM mode to detail the keys pressed, then in MAN mode to demonstrate "cleaner" output.

```
             XEQ "AMORT"
AUTO: SF 02,THEN PRESS E
                   XEQ E
******
AM. SCHEDULE
------
I(%)= 0.00?
          11.00  XEQ b
I= 0.92 %
PV= 0.00?
        35000.00    RUN
PV= 35000.00
PMT= 0.00?
          343.04    RUN
PMT= -343.04
------

KEY K, R/S
            1.00    RUN
PAYMENT: 1
INT= -320.83
PRIN= -22.21
BAL= 34977.79

R/S OR K, R/S
           12.00    RUN
PMTS. 2--13
ΣINT= -3833.57
ΣPRIN= -282.91
BAL= 34694.88

R/S OR K, R/S
                    RUN
PMTS. 14--25
ΣINT= -3800.86
ΣPRIN= -315.62
BAL= 34379.26

R/S OR K, R/S
           11.00    RUN
PMTS. 26--36
ΣINT= -3452.12
ΣPRIN= -321.32
BAL= 34057.94

R/S OR K, R/S
```

```
******
AM. SCHEDULE
------
I= 0.92 %
PV= 35000.00
PMT= -343.04
------

PAYMENT: 1
INT= -320.83
PRIN= -22.21
BAL= 34977.79

PMTS. 2--13
ΣINT= -3833.57
ΣPRIN= -282.91
BAL= 34694.88

PMTS. 14--25
ΣINT= -3800.86
ΣPRIN= -315.62
BAL= 34379.26

PMTS. 26--36
ΣINT= -3452.12
ΣPRIN= -321.32
BAL= 34057.94
```

E. Program procedure and examples (continued)

III. Comments

1. If the output answers appear too fast for recording, they can be recalled from the assigned registers:

   R 01 = current grouped interest

   R 02 = current grouped principal reduction

   R 03 = current balance

   Alternatively, the R/S key can be pressed during any display, thus "freezing" the answer; pressing R/S again will re-start the program at the point it was interrupted.

2. As should be evident from the examples, input and output data make use of the conventional signs from the viewpoint of the borrower, that is, money received (the loan amount and the current balance) is positive, whereas money paid (amount of interest, payments, and principal reduction) is negative.

3. Contrary to the corresponding HP-11C and HP-12C programs, the AMORT program <u>does</u> check the input for proper sign, and if necessary changes the sign, thus ensuring sensible results.

F. Background and formulas used

(Same as the used in the corresponding HP-11C program, AMORT78-11)

G. Program listing and barcode for AMORT.


```
3:00PM 05/23
01♦LBL "AMORT"
02 CLRG
03 CF 02
04 "AUTO: SF 02,"
05 "⊢THEN PRESS E"
06 PROMPT
07♦LBL b
08 12
09 /
10 GTO B
11♦LBL E
12 FS? 02
13 XEQ 96
14 FS? 55
15 XEQ 99
16 "AM. SCHEDULE"
17 AVIEW
18 XEQ 98
19 CF 29
20 FIX 2
21 CF 22
22 0
23 STO 06
24 "I(%)= "
25 ARCL 04
26 "⊢?"
27 TONE 9
28 PROMPT
29♦LBL B
30 FS?C 22
31 STO 04
32 "I= "
33 ARCL 04
34 "⊢ %"
35 AVIEW
36 PSE
37 "PV= "
38 ARCL 03
39 "⊢?"
40 TONE 9
41 PROMPT
42♦LBL C
43 FS?C 22
44 STO 03
45 RCL 03
46 X<0?
47 CHS
48 STO 03
49 "PV= "
50 ARCL 03
51 AVIEW
52 PSE
53 "PMT= "
54 ARCL 05
55 "⊢?"
56 TONE 9
57 PROMPT
58♦LBL D
59 FS?C 22
60 STO 05
61 RCL 05
62 X>0?
63 CHS
64 STO 05
65 "PMT= "
66 ARCL 05
67 AVIEW
68 FS? 55
69 XEQ 98
70 ADV
71 PSE
72 "KEY K, R/S"
73 TONE 7
74 TONE 7
75 PROMPT
76♦LBL 03
77 FS?C 22
78 STO 00
79 RCL 00
80 STO 13
81 0
82 STO 01
83 STO 02
84♦LBL 01
85 RCL 03
86 CHS
87 RCL 04
88 %
89♦LBL 06
90 FIX 2
91 RND
92 ST+ 01
93 CHS
94 RCL 05
95 +
96 ST+ 03
97 ST+ 02
98 1
99 ST+ 06
100 DSE 13
101 GTO 01
102♦LBL 05
103 CF 03
104 FIX 0
105 1
106 RCL 00
107 X=Y?
108 SF 03
109 CHS
110 RCL 06
111 +
112 1
113 +
114 FS? 03
115 "PAYMENT: "
116 FC? 03
117 "PMTS. "
118 ARCL X
119 FS? 03
120 GTO 11
121 "⊢--"
122 ARCL 06
123♦LBL 11
124 AVIEW
125 PSE
126 FIX 2
127 "ΣINT= "
128 FS? 03
129 "INT= "
130 ARCL 01
131 AVIEW
132 FS? 55
133 GTO 12
134 PSE
135 PSE
136♦LBL 12
137 "ΣPRIN= "
138 FS? 03
139 "PRIN= "
140 ARCL 02
141 AVIEW
142 FS? 55
143 GTO 13
144 PSE
145 PSE
146♦LBL 13
147 "BAL= "
148 ARCL 03
149 AVIEW
150 ADV
151 FS? 55
152 GTO 14
153 PSE
154 PSE
155♦LBL 14
156 CF 22
157 RCL 00
158 FS?C 04
159 GTO 99
160 FS? 02
161 GTO 97
162 "R/S OR K, R/S"
163 TONE 0
164 TONE 0
165 PROMPT
166 GTO 03
167♦LBL 95
168 SF 04
169 RCL 09
170 RCL 06
171 -
172 STO 00
173 RTN
174♦LBL 96
175 "FINAL N?"
176 TONE 9
177 PROMPT
178 STO 09
179 RTN
180♦LBL 97
181 RCL 06
182 RCL 09
183 X=Y?
184 GTO 99
185 X<>Y
186 -
187 RCL 00
188 X>Y?
189 XEQ 95
190 GTO 03
191♦LBL 98
192 SF 12
193 "-------"
194 AVIEW
195 CF 12
196 RTN
197♦LBL 99
198 SF 12
199 "******"
200 AVIEW
201 CF 12
202 END
```

HP-41 Program Barcode: AMORT

Program Registers Required: 67

ROW 1: LINES 1 - 3

ROW 2: LINES 3 - 4

ROW 3: LINES 4 - 5

ROW 4: LINES 5 - 8

ROW 5: LINES 9 - 13

ROW 6: LINES 14 - 16

ROW 7: LINES 16 - 18

ROW 8: LINES 19 - 24

ROW 9: LINES 24 - 27

ROW 10: LINES 28 - 33

ROW 11: LINES 33 - 37

ROW 12: LINES 37 - 42

ROW 13: LINES 43 - 49

ROW 14: LINES 49 - 53

ROW 15: LINES 54 - 59

ROW 16: LINES 59 - 65

ROW 17: LINES 65 - 71

ROW 18: LINES 72 - 72

HP-41 Program Barcode: AMORT (continued)

ROW 19: LINES 73 - 80

ROW 20: LINES 81 - 90

ROW 21: LINES 91 - 98

ROW 22: LINES 99 - 104

ROW 23: LINES 105 - 114

ROW 24: LINES 114 - 115

ROW 25: LINES 116 - 118

ROW 26: LINES 119 - 123

ROW 27: LINES 124 - 127

ROW 28: LINES 128 - 131

ROW 29: LINES 132 - 137

ROW 30: LINES 137 - 139

ROW 31: LINES 139 - 145

ROW 32: LINES 146 - 150

ROW 33: LINES 151 - 158

ROW 34: LINES 158 - 162

ROW 35: LINES 162 - 162

ROW 36: LINES 162 - 168

HP-41 Program Barcode: AMORT (continued)

ROW 37: LINES 168 - 175

ROW 38: LINES 175 - 179

ROW 39: LINES 180 - 187

ROW 40: LINES 188 - 193

ROW 41: LINES 193 - 197

ROW 42: LINES 197 - 200

ROW 43: LINES 201 - 202

Program No. 41F044

Calculator: HP-41

Program Name: AM7841 (**AM**ortization schedules and similar financial calculations using the rule of **78**s with the HP-**41**)

Author: Thomas W. Beers

Date: June, 1986

Purpose: To provide the usual entries in an amortization schedule (payment number, interest paid, reduction in principal, and outstanding balance) assuming the input values are

k = the number of periods over which the amount of interest and the reduction in principal are to be summed (can be 1 for the complete schedule, month by month or year by year)

I = periodic interest rate as a percent

PV = initial value of the loan, the amount borrowed

PMT = periodic payment (input as a negative)

The program provides the option of calculating similar entries for a schedule which assumes the rule of 78s interest allocation procedure, in which case the required input values are PV, PMT, and k as defined above, and

n = the total number of periods (usually months) in the loan

and

INT = the total finance charge for the loan, i.e., the sum of all the periodic interest payments (input as a negative)

A printed schedule can be obtained when a printer (HP-82143A or HP-82162A) is connected, and a continuous printout is optionally available (by setting flag 02) if the size of the group (k) is uniform.

SPECIAL NOTE TO THE USER:

Since AM7841 is so similar to AMORT78-11, the directions of use and examples provided there can be followed. Also, the bulk of the register assignments and the various labels are very similar to that used in the AMORT program for the HP-41. For these reasons a detailed writeup was not considered necessary.

Program listings, printed results of the examples used in AMORT78-11, and program barcode are found on the following pages.

A. Printed results of examples using data from AMORT78-11

1. Example 11-1, first in NORM printer mode to show keystrokes, then in MAN printer mode for cleaner looking results.

```
            XEQ "AM7841"
AUTO: SF 02,PRESS E OR e
                   CF 02
                   XEQ E

******
AM. SCHEDULE
------
I(%)= 0.00?
            11.00  XEQ b
I= 0.92 %
PV= 0.00?
         35000.00  XEQ C
PV= 35000.00
PMT= 0.00?
           343.04  XEQ D
PMT= -343.04
------

KEY K, R/S
             1.00    RUN
PAYMENT: 1
INT= -320.83
PRIN= -22.21
BAL= 34977.79

R/S OR K, R/S
                     RUN
PAYMENT: 2
INT= -320.63
PRIN= -22.41
BAL= 34955.38

R/S OR K, R/S
                     RUN
PAYMENT: 3
INT= -320.42
PRIN= -22.62
BAL= 34932.76

R/S OR K, R/S
```

```
******
AM. SCHEDULE
------
I= 0.92 %
PV= 35000.00
PMT= -343.04
------

PAYMENT: 1
INT= -320.83
PRIN= -22.21
BAL= 34977.79

PAYMENT: 2
INT= -320.63
PRIN= -22.41
BAL= 34955.38

PAYMENT: 3
INT= -320.42
PRIN= -22.62
BAL= 34932.76
```

2. Example 11-2, 11-3, and 11-4 in MAN printer mode.

```
******
AM. SCHEDULE
------
I= 0.92 %
PV= 35000.00
PMT= -343.04
------

PAYMENT: 1
INT= -320.83
PRIN= -22.21
BAL= 34977.79

PMTS. 2--13
ΣINT= -3833.57
ΣPRIN= -282.91
BAL= 34694.88

PMTS. 14--25
ΣINT= -3800.86
ΣPRIN= -315.62
BAL= 34379.26

PMTS. 26--36
ΣINT= -3452.12
ΣPRIN= -321.32
BAL= 34057.94
```

```
******
RULE OF 78s
------
INT= -1000.00
PV= 5000.00
PMT= -500.00
------

PAYMENT: 1
INT= -153.85
PRIN= -346.15
BAL= 4653.85

PAYMENT: 2
INT= -141.03
PRIN= -358.97
BAL= 4294.88

PAYMENT: 3
INT= -128.21
PRIN= -371.79
BAL= 3923.09

PMTS. 4--9
ΣINT= -499.98
ΣPRIN= -2500.02
BAL= 1423.07
```

```
******
RULE OF 78s
------
INT= -7720.00
PV= 20000.00
PMT= -770.00
------

PMTS. 1--12
ΣINT= -4242.52
ΣPRIN= -4997.48
BAL= 15002.52

PMTS. 13--24
ΣINT= -2573.34
ΣPRIN= -6666.66
BAL= 8335.86

PMTS. 25--36
ΣINT= -904.14
ΣPRIN= -8335.86
BAL= 0.00
```

3. Example 11-4 using the automatic printout option (flag 02 set); results shown in both NORM and MAN printer modes.

```
            XEQ "AM7841"
AUTO: SF 02,PRESS E OR e
                   SF 02
                   XEQ e
FINAL N?
            36.00    RUN
******
RULE OF 78s
------
INT($)= 0.00?
          7720.00  XEQ B
INT= -7720.00
PV= 0.00?
         20000.00  XEQ C
PV= 20000.00
PMT= 0.00?
           770.00  XEQ D
PMT= -770.00
------

KEY N↑K, R/S
            36.00 ENTER↑
            12.00    RUN
PMTS. 1--12
ΣINT= -4242.52
ΣPRIN= -4997.48
BAL= 15002.52

PMTS. 13--24
ΣINT= -2573.34
ΣPRIN= -6666.66
BAL= 8335.86

PMTS. 25--36
ΣINT= -904.14
ΣPRIN= -8335.86
BAL= 0.00

******
```

```
******
RULE OF 78s
------
INT= -7720.00
PV= 20000.00
PMT= -770.00
------

PMTS. 1--12
ΣINT= -4242.52
ΣPRIN= -4997.48
BAL= 15002.52

PMTS. 13--24
ΣINT= -2573.34
ΣPRIN= -6666.66
BAL= 8335.86

PMTS. 25--36
ΣINT= -904.14
ΣPRIN= -8335.86
BAL= 0.00

******
```

B. Program listing and barcode for AM7841.


```
01♦LBL "AM7841"
02 CLRG
03 CF 02
04 "AUTO: SF 02,"
05 "⊦PRESS E OR e"
06 PROMPT
07♦LBL e
08 SF 00
09 FS? 02
10 XEQ 96
11 GTO 10
12♦LBL b
13 12
14 /
15 GTO B
16♦LBL E
17 CF 00
18 FS? 02
19 XEQ 96
20♦LBL 10
21 FS? 55
22 XEQ 99
23 "AM. SCHEDULE"
24 FS? 00
25 "RULE OF 78s"
26 AVIEW
27 XEQ 98
28 CF 29
29 FIX 2
30 CF 22
31 0
32 STO 06
33 "I(%)= "
34 FS? 00
35 "INT($)= "
36 ARCL 04
37 "⊦?"
38 TONE 9
39 PROMPT
40♦LBL B
41 FS?C 22
42 STO 04
43 "I= "
44 FS? 00
45 XEQ 20
46 ARCL 04
47 FC? 00
48 "⊦ %"
49 AVIEW
50 PSE
51 "PV= "
52 ARCL 03
53 "⊦?"
54 TONE 9
55 PROMPT
56♦LBL C
57 FS?C 22
58 STO 03
59 RCL 03
60 X<0?
61 CHS
62 STO 03
63 "PV= "
64 ARCL 03
65 AVIEW
66 PSE
67 "PMT= "
68 ARCL 05
69 "⊦?"
70 TONE 9
71 PROMPT
72♦LBL D
73 FS?C 22
74 STO 05
75 RCL 05
76 X>0?
77 CHS
78 STO 05
79 "PMT= "
80 ARCL 05
81 AVIEW
82 FS? 55
83 XEQ 98
84 ADV
85 PSE
86 "KEY K, R/S"
87 FS? 00
88 "KEY N↑K, R/S"
89 TONE 7
90 TONE 7
91 PROMPT
92 FS? 00
93 GTO A
94♦LBL 03
95 FS?C 22
96 STO 00
97 RCL 00
98 STO 13
99 0
100 STO 01
101 STO 02
102 FS? 00
103 GTO 04
104♦LBL 01
105 RCL 03
106 CHS
107 RCL 04
108 %
109♦LBL 06
110 FIX 2
111 RND
112 ST+ 01
113 CHS
114 RCL 05
115 +
116 ST+ 03
117 ST+ 02
118 1
119 ST+ 06
120 FS? 00
121 RTN
122 DSE 13
123 GTO 01
124♦LBL 05
125 CF 03
126 FIX 0
127 1
128 RCL 00
129 X=Y?
130 SF 03
131 CHS
132 RCL 06
133 +
134 1
135 +
136 FS? 03
137 "PAYMENT: "
138 FC? 03
139 "PMTS. "
140 ARCL X
141 FS? 03
142 GTO 11
143 "⊦--"
144 ARCL 06
145♦LBL 11
146 AVIEW
147 PSE
148 FIX 2
149 "ΣINT= "
150 FS? 03
```

B. Program listing and barcode for AM7841 (continued).


```
151 "INT= "
152 ARCL 01
153 AVIEW
154 FS? 55
155 GTO 12
156 PSE
157 PSE
158♦LBL 12
159 "ΣPRIN= "
160 FS? 03
161 "PRIN= "
162 ARCL 02
163 AVIEW
164 FS? 55
165 GTO 13
166 PSE
167 PSE
168♦LBL 13
169 "BAL= "
170 ARCL 03
171 AVIEW
172 ADV
173 FS? 55
174 GTO 14
175 PSE
176 PSE
177♦LBL 14
178 CF 22
179 RCL 00
180 FS? 02
181 GTO 97
182 "R/S OR K, R/S"
183 TONE 0
184 TONE 0
185 PROMPT
186 GTO 03
187♦LBL A
188 STO 00
189 STO 13
190 X<>Y
191 STO 07
192 1
193 +
194 RCL 07
195 *
196 2
197 /
198 STO 08
199 0
200 STO 01
201 STO 02
202♦LBL 04
203 RCL 07
204 X=0?
205 GTO 05
206 RCL 08
207 /
208 RCL 04
209 *
210 XEQ 06
211 ST- 07
212 DSE 13
213 GTO 04
214 GTO 05
215♦LBL 20
216 RCL 04
217 X>0?
218 CHS
219 STO 04
220 "INT= "
221 RTN
222♦LBL 96
223 "FINAL N?"
224 TONE 9
225 PROMPT
226 STO 09
227 RTN
228♦LBL 97
229 RCL 06
230 RCL 09
231 X<=Y?
232 GTO 99
233 GTO 03
234♦LBL 98
235 SF 12
236 "------"
237 AVIEW
238 CF 12
239 RTN
240♦LBL 99
241 SF 12
242 "******"
243 AVIEW
244 CF 12
245 END
```

HP-41 Program Barcode: AM7841

Program Registers Required: 82

ROW 1: LINES 1 - 2

ROW 2: LINES 3 - 4

ROW 3: LINES 4 - 5

ROW 4: LINES 5 - 8

ROW 5: LINES 8 - 13

ROW 6: LINES 13 - 18

ROW 7: LINES 19 - 23

ROW 8: LINES 23 - 23

ROW 9: LINES 24 - 25

ROW 10: LINES 25 - 29

ROW 11: LINES 30 - 33

ROW 12: LINES 34 - 35

ROW 13: LINES 36 - 41

ROW 14: LINES 41 - 45

ROW 15: LINES 46 - 51

ROW 16: LINES 51 - 54

ROW 17: LINES 55 - 63

ROW 18: LINES 63 - 67

HP-41 Program Barcode: AM7841 (continued)

ROW 19: LINES 67 - 71

ROW 20: LINES 72 - 79

ROW 21: LINES 79 - 83

ROW 22: LINES 83 - 86

ROW 23: LINES 86 - 88

ROW 24: LINES 88 - 90

ROW 25: LINES 91 - 97

ROW 26: LINES 98 - 106

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ROW 29: LINES 122 - 129

ROW 30: LINES 130 - 137

ROW 31: LINES 137 - 139

ROW 32: LINES 139 - 142

ROW 33: LINES 142 - 148

ROW 34: LINES 148 - 151

ROW 35: LINES 151 - 155

ROW 36: LINES 155 - 159

HP-41 Program Barcode: AM7841 (continued)

ROW 37: LINES 159 - 162

ROW 38: LINES 162 - 169

ROW 39: LINES 169 - 174

ROW 40: LINES 174 - 181

ROW 41: LINES 181 - 182

ROW 42: LINES 182 - 186

ROW 43: LINES 187 - 196

ROW 44: LINES 197 - 206

ROW 45: LINES 207 - 213

ROW 46: LINES 213 - 220

ROW 47: LINES 220 - 223

ROW 48: LINES 223 - 228

ROW 49: LINES 228 - 234

ROW 50: LINES 235 - 238

ROW 51: LINES 238 - 242

ROW 52: LINES 242 - 245

ANALOGOUS HP-41 PROGRAM DESCRIPTIONS

Prior to the existence of the HP-11C, programs were written for the HP-41 which address the subject matter covered by the first five 11C programs presented in this manual. HP-41 users who have a need for such programs may find the following descriptions useful. Copies of the programs as separates[1/] can be ordered as described on page 213.

A. HP-41 Analogies to the HP-11C program SLR:

1. LR (**L**inear **R**egression), Program No. 41F018.

Provides simple linear regression calculations which simulate the L.R., $\hat{y}$, and r keys found on the HP-34C, HP-11C and HP-15C. Specifically, one may readily calculate the following, for X and Y data summarized by the Σ+ key:

a, the Y-intercept

b, the slope of the least-squares fitted line

b', the slope assuming the line is forced through the origin

r, the simple correlation coefficient

r^2, the coefficient of determination

t_r, the calculated t to test the hypothesis of zero correlation

$\hat{Y}$, a predicted value of Y for any keyed-in X

Additional statistics such as confidence interval estimates and standard errors can be calculated using the program SLR (Program No. 41F019), but for brevity (96 program steps, 176 bytes, one magnetic card) only the listed items are included in "LR".

[1/] Programs LR, SLR, MSLR, LOGVOL, and SSRS are also included (without barcode) in the 1983 publication, "A Chronology of HP-41C Programs for Use and Example", by T.W. Beers.

2. SLR (**S**imple **L**inear **R**egression), Program No. 41F019.

Extends the simple linear regression calculations achieved by "LR" (Program No. 41F018) to provide, for ungrouped X and Y data summarized by the + key:

(1) a, b, b', r, r^2, t_r, and $\hat{Y}$ (see LR program);

(2) standard errors: s_{yx}, s_a, s_b, $s_{b'}$;

(3) confidence interval estimates assuming mean Y and assuming individual Y for given X_o;

and (4) Student's t to test the following hypotheses:

$$H_o: \rho = 0,\ H_o: \alpha = 0,\ H_o: \beta = 0,\ H_o: \beta' = 1.$$

Care was taken to make the program "printer compatible" and an automatic print-out mode (flag 00 set) is provided.

3. MSLR (**M**aster **S**imple **L**inear **R**egression), Program No. 41F020.

MSLR is a steering program to be used in conjunction with SLR (Program No. 41F019) to accommodate ungrouped, grouped, or weighted data and to extend the prompting, correction, and/or deletion capabilities of that program. With the insertion of appropriate subroutines it is anticipated that MSLR can also be used to transform the input data and achieve linear approximations to certain non-linear models.

Detailed examples of ungrouped and grouped cases are provided with printer output.

The deletion of data sets, purposefully or to correct for a keyboard error, is provided for and described in detail.

B. HP-41 Analogy to the HP-11C program LVC:

LOGVOL (**LOG** **VOL**umes), Program No. 41F008.

Written to calculate log volumes by Doyle, Scribner, Int. 1/4-inch, Int. 1/8-inch rules and cubic content by Huber, Smalian, and Newton formulas -- United States or Metric units.

Metric or U.S. units input and output are readily handled as is output conversion from one to the other.

C. <u>HP-41 Analogy to the HP-11C program PLS</u>:

PRABLA (obvious contraction of PARABOLA), Program No. 41F034.

Written to calculate the gross and corrected sums of squares (SS) and sums products (SP), regression coefficients, and other related statistics for the least squares fit to the "parabolic" model:

$Y = b_o + b_1X_1 + b_2X_2$, with the following optional definition of the Xs:

A. $X_2 = X_1^2$

B. X_1 and X_2 as non-functionally related variables

C. $X_2 = X_1^2$, and the values of X_1 are equally spaced between arbitrary lower and upper limits.

The program, by means of flags, can process data that are ungrouped, grouped (F1 set), or weighted (F1 + F2 set).

D. <u>HP-41 Analogy to the HP-11C program ABCD</u>:

COMPAS (obvious abbreviation for COMPASS), Program No. 41F038.

Written to achieve the conversion of angles measured from the true north meridian to angles measured from magnetic north and vice versa. Angles can be expressed as bearings (such as S 25 E) or in azimuth units from north (such as 125 azimuth degrees) and can be in degrees, minutes, seconds format (e.g., dd.mmss) or the decimal degrees format (e.g., dd.dddddd).

The program also has facility to convert from one format tothe other (such as from decimal to dms) and to convert bearings to azimuth angles and vice versa.

E. <u>HP-41 Analogy to the HP-11C program SIMSTRAT</u>:

SSRS (<u>S</u>imple and <u>S</u>tratified <u>R</u>andom <u>S</u>ampling), Program No. 41F017.

Written to summarize data from either a simple or stratified sample, obtaining within stratum and overall estimates of the mean and standard error and, optionally, user-specified confidence intervals and other sample statistics. Provision is made to use or not (flag 00 set) a finite population correction.

PUBLICATION AVAILABILITY INFORMATION

Copies of the following publications are available and can be ordered from the author as indicated below.

1. A Chronology of HP-41C Programs for Use and Example. 299 pages, including 29 statistical and forestry related programs (without barcode). Price: $33.00 plus $3.00 postage.

2. OS-41, An Operating System for the HP-41 and Peripherials. 472 pages, including over 70 programs (with barcode) especially useful to move programs, and either numeric or alphabetic data back and forth among main memory, extended memory, magnetic card, tape and disc storages, printers, and barcode. Price: $42.00 plus $3.00 postage.

3. Separates of the following HP-41 "analogous" programs:

   LR (Linear Regression) 7 pages, plus barcode.
   SLR (Simple Linear Regression) 16 pages, plus barcode.
   MSLR (Master Simple Linear Regression) 18 pages, plus barcode.
   LOGVOL (LOG VOLumes) 16 pages, plus barcode.
   PRABLA (Parabola) 17 pages, plus barcode.
   COMPAS (Compass) 24 pages, plus barcode.
   SSRS (Simple and Stratified Random Sampling) 13 pages, plus barcode.

   Price per separate (including program listings and barcode): $5.00 each, plus $3.00 postage and handling for up to 10 separates.

4. Additional copies of the present manual, Substantial Programs for the HP-11C and HP-15C. Price $17.00 plus $3.00 postage.

Copies of the above may be ordered from

Thomas W. Beers
1808 Summit Drive
West Lafayette, IN 47906

Please make checks or money order payable to Thomas W. Beers.

Indiana residents add 5% sales tax.

Overseas orders should double the postage charge.

ORDERS ORIGINATING OUTSIDE THE UNITED STATES MUST BE PREPAID IN U.S. DOLLARS AND BE DRAWN ON A UNITED STATES BANK.

COLOR KEY

Blue -- Start of HP-11C program descriptions

Gold -- HP-12C program

Green -- HP-41C programs

SLR

LVC

PLS

ABCD

SIMSTRAT

FICALC

AREA

MUGO

AMORT

FINC78

MAIN PROGRAMS